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Physics Unit 1 (Introduction to Physics)

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CHAPTER 1 Definition of Physics

- State a definition of physics.

This is one of several large parabolic antennas at the Goldstone complex of the worldwide Deep Space Network that Jet Propulsion Laboratory manages for NASA. It spans 112 feet from rim to rim and stands nine stories tall. NASA used it for years to communicate with ships and devices in solar system exploration missions such as the Mariner, Voyager, and Galileo. When newer antennas replaced this antenna in communications use, it was converted to a radio telescope.

Definition of Physics

What is **physics**? Here is a definition you might find in a dictionary:

> Physics is the science of matter and energy and of interactions between the two. Physics is grouped into traditional fields such as acoustics, optics, mechanics, thermodynamics, and electromagnetism, as well as in modern extensions including atomic and nuclear physics, cryogenics, solid-state physics, particle physics, and plasma physics.

This is an accurate definition, but it doesn't really mean much to a beginning physics student, especially if he/she doesn't know definitions for acoustics, optics, cryogenics, etc.

Physics is the branch of science that studies the physical world using objects as small as sub-atomic particles and as large as galaxies. It studies the natures of matter and energy and how they interact. Physicists are inquisitive people who want to know the causes of what they see. How does the moon move? Why does the moon move? Why do the stars shine? Why do your hands get warm when you rub them together? Physicists, like all scientists, hope to find explanations that describe more than one phenomenon and offer a better understanding of how the universe works. Perhaps physics should be defined as the search for understanding of how the universe works.

Summary

- Physics is the branch of science that studies matter and energy and how they interact.

Practice

Use this resource to answer the questions below.

http://www.youtube.com/watch?v=AhR0DKspPos

1. In this video the definition of "physics" is, The Study of _____________.

Review

1. Give a definition of physics.
2. Can you think of examples of where you might have observed physics in action in your home or in your environment?

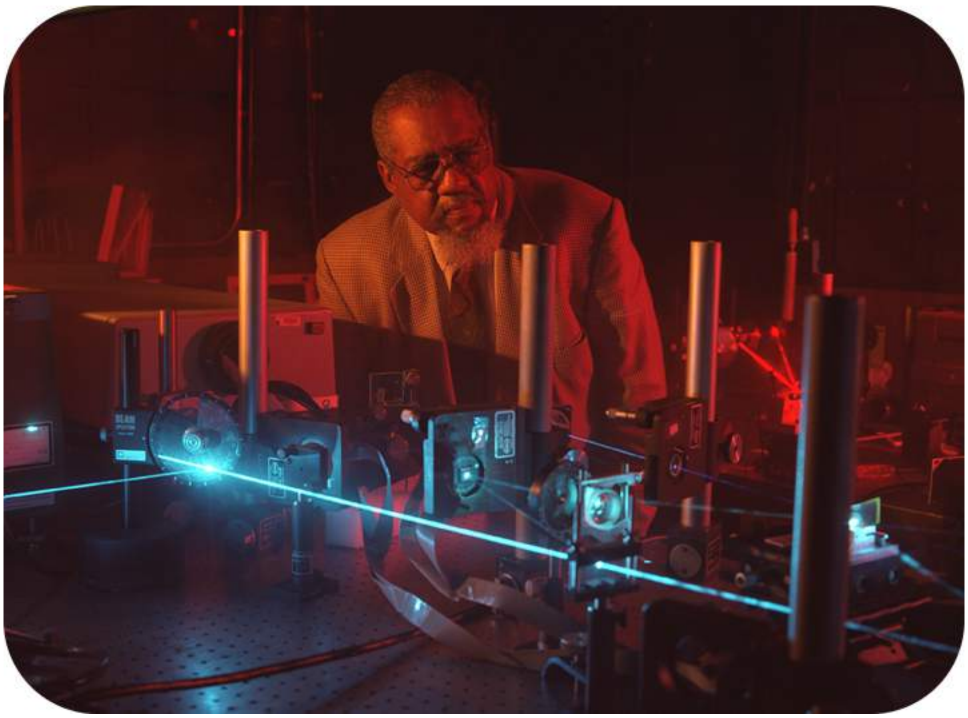
- **physics**: Physics is the science of matter and energy and of interactions between the two, grouped in traditional fields such as acoustics, optics, mechanics, thermodynamics, and electromagnetism, as well as in modern extensions including atomic and nuclear physics, cryogenics, solid-state physics, particle physics, and plasma physics.

References

1. Courtesy of NASA/JPL-Caltech. http://www.jpl.nasa.gov/news/news.php?feature=560. Public Domain

CHAPTER 2 Scientific Method

- State the steps in the scientific method.
- Define hypothesis.
- Define theory.
- Define law.



Don Frazier, a NASA chemist, conducting an experiment using a laser imaging system.

In science, we need to make observations on various phenomena in order to either form or test hypotheses. If we can find the phenomenon occurring in nature to observe, we are fortunate –but frequently, we must "arrange" for the phenomenon to occur at a time and in a place of convenience for us –convenient because it is a place where we have all of our measuring equipment. When we cause a phenomenon to occur in order to observe it, we call the activity an experiment. We use the experiment to help verify or falsify the validity of a hypothesis. Experiments vary greatly in their goal and scale, but always rely on repeatable procedure and logical analysis of the results.

The Scientific Method

The **scientific method** is the process by which science acquires new knowledge and thus increases our understanding of the universe. To understand why the method is so important, it is useful to consider the success or lack of success of methods used to acquire knowledge about the physical world *before* the development of the scientific method. One major method used was reliance on authority. This may have been the authority of a church, government, or particular individuals known to be very intelligent (such as many of the ancient Greek philosophers).

Prior to the invention of the scientific method, people based their actions on explanations of the physical world provided by authorities. When these activities failed because the explanations were incorrect, many people began seeking other explanations. An early case of reliance on authority proving incorrect was when Galileo's disproved

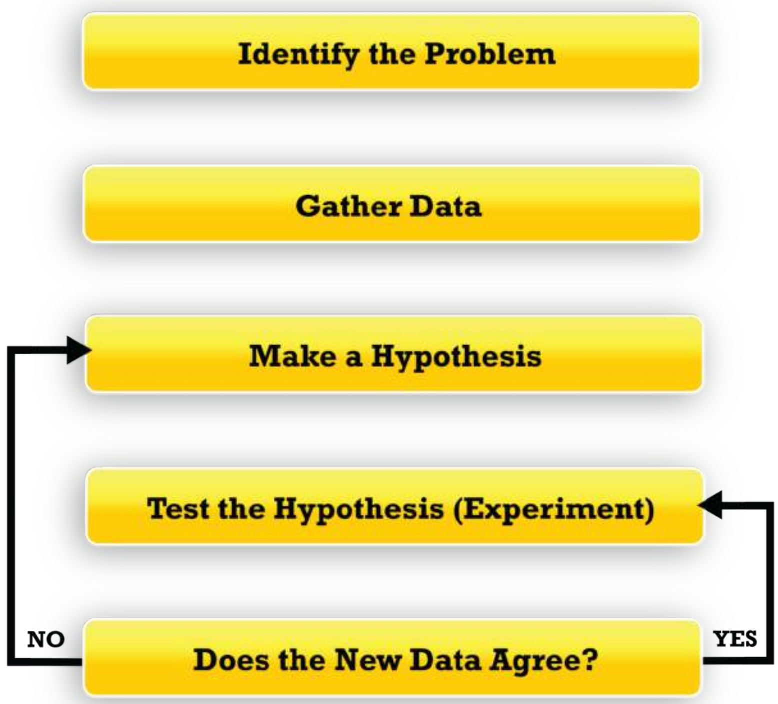
Aristotle's ideas about falling objects (as discussed in the introduction to this chapter). Reliance on authority did not produce a successful result, but observation of nature did.

In another failure of relying on authority, one particular church said that the earth was the center of the universe and did not move. It was also Galileo who proved this concept false, though it nearly cost him his life. Authorities were also incorrect when leaders claimed that smallpox could be avoided by making loud noises, such as with bells and cannons. Hundreds of thousands of people died of smallpox while the bells and cannons boomed away.

Scientists could see that authority, opinion, and superstition were keeping seekers from discovering how the physical world truly functioned. Over many years, a procedure was devised that produced greater success. Using the new method, astronomers could correctly predict where individual stars would be on a given night. The process of vaccination was discovered, saving millions of lives. The sciences of chemistry, physics, and biology began to move ahead. Using the scientific method, mankind has learned more about the physical world in the last 200 years than was learned in the previous 5000 years.

Once the **scientific method** was devised, the *observation* of nature was revealed the key to true understanding of nature. All theories must be consistent with observations of the phenomenon in question. Of course, it goes without saying that accurate observations must be recorded without fear or favor. Scientists cannot allow public opinion or governmental pressure to affect the recording of observations.

Even though some of the steps do not occur at all, and sometimes the steps occur in a different order, it is still worthwhile to list the steps in the scientific method.



Three very important points about the scientific method are:

1. Experimental data/results must be reproduced and verified by other scientists.
2. Theories must agree with all observations made on the phenomenon under study.
3. Theories are continually tested . . . forever.

The scientific method involves making observations on the phenomenon being studied, suggesting explanations for the observations, and testing the suggested explanations (also called **hypotheses**) by making new observations. Hypotheses are a sort of first guess in terms of explanations for observations.

After many **experiments** and tests in which results support the hypothesis, the hypothesis gradually becomes a **theory**. Theories remain theories forever and are constantly retested with every new observation. Theories never become "facts" or **laws**.

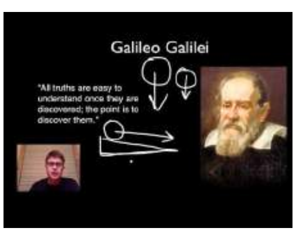
In science, a law is a mathematical relationship that is determined to exist between observations under a given set of conditions. The gas laws are excellent examples of scientific laws. The gas laws are mathematical relationships that exist between the pressure, volume, and absolute temperature of a gas under certain conditions. There is a very fundamental difference between observations of the physical world and explanations of the nature of the physical world. Hypotheses and theories are explanations, whereas laws and measurements are observational.

Summary

- Early methods of learning about the physical world, including reliance on authority, opinion, and superstition were unsuccessful.
- The scientific method was successful in helping humankind begin to effectively understand the physical world.
- The scientific method consists of making observations, suggesting a possible explanation (hypothesis), testing the hypothesis with new observations, making a new hypothesis if the new observations contradict the old hypothesis, or continuing to test the hypothesis if the observations agree.
- A hypothesis is a tentative explanation that can be tested by further observation.
- A theory is a hypothesis that has been supported with repeated testing.
- A scientific law is a statement that summarizes the results of many observations.
- Experimental data must be verified by reproduction from other scientists.
- Theories must agree with all observations made on the phenomenon under study.
- Theories are continually tested . . . forever.

Practice

Use this resource to answer the questions that follow.



MEDIA
Click image to the left for more content.

1. Which two Greek philosophers are mentioned in the video?
2. Which scientist disproved Aristotle's idea that heavy objects fall faster than lighter objects?

Review

1. A scientific investigation is not valid unless every step in the scientific method is present and carried out in the exact order listed in this lesson.
   (a) True
   (b) False
2. When a theory has been known for a long time, it becomes a law.

(a) True
(b) False

3. Which of the following is closest in meaning to the word "hypothesis"?
   (a) Fact
   (b) Law
   (c) Formula
   (d) Suggested explanation
   (e) Conclusion
4. Why do scientists sometimes discard theories?
   (a) The steps in the scientific method were not followed.
   (b) Public opinion disagrees with the theory.
   (c) The theory is opposed by the church.
   (d) Contradictory observations are found.
   (e) Congress voted against it.
5. If a hypothesis is rejected by the observations from an experiment, then the experiment
   (a) May have been a success.
   (b) Was a failure.
   (c) Must have been poorly designed.
   (d) Didn't follow the scientific method.

- **scientific method:** Consists of making observations on the phenomenon needing explanation, suggesting a possible explanation for the observations (hypothesis), testing the suggested explanation with new observations, make a new hypothesis if the new observations contradict the old hypothesis, or continue to test the hypothesis if the observations agree.
- **hypothesis:** A tentative explanation that can be tested by further observation.
- **theory:** A hypothesis that has been supported with repeated testing.
- **law:** A statement that summarizes, often as a mathematical relationship, the results of many observations.
- **experiment:** When we cause a phenomenon to occur in order to observe it, we call the activity an experiment. We use the experiment to help verify or falsify the validity of a hypothesis.

References

1. Courtesy of NASA. http://www.nasa.gov/centers/marshall/about/marshallfaces/frazier.html. Public Domain
2. CK-12 Foundation - Christopher Auyeung. . CC-BY-NC-SA 3.0

CHAPTER 3 Measurement

- Distinguish between qualitative and quantitative observations.
- Distinguish between fundamental and derived quantities.
- State the SI units for length, mass, and time.
- Make requested unit conversions in the SI system.

Measurements are a basic necessity in science. Scientists have designed thousands of meters and other measuring tools to help in the vital process of measuring. In this image of the control panel of the space shuttle Atlantis, we see dozens of readouts from measuring systems.

Measurement

Observation is an integral part of the scientific method. Hypotheses are accepted or rejected based on how well they explain observations. Some observations have numbers associated with them and some do not. An observation such as "the plant turned brown" is called a qualitative observation because it does not have any numbers associated with it. An observation such as "the object moved 200 meters" is called a quantitative observation because it contains a number. Quantitative observations are also called **measurements.** The numerical component of the observation is obtained by measurement, i.e. comparing the observation to some standard even if the comparison is an estimate. In terms of value to a scientist, all observations are useful but quantitative observations are much more useful. Whenever possible, you should make quantitative rather than qualitative observations, even if the measurement is an estimate.

Consider the following pair of observations.

1. When the volume of a gas is decreased, its pressure is increased.
2. When the volume of a gas is reduced from 2.0 liters to 1.0 liter, the pressure increases from 3.0 atm to 6.0 atm.

It should be easy to see that a great deal more information in available in the second observation.

Since accurate measurement if a vital tool in doing science, it becomes obvious that a consistent set of units for measurement is necessary. Physicists throughout the world use the **Internation System of Units** (also called the SI system). The SI system is basically the metric system which is a convenient system because units of different size are related by powers of 10. The system has physical standards for length, mass, and time. These are called **fundamental** units because they have an actual physical standard. There are two other fundamental units that you will earn later.

The standard SI unit for length is the meter. Originally, the definition of the meter was the distance between two scratches on a length of metal. The standard length of metal was stored in a secure vault under controlled conditions of temperature, pressure, and humidity. Most countries had their own copies of the standard meter and many copies were made for actual use. Later, the standard was redefined as one ten-millionth of the distance from the north pole to the equator measured along a line that passed through Lyons, France. In 1960, the standard was redefined again as a multiple of a wavelength of light emitted by krypton-86. In 1982, the standard was redefined yet again as the distance light travels in 1/299792458 second in a vacuum.

The standard unit of time, the second, was once defined as a fraction of the time it takes the earth to complete it orbit around the sun but has now been redefined in terms of the frequency of one type of radiation emitted by a cesium-133 atom.

The standard unit for mass is the kilogram. This standard is a mass of platinum-iridium metal cylinder kept near Paris, France. Other countries, of course, keep copies.

Units that are expressed using combinations of fundamental units are called **derived** units. For example, length is a fundamental unit measured in meters, time is a fundamental unit measured in seconds, and speed is a derived unit measured in meters/second.

As mentioned earlier, the SI system is a decimal system. Prefixes are used to change SI units by powers of ten. Thus, one hundredth of a meter is a centimeter and one thousandth of a gram is a milligram. The metric units for all quantities use the same prefixes. One thousand meters is a kilometer and one thousand grams is a kilogram. The common prefixes are shown in the **Table** 3.1.

TABLE 3.1: Prefixes Used with SI Units

Prefix	Symbol	Fractions	Example
pico	p	1×10^{-12}	picometer (pm)
nano	n	1×10^{-9}	nanometer (nm)
micro	μ	1×10^{-6}	micgrogram (μg)
milli	m	1×10^{-3}	milligram (mg)
centi	c	1×10^{-2}	centimeter (cm)
deci	d	1×10^{-1}	decimeter (dm)
		Multiples	
tera	T	1×10^{12}	terameter (Tm)
giga	G	1×10^{9}	gigameter (Gm)
mega	M	1×10^{6}	megagram (Mg)
kilo	k	1×10^{3}	kilogram (kg)
hecto	h	1×10^{2}	hectogram (hg)
deka	da	1×10^{1}	dekagram (dag)

Equivalent measurements with different units can be shown as equalities such as 1 meter = 100 centimeters. Each of the prefixes with each of the quantities has equivalency statements. For example, 1 gigameter = 1×10^9 meters and 1 kilogram = 1000 grams. These equivalencies are used as **conversion factors** when units need to be converted.

Example: Convert 500. millimeters to meters.

Solution:

The equivalency statement for millimeters and meters is 1000 mm = 1 m.

To convert 500. mm to m, we multiply 500. mm by a conversion factor that will cancel the millimeter units and generate the meter units. This requires that the conversion factor has meters in the numerator and millimeters in the denominator.

$$(500.\ \text{mm})\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right) = 0.500\ \text{m}$$

This conversion factor is constructed from the equivalency statement 1000 mm = 1 m.

Example: Convert 11 μg to mg.

Solution:

We need two equivalency statements because we need two conversion factors.

$1 \times 10^9\ \mu\text{g} = 1\ \text{g}$ and $1000\ \text{mg} = 1\ \text{g}$

$(11\ \mu\text{g})\left(\frac{1\ \text{g}}{1 \times 10^{-9}\ \mu\text{g}}\right)\left(\frac{1000\ \text{mg}}{1\ \text{g}}\right) = 1.1 \times 10^{-5}\ \text{mg}$

The first conversion factor converts from micrograms to grams and the second conversion factor converts from grams to milligrams.

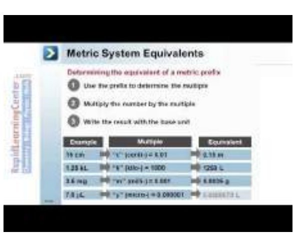
Summary

- Measurements (quantitative observations) are more useful than qualitative observations.
- You should make measurements, even estimated ones, whenever possible.
- The system of units for measurements in physics is the SI system.
- At this time, the fundamental quantities are length, mass, and time.
- The SI unit for length is the meter and the standard meter is the distance light travels in $\frac{1}{299792458}$ second in a vacuum.
- The SI unit for time is the second and the standard second is based on of the frequency of one type of radiation emitted by a cesium-133 atom.
- The SI unit for mass is the kilogram and is based on a mass stored in France.
- Prefixes are used to change SI units by powers of ten.
- Equivalencies are used as conversion factors when units need to be converted.

Practice

Use this resource to answer the following questions.

http://www.youtube.com/watch?v=mSxcLzQAbkI



MEDIA

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1. Name two areas of study within the field of physics.

2. What is the definition of a "base unit" in measurement systems?
3. Name one commonly used non-SI unit.

Review

1. Which of the following are quantitative observations?
   (a) The sky is blue.
   (b) The toy car is about 3 inches long.
   (c) It is 250,000 miles from the earth to the moon.
   (d) The wooden cart has a mass of 18.654 g.
   (e) When at rest, the pendulum points toward the center of the earth.
2. Convert 76.2 kilometers to meters.
3. Convert 76.2 picograms to kilograms.
4. Convert 1 day into seconds.

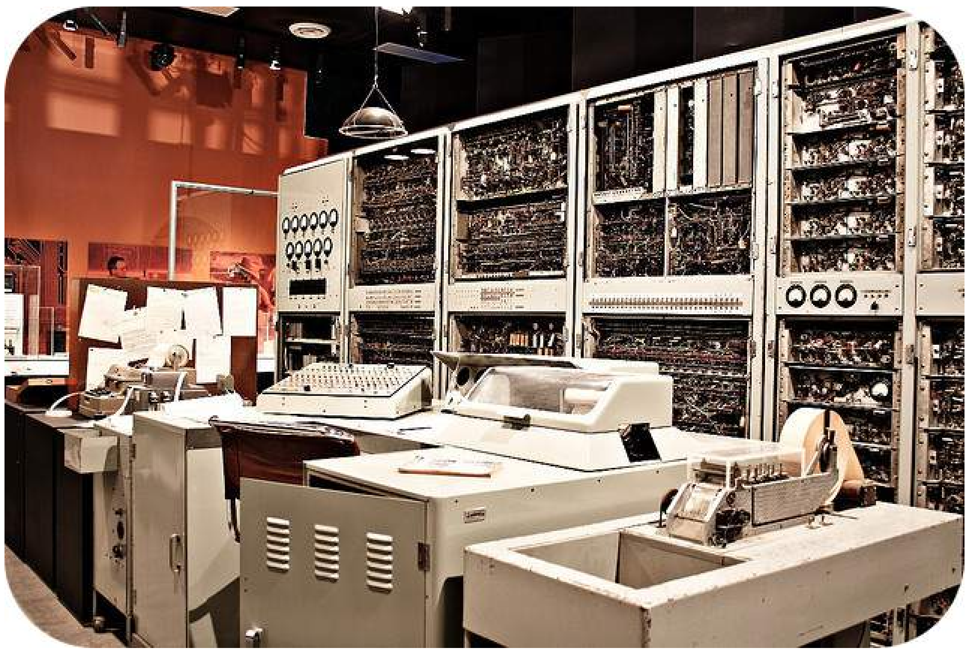
- **measurement:** The process or the result of determining the ratio of a physical quantity, such as a length, time, temperature etc., to a unit of measurement, such as the meter, second or degree Celsius.
- **the SI system of units:** A complete metric system of units of measurement for scientists; fundamental quantities are length (meter) and mass (kilogram) and time (second) and electric current (ampere) and temperature (kelvin) and amount of matter (mole) and luminous intensity (candela).
- **fundamental quantity vs derived quantity:** In the language of measurement, *quantities* are quantifiable aspects of the world, such as time, distance, velocity, mass, temperature, energy, and weight, and *units* are used to describe their measure. Many of these quantities are related to each other by various physical laws, and as a result the units of some of the quantities can be expressed as products (or ratios) of powers of other units (e.g., momentum is mass times velocity and velocity is measured in distance divided by time). Those quantities whose units are expressed in terms of other units are regarded as derived quantities. Those that cannot be so expressed in terms of other units are regarded as "fundamental" quantities.
- **conversion factor:** A numerical factor used to multiply or divide a quantity when converting from one system of units to another.

References

1. Courtesy of NASA. http://spaceflight.nasa.gov/gallery/images/shuttle/sts-101/html/jsc2000e10522.html. Public Domain
2. CK-12 Foundation - Richard Parsons. . CC-BY-NC-SA 3.0

CHAPTER 4 Mathematics Tools for Physics

- Convert given numbers from expanded form to exponential form and vice versa.
- Carry out mathematical operations with exponential numbers.
- Identify the number of significant figures in given measurements.
- Carry proper significant figures through math operations.



Scientists use computers to aid with all the mathematics needed to do science. Computers take and store measurement readings and do hours of mathematical calculations. The image is of the CSIRAC (Council for Scientific and Industrial Research Automatic Computer), Australia's first digital computer, first run in November 1949. This room-filling computer was able to do less than the personal computers of today.

Mathematics Tool Kit

What is the place of mathematics in physics?

It has been said by beginning physics students that physics is just another math class. Well, there is a lot of math in physics but none of the concepts and theories of physics can be derived strictly from mathematics. If you wish to build a large complex mechanical structure, it is pointless to begin without wrenches and screwdrivers. To build mechanical structures, you must have tools. One of the primary tools for working in physics is mathematics. At the high school level, physics requires algebra, geometry, and trigonometry. If you go to higher-level physics, calculus is also required. Calculus was, in fact, invented by Isaac Newton specifically for doing physics. If you study higher mathematics, you will find, when you look at the practice problems at the end of the chapter of the math book, you are looking at physics problems.

Exponential Notation

In the definition of physics, it was noted that physics deals with objects as small as sub-atomic particles and as large as galaxies. It should be clear that physicists deal with extremely small numbers like the mass of a lead atom, 0.00000000000000000000034 gram, and extremely large numbers like the distance from our galaxy to the Andromeda galaxy, 2.5 million light years, which is 25,000,000,000,000,000,000 kilometers.

These numbers are difficult to write and even more difficult when calculations must be done. It is much more convenient to write and calculate with such extreme numbers if they are written in exponential form. In exponential form, the mass of a lead atom is 3.4×10^{-34} g, and the distance from our galaxy to the Andromeda galaxy is 2.5×10^{19} km.

A number is expressed in exponential form by moving the decimal so that exactly one non-zero digit is on the left of the decimal and the exponent of 10 will be the number of places the decimal was moved. If the decimal is moved to the left, the exponent is positive and if the decimal was moved to the right, the exponent is negative. All **significant figures** are maintained in **exponential notation**.

Example: Express 13,700,000,000 in exponential form.

Solution: Since the decimal will be moved to the left 10 places, the exponent will be 10. So, the correct exponential form is 1.37×10^{10}.

Example: Express 0.000000000000000074 in exponential form.

Solution: Since the decimal will be moved to the right 17 places, the exponent will be -17. So the correct exponential form is 7.4×10^{-17}.

Example: Express the number 8.43×10^5 in expanded form.

Solution: 10^5 is 100,000 so 8.43×10^5 is $8.43 \times 100{,}000$ or 843,000.

Operations with Exponential Numbers

In order to add or subtract exponential numbers, the exponents must be the same. If the exponents are not the same, one of the numbers must be changed so that the exponents are the same. Once the exponents are the same, the numbers are added and the same exponents are carried.

Example: Add 5.0×10^5 and 4.0×10^4.

Solution: In order to add these numbers, we can change 4.0×10^4 to 0.40×10^5 and then add 0.40×10^5 to 5.0×10^5 which yields 5.4×10^5.

When you multiply exponential numbers, the numbers multiply and the exponents add.

Example: Multiply 5.0×10^5 and 4.0×10^4.

Solution: $(5.0 \times 10^5)(4.0 \times 10^4) = (5.0)(4.0) \times 10^{5+4} = 20 \times 10^9 = 2 \times 10^{10}$

Example: Multiply 6.0×10^3 and 2.0×10^{-5}.

Solution: $(6.0 \times 10^3)(2.0 \times 10^{-5}) = 12 \times 10^{3-5} = 12 \times 10^{-2} = 1.2 \times 10^{-1} = 0.12$

When you divide exponential numbers, the numbers are divided and the exponent of the divisor is subtracted from the exponent of the dividend.

Example: Divide 6.0×10^3 by 2.0×10^{-5}.

Solution: $\frac{6.0 \times 10^3}{2.0 \times 10^{-5}} = 3.0 \times 10^{3-(-5)} = 3.0 \times 10^8$

Significant Figures

The numbers you use in math class are considered to be exact numbers. When you are given the number 2 in a math problem, it does not mean 1.999 rounded up to 2, nor does it mean 2.00001 rounded down to 2. In math class, the number 2 means exactly 2.000000... with an infinite number of zeros –a perfect 2! Such numbers are produced only by definition, *not* by measurement. We can define 1 foot to contain exactly 12 inches with both numbers being perfect numbers, but we cannot measure an object to be exactly 12 inches long. In the case of measurements, we can only read our measuring instruments to a limited number of subdivisions. We are limited by our ability to see smaller and smaller subdivisions, and we are limited by our ability to construct smaller and smaller subdivisions on our measuring devices. Even with the use of powerful microscopes to construct and read our measuring devices, we eventually reach a limit. Therefore, although the actual measurement of an object may be a perfect 12 inches, we cannot prove it to be so. Measurements do not produce perfect numbers; the only perfect numbers in science are defined numbers, such as conversion factors.

It is very important to recognize and report the limitations of a measurement along with the magnitude and unit of the measurement. Many times, the measurements made in an experiment are analyzed for regularities. If the numbers reported show the limits of the measurements, the regularity, or lack thereof, becomes visible.

Consider the following table of the pressures and volumes of a gas sample and the calculated PV product.

TABLE 4.1:

Pressure	Volume	Pressure $\times$ Volume (P $\times$ V)
4.01 atm	6.03 L	24.1803 L-atm
3.02 atm	7.99 L	24.1298 L-atm
6.04 atm	3.98 L	24.0392 L-atm
11.98 atm	1.99 L	23.8402 L-atm

Now look at this same set of data when we are told that all the measurements have only two **significant figures** and all the numbers must be rounded to two places.

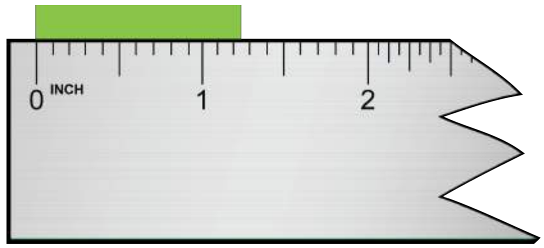
TABLE 4.2:

Pressure	Volume	Pressure $\times$ Volume (P $\times$ V)
4.0 atm	6.0 L	24 L-atm
3.0 atm	8.0 L	24 L-atm
6.0 atm	4.0 L	24 L-atm
12 atm	2.0 L	24 L-atm

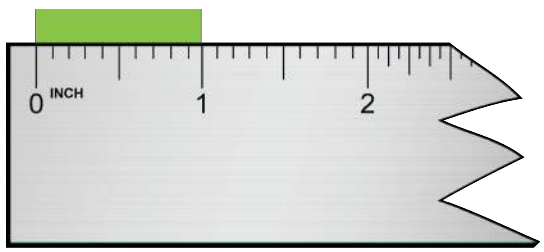
When the numbers are expressed with proper number of significant figures, a regularity appears that was not apparent before.

Rules for Determining Significant Figures

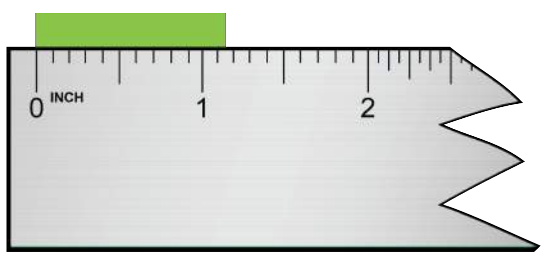
Significant figures are all of the digits that can be known with certainty in a measurement plus an estimated last digit. Significant figures provide a system to keep track of the limits of the original measurement. To record a measurement, you must write down all the digits actually measured, including measurements of zero, and you must *not* write down any digit not measured. The only real difficulty with this system is that zeros are sometimes used as measured digits, while other times they are used to locate the decimal point.



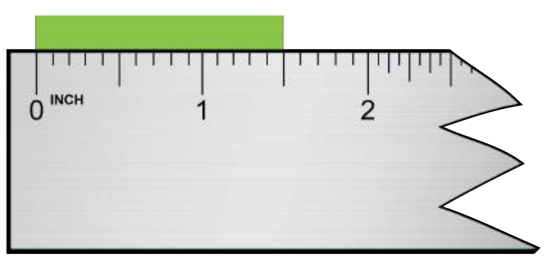
In the sketch shown above, the correct measurement is greater than 1.2 inches but less than 1.3 inches. It is proper to estimate one place beyond the calibrations of the measuring instrument. This ruler is calibrated to 0.1 inches, so we can estimate the hundredths place. This reading should be reported as 1.25 or 1.26 inches.



In this second case (sketch above), it is apparent that the object is, as nearly as we can read, 1 inch. Since we know the tenths place is zero and can estimate the hundredths place to be zero, the measurement should be reported as 1.00 inch. It is vital that you include the zeros in your reported measurement because these are measured places and are significant figures.



This measurement is read as 1.15 inches, 1.16 inches, or perhaps even 1.17 inches.



This measurement is read as 1.50 inches.

In all of these examples, the measurements indicate that the measuring instrument had subdivisions of a tenth of an inch and that the hundredths place is estimated. There is some uncertainty about the last, and only the last, digit.

In our system of writing measurements to show significant figures, we must distinguish between measured zeros and place-holding zeros. Here are the rules for determining the number of significant figures in a measurement.

Rules for Determining the Number of Significant Figures:

1. All non-zero digits are significant.
2. All zeros between non-zero digits are significant.
3. All beginning zeros are *not* significant.
4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal (the decimal point is not written in).

Examples of the Significant Figure Rules:

1. All non-zero digits are significant.

    543 has 3 significant figures.

    22.437 has 5 significant figures.

    1.321754 has 7 significant figures.

2. All zeros between non-zero digits are significant.

    7,004 has 4 significant figures.

    10.3002 has 6 significant figures.

    103 has 3 significant figures.

3. All beginning zeros are *not* significant.

    0.00000075 has 2 significant figures.

    0.02 has 1 significant figure.

    0.003003 has 4 significant figures.

4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal.

    37.300 has 5 significant figures.

    33.00000 has 7 significant figures.

    100. has 3 significant figures.

    100 has 1 significant figure.

    302,000 has 3 significant figures.

    1,050 has 3 significant figures.

Significant Figures in Addition and Subtraction

The answer to an addition or subtraction operation must not have any digits further to the right than the shortest addend. In other words, the answer should have as many decimal places as the addend with the smallest number of decimal places.

Example:

$$\begin{array}{r} 13.3843\ \mathbf{cm} \\ 1.012\ \mathbf{cm} \\ +\ 3.22\ \mathbf{cm} \\ \hline 17.6163\ \mathbf{cm} = 17.62\ \mathbf{cm} \end{array}$$

Notice that the top addend has a 3 in the last column on the right, but neither of the other two addends have a number in that column. In elementary math classes, you were taught that these blank spaces can be filled in with zeros and the answer would be 17.6163 cm. In the sciences, however, these blank spaces are unknown numbers, *not* zeros. Since they are unknown numbers, you cannot substitute any numbers into the blank spaces. As a result, you cannot know the sum of adding (or subtracting) any column of numbers that contain an unknown number. When you add the columns of numbers in the example above, you can only be certain of the sums for the columns with known numbers in each space in the column. In science, the process is to add the numbers in the normal mathematical process and then round off all columns that contain an unknown number (a blank space). Therefore, the correct answer for the example above is 17.62 cm and has only four significant figures.

Multiplication and Division

The answer for a multiplication or division operation must have the same number of significant figures as the factor with the least number of significant figures.

Example: $(3.556\text{ cm})(2.4\text{ cm}) = 8.5344\text{ cm}^2 = 8.5\text{ cm}^2$

The factor 3.556 cm has four significant figures, and the factor 2.4 cm has two significant figures. Therefore the answer must have two significant figures. The mathematical answer of 8.5344 cm^2 must be rounded back to 8.5 cm^2 in order for the answer to have two significant figures.

Example: $(20.0\text{ cm})(5.0000\text{ cm}) = 100.00000\text{ cm}^2 = 100.\text{ cm}^2$

The factor 20.0 cm has three significant figures, and the factor 5.0000 cm has five significant figures. The answer must be rounded to three significant figures. Therefore, the decimal must be written in to show that the two ending zeros are significant. If the decimal is omitted (left as an understood decimal), the two zeros will not be significant and the answer will be wrong.

Example: $(5.444\text{ cm})(22\text{ cm}) = 119.768\text{ cm}^2 = 120\text{ cm}^2$

In this case, the answer must be rounded back to two significant figures. We cannot have a decimal after the zero in 120 cm^2 because that would indicate the zero is significant, whereas this answer must have exactly two significant figures.

Summary

- Mathematics is a major tool for doing physics.
- The very large and very small measurements in physics make it useful to express numbers in exponential notation.
- It is also necessary in physics to do calculations with exponential numbers.
- There is uncertainty in all measurements.
- The use of significant figures is one way to keep track of uncertainty.
- Measurements must be written is the proper number of significant figures and the results of calculations must show the proper number of significant figures.
- Rules for Determining the Number of Significant Figures:

- All non-zero digits are significant.
- All zeros between non-zero digits are significant.
- All beginning zeros are *not* significant.
- Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal (the decimal point is not written in).

Practice

Use these resources to answer the questions that follow.

The following url is for a video that teaches the concepts of exponential notation: http://www.youtube.com/watch?v=ACZJMjt6qFk.

The following url is for a video that teaches the concepts of significant figures: http://www.youtube.com/watch?v=_jZAe9xeTFA.

1. Why do we use scientific notation?
2. What is the purpose of significant figures?
3. Are leading zeros significant or not significant?"

Review

1. Write the following numbers in proper exponential form.
   (a) 3,120
   (b) 0.00000341
2. Write the following numbers in expanded form.
   (a) 4.35×10^6
   (b) 6.1×10^{-4}
3. How many significant figures are in the following numbers?
   (a) 2.3
   (b) 17.95
   (c) 9.89×10^3
   (d) 170
   (e) 1.02
4. Perform the following calculations and give your answer with the correct number of significant figures:
   (a) $10.5 + 11.62$
   (b) $0.01223 + 1.01$
   (c) $19.85 - 0.0113$
5. Perform the following calculations and give your answer with the correct number of significant figures:
   (a) 0.1886×12
   (b) $2.995 \div 0.16685$
   (c) 910×0.18945

- **exponential notation**: A method of writing or displaying numbers in terms of a decimal number between 1 and 10 multiplied by a power of 10. The scientific notation of 10,492, for example, is 1.0492×10^4.
- **significant digits**: The significant figures (also known as significant digits, and often shortened to sig figs) of a number are those digits that carry meaning contributing to its precision. This includes all digits *except*:
  - leading and trailing zeros which are merely placeholders to indicate the scale of the number.

– spurious digits introduced, for example, by calculations carried out to greater precision than that of the original data, or measurements reported to a greater precision than the equipment supports.

References

1. Serg C (Flickr: keko.). http://www.flickr.com/photos/k3k0/3975819685/. CC-BY 2.0
2. CK-12 Foundation - Christopher Auyeung. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Christopher Auyeung. . CC-BY-NC-SA 3.0
4. CK-12 Foundation - Christopher Auyeung. . CC-BY-NC-SA 3.0
5. CK-12 Foundation - Christopher Auyeung. . CC-BY-NC-SA 3.0

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Physics Unit 2 (One Dimensional Motion)

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CHAPTER 1 Motion

- Define motion.
- Explain how frame of reference is related to motion.

The wings of this hummingbird are moving so fast that they're just a blur of motion. You can probably think of many other examples of things in motion. If you can't, just look around you. It's likely that you'll see something moving, and if nothing else, your eyes will be moving. So you know from experience what motion is. No doubt it seems like a fairly simple concept. However, when you read this article, you'll find out that it's not quite as simple as it seems.

Defining Motion

In science, **motion** is defined as a change in position. An object's position is its location. Besides the wings of the hummingbird in opening image, you can see other examples of motion in the **Figure** 1.1. In each case, the position of something is changing.

Q: In each picture in the **Figure** 1.1, what is moving and how is its position changing?

A: The train and all its passengers are speeding straight down a track to the next station. The man and his bike are racing along a curving highway. The geese are flying over their wetland environment. The meteor is shooting through the atmosphere toward Earth, burning up as it goes.

Frame of Reference

There's more to motion than objects simply changing position. You'll see why when you consider the following example. Assume that the school bus pictured in the **Figure** 1.2 passes by you as you stand on the sidewalk. It's obvious to you that the bus is moving, but what about to the children inside the bus? The bus isn't moving relative to them, and if they look at the other children sitting on the bus, they won't appear to be moving either. If the ride

FIGURE 1.1

FIGURE 1.2

is really smooth, the children may only be able to tell that the bus is moving by looking out the window and seeing you and the trees whizzing by.

This example shows that how we perceive motion depends on our frame of reference. **Frame of reference** refers to something that is not moving with respect to an observer that can be used to detect motion. For the children on the bus, if they use other children riding the bus as their frame of reference, they do not appear to be moving. But if they use objects outside the bus as their frame of reference, they can tell they are moving. The video at the URL below illustrates other examples of how frame of reference is related to motion.

http://www.youtube.com/watch?v=7FYBG5GSklU

MEDIA

Click image to the left for more content.

Q: What is your frame of reference if you are standing on the sidewalk and see the bus go by? How can you tell that the bus is moving?

A: Your frame of reference might be the trees and other stationary objects across the street. As the bus goes by, it momentarily blocks your view of these objects, and this helps you detect the bus' motion.

Summary

- Motion is defined as a change of position.
- How we perceive motion depends on our frame of reference. Frame of reference refers to something that is not moving with respect to an observer that can be used to detect motion.

Vocabulary

- **frame of reference**: Something that is not moving with respect to an observer that can be used to detect motion.
- **motion**: Change in position.

Practice

Do the frame of reference activity at the following URL. Watch the introduction and then do the nine trials. Repeat any trial you answer incorrectly until you get the correct answer.

http://www.amnh.org/learn/pd/physical_science/week2/frame_reference.html

Review

1. How is motion defined in science?
2. Describe an original example that shows how frame of reference influences the perception of motion.

References

1. Train: John H. Gray; Bike: Flickr:DieselDemon; Geese: Don McCullough; Meteor: Ed Sweeney (Flickr:Navicore). . CC BY 2.0
2. Bus: Flickr:torbakhopper; Children: Flickr:woodleywonderworks. . CC BY 2.0

CHAPTER 2 Position and Displacement

- Define and give an example of a frame of reference.
- Describe the difference between distance and displacement.
- Identify the position, distance, and displacements in various descriptions of motions.

In stockcar races, the winners frequently travel a distance of 500 miles but at the end of the race, their displacement is only a few feet from where they began.

Position, Distance, and Displacement

In order to study how something moves, we must know where it is. For straight line motion, it is easy to visualize the object on a number line. The object may be placed at any point on the number line either in the positive numbers or the negative numbers. It is common to choose the original position of the object to be on the zero mark. In making the zero mark the reference point, you have chosen a frame of reference. The position of an object is the separation between the object and the reference point.

When an object moves, we often refer to the amount it moves as the **distance**. Distance does not need a reference point and does not need a direction. If an automobile moves 50 kilometers, the distance traveled is 50 kilometers regardless of the starting point or the direction of movement. If we wish to find the final position of the automobile, however, just having the distance traveled will not allow us to determine the final position. In order to find the final position of the object, we need to know the starting point and the direction of the motion. The change in the position of the object is called its **displacement**. The displacement must include a direction because the final position may be either in the positive or negative direction along the number line from the initial position. The displacement is a vector quantity and vectors are discussed in another section.

Summary

- The length traveled by an object moving in any direction or even changing direction is called distance.

- The location of an object in a frame of reference is called position.
- For straight line motion, positions can be shown using a number line.
- The separation between original and final position is called displacement.

Practice

The following url is for a discussion of the difference between distance and displacement.

http://www.tutorvista.com/content/physics/physics-i/motion/distance-and-displacement.php

Use this resource to answer the questions that follow.

MEDIA

Click image to the left for more content.

1. What is the vector equivalent of the scalar "distance"?
2. What is the vector equivalent of the scalar "speed"?

Review

1. Explain the difference between distance and displacement in your own words.
2. Suppose that John lives on a square block that is 180 yards per side, and in the evenings, he walks with his dog around the block after dinner for a little exercise.
   (a) If John walks once around the block, what distance does he travel?
   (b) If John walks around the block, what is his displacement at the end?
3. Joanna's house is 8000 feet due west of her school. If her house is assigned the position of zero and her school is assigned the possition of +8000, what would Joanna's position be if she walked 100 feet west of her house?

- **distance:** The space between two objects but this is not adequate when considering the *distance travelled*. The distance travelled cannot be negative and can never get smaller –in this sense, distance is the total length of path traversed by the moving body irrespective of direction.
- **displacement:** The vector from the initial position to a subsequent position assumed by a body.

References

1. Image copyright Action Sports Photography, 2013. http://www.shutterstock.com. Used under license from Shutterstock.com

CHAPTER 3 Velocity

- Distinguish between velocity and speed.
- Represent velocity with vector arrows.
- Describe objects that have different velocities.
- Show how to calculate average velocity when direction is constant.

Ramey and her mom were driving down this highway at 45 miles per hour, which is the speed limit on this road. As they approached this sign, Ramey's mom put on the brakes and started to slow down so she could safely maneuver the upcoming curves in the road. This speed limit sign actually represents two components of motion: speed and direction.

Speed and Direction

Speed tells you only how fast or slow an object is moving. It doesn't tell you the direction the object is moving. The measure of both speed and direction is called **velocity**. Velocity is a vector. A **vector** is measurement that includes both size and direction. Vectors are often represented by arrows. When using an arrow to represent velocity, the length of the arrow stands for speed, and the way the arrow points indicates the direction. If you're still not sure of the difference between speed and velocity, watch the cartoon at this URL:

http://www.youtube.com/watch?v=mDcaeO0WxBI

MEDIA

Click image to the left for more content.

Using Vector Arrows to Represent Velocity

The arrows in the **Figure** 3.1 represent the velocity of three different objects. Arrows A and B are the same length but point in different directions. They represent objects moving at the same speed but in different directions. Arrow C is shorter than arrow A or B but points in the same direction as arrow A. It represents an object moving at a slower speed than A or B but in the same direction as A.

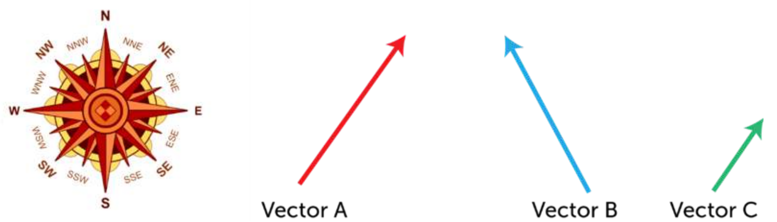


FIGURE 3.1

Differences in Velocity

Objects have the same velocity only if they are moving at the same speed and in the same direction. Objects moving at different speeds, in different directions, or both have different velocities. Look again at arrows A and B from the **Figure** 3.1. They represent objects that have different velocities only because they are moving in different directions. A and C represent objects that have different velocities only because they are moving at different speeds. Objects represented by B and C have different velocities because they are moving in different directions and at different speeds.

Q: Jerod is riding his bike at a constant speed. As he rides down his street he is moving from east to west. At the end of the block, he turns right and starts moving from south to north, but he's still traveling at the same speed. Has his velocity changed?

A: Although Jerod's speed hasn't changed, his velocity has changed because he is moving in a different direction.

Q: How could you use vector arrows to represent Jerod's velocity and how it changes?

A: The arrows might look like this (see **Figure** 3.2):

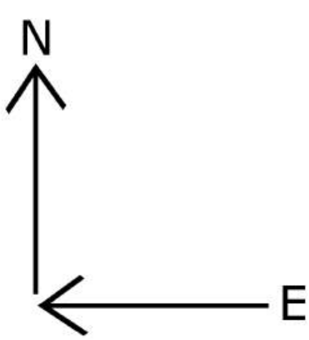


FIGURE 3.2

Calculating Average Velocity

You can calculate the average velocity of a moving object that is not changing direction by dividing the distance the object travels by the time it takes to travel that distance. You would use this formula:

$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

This is the same formula that is used for calculating average speed. It represents velocity only if the answer also includes the direction that the object is traveling.

Let's work through a sample problem. Toni's dog is racing down the sidewalk toward the east. The dog travels 36 meters in 18 seconds before it stops running. The velocity of the dog is:

$$\begin{aligned}\text{velocity} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{36 \text{ m}}{18 \text{ s}}\\ &= 2 \text{ m/s east}\end{aligned}$$

Note that the answer is given in the SI unit for velocity, which is m/s, and it includes the direction that the dog is traveling.

Q: What would the dog's velocity be if it ran the same distance in the opposite direction but covered the distance in 24 seconds?

A: In this case, the velocity would be:

$$\begin{aligned}\text{velocity} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{36 \text{ m}}{24 \text{ s}}\\ &= 1.5 \text{ m/s west}\end{aligned}$$

Summary

- Velocity is a measure of both speed and direction of motion. Velocity is a vector, which is a measurement that includes both size and direction.
- Velocity can be represented by an arrow, with the length of the arrow representing speed and the way the arrow points representing direction.
- Objects have the same velocity only if they are moving at the same speed and in the same direction. Objects moving at different speeds, in different directions, or both have different velocities.
- The average velocity of an object moving in a constant direction is calculated with the formula: velocity $= \frac{\text{distance}}{\text{time}}$. The SI unit for velocity is m/s, plus the direction the object is traveling.

Vocabulary

- **vector**: Measure such as velocity that includes both size and direction; may be represented by an arrow.
- **velocity**: Measure of both speed and direction of motion.

Practice

At the following URL, review how to calculate speed and velocity, and work through the sample problems. Then solve the 10 practice problems. http://www2.franciscan.edu/academic/mathsci/mathscienceintegation/MathScienceIntegation-827.htm

Review

1. What is velocity?
2. How does velocity differ from speed? Why is velocity a vector?
3. Explain how an arrow can be used to represent velocity.
4. Use vector arrows to represent the velocity of a car that travels north at 50 mi/h and then travels east at 25 mi/h.
5. Another car travels northwest for 2 hours and covers a distance of 90 miles. What is the average velocity of the car?

References

1. Christopher Auyeung (CK-12 Foundation); Compass: Seamus McGill. . CC BY-NC 3.0; Compass: Public Domain
2. . . CC BY-NC 3.0

CHAPTER 4 Average Velocity

- Explain the difference between speed and velocity.
- Define the concept of average velocity.
- Given displacement and time, calculate average velocity.
- Solve for any variable in the equation $V_{\text{ave}} = \frac{\Delta x}{\Delta t}$

Test Pilot Neil Armstrong (later to become a famous astronaut) is seen here next to the X-15 ship after a research flight. Armstrong made his first X-15 flight on November 30, 1960. This was the first X-15 flight to use the ball nose, which provided accurate measurement of air speed at hypersonic speeds. The servo-actuated ball nose can be seen in this photo in front of Armstrong's right hand. The X-15 employed a non-standard landing gear. It had a nose gear with a wheel and tire, but the main landing consisted of skids mounted at the rear of the vehicle. In the photo, the left skid is visible, as are marks on the lakebed from both skids. Because of the skids, the rocket-powered aircraft could only land on a dry lakebed, not on a concrete runway. The X-15 weighed about 14,000 lb empty and approximately 34,000 lb at launch. The X-15 was flown over a period of nearly 10 years – June 1959 to Oct. 1968 – and set the world's unofficial speed record of 4,520 mph.

Average Velocity

The terms **speed** and **velocity** are used interchangeably in ordinary language. In physics, however, we make a distinction between the two. Essentially both words refer to how fast an object is moving. Speed is the number we read off the speedometer of a car. It indicates how fast the car is moving at any instant but given no indication of the direction it is moving. For a particular time interval, average speed would be calculated by dividing the distance travelled by the time interval of travel. Velocity**,** in physics, is a vector, meaning that it must have a direction as well as a magnitude. Furthermore, the average velocity is defined in terms of displacement rather than distance. Average velocity would be calculated by dividing the displacement by the time interval where displacement is the change in position of the object.

To show the distinction, we could calculate the average speed and the average velocity of a person who walks 50 m to the east, then turns around and walks 50 m to the west. The total time interval is 20 seconds. The distance traveled in this trip is 100 m but the displacement is zero.

The average speed would be calculated by dividing 100 m by 20 s with a result of 5 m/s. The average velocity, on the other hand, would be calculated by dividing 0 m by 20 s giving a result of 0 m/s.

Neither average speed nor average velocity implies a constant rate of motion. That is to say, an object might travel at 10 m/s for 10 s and then travel at 20 m/s for 5 s and then travel at 100 m/s for 5s. This motion would cover a distance of 700 m in 20 s and the average speed would be 35 m/s. We would report the average speed during this 20 s interval to be 35 m/s and yet at no time during the interval was the speed necessarily 35 m/s. The concept of constant velocity is very different from average velocity. If an object traveled at 35 m/s for 20 s, it would travel the same distance in the same time as the previous example but in the second case, the speed of the object would always be 35 m/s.

Example: The position of a runner as a function of time is plotted as moving along the x-axis of a coordinate system. During a 3.00 s time interval, the runner's position changes from $x_1 = 50.0$ m to $x_2 = 30.5$ m. What was the runner's average velocity.

Solution:

Displacement $= 30.5 \text{ m} - 50.0 \text{ m} = -19.5 \text{ m}$ (the object was traveling back toward zero)

$\Delta t = 3.00 \text{ s}$

$v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{-19.5 \text{ m}}{3.00 \text{ s}} = -6.50 \text{ m/s}$

Summary

- Average speed is distance divided by time.
- Average velocity is displacement divided by time.

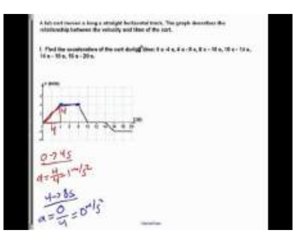
Practice

The url below is a physics classroom discussion of speed versus velocity with a short animation.

http://www.physicsclassroom.com/Class/1DKin/U1L1d.cfm

Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=BWP1tN7PZps



MEDIA

Click image to the left for more content.

1. The velocity versus time graph in the video is divided into six sections. In how many of these sections is the velocity constant?
2. In how many sections of the graph is the velocity zero?
3. What does the area under the curve of a velocity versus time graph represent?

Review

1. On a one day vacation, Jane traveled 340 miles in 8.0 hours. What was her average speed?
2. An object on a number line moved from $x = 12$ m to $x = 124$ m and moved back to $x = 98$ m. The time interval for all the motion was 10. s. What was the average velocity of the object?
3. An object on a number line moved from x = 15 cm to x = 165 cm and then moved back to x = 25 cm all in a time of 100 seconds.
   (a) What was the average velocity of the object?
   (b) What was the average speed of the object?

- **speed:** Distance travelled per unit time.
- **velocity:** A vector measurement of the rate and direction of motion or, in other terms, the rate and direction of the change in the position of an object.

References

1. Courtesy of NASA. http://commons.wikimedia.org/wiki/File:Pilot_Neil_Armstrong_and_X-15.jpg. Public Domain

CHAPTER 5 Instantaneous Velocity

- Define instantaneous velocity.
- Plot and interpret position vs time graphs.
- Determine the slope of a curve on a position vs time graph.

In a footrace such as the one shown here, the initial velocity of a runner is zero. The runner increases his velocity out of the starting blocks and his velocity continues to increase as the race proceeds. For the well-trained athlete, his highest velocity is maintained through the finish line.

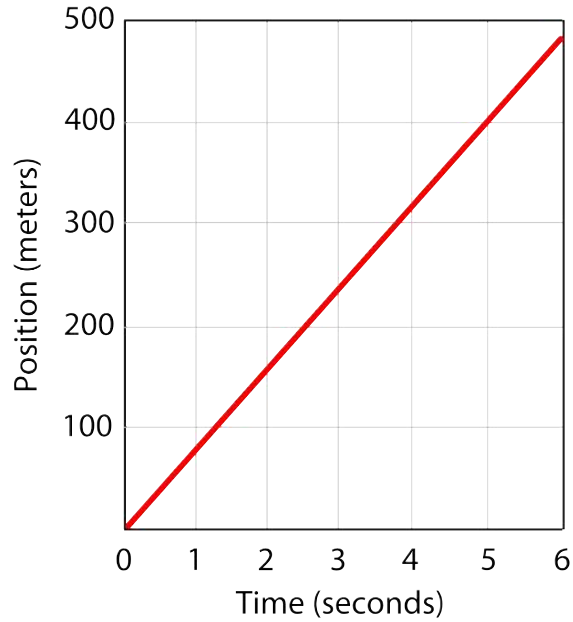
Instantaneous Velocity

The **instantaneous velocity** of an object is the precise velocity at a given moment. It is a somewhat difficult quantity to determine unless the object is moving with constant velocity. If the object is moving with constant velocity, then the instantaneous velocity at every instant, the average velocity, and the constant velocity are all exactly the same.

Position vs Time Graphs

Consider a position versus time graph for an object starting at $t = 0$ and $x = 0$ that has a constant velocity of 80. m/s.

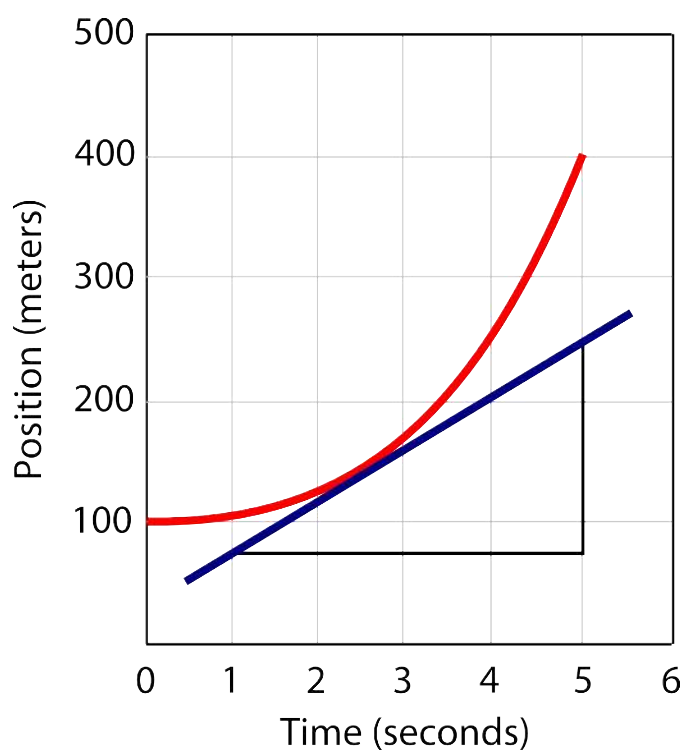
Position vs Time	
Time (s)	Position (m)
0	0
1	80.
2	160
3	240
4	320
5	400
6	480



The velocity of an object can be found from a position vs time graph. On a position vs time graph, the displacement is the vertical separation between two points and the time interval is the horizontal separation. The ratio of displacement to time interval is the average velocity. The ratio of the vertical separation to the horizontal separation is also the slope of the line. Therefore, the slope of straight line is the average velocity. The slope at any given time is the instantaneous velocity. For the motion pictured above,

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta d}{\Delta t} = \frac{400.\ \text{m}}{5.0\ \text{s}} = 80.\ \text{m/s}$

For accelerated motion (the velocity is constantly changing), the position vs time graph will be a curved line. The slope of the curved line at any point will be the instantaneous velocity at that time. If we were using calculus, the slope of a curved line could be calculated. Since we are not using calculus, we can only approximate the slope of curved line by laying a straight edge along the curved line and guessing at the slope.



In the image above, the red line is the position vs time graph and the blue line is an approximated slope for the line at $t = 2.5$ seconds. The rise for this slope is approximately 170 m and the time interval (run) is 4.0 seconds. Therefore, the approximated slope is 43 m/s.

Summary

- The slope of a position versus time graph is the velocity. For a constant velocity motion, the slope gives the constant velocity, the average velocity, and the instantaneous velocity at every point. For constant acceleration motion, the slope of the position versus time curve gives only the instantaneous velocity at that point.

Practice

Draw a velocity versus time graph for an object whose constant velocity is 15 m/s and whose position starts at $x = 0$ when $t = 0$. Graph the motion for the first 5.0 seconds.

Use this resource to answer the questions that follow.

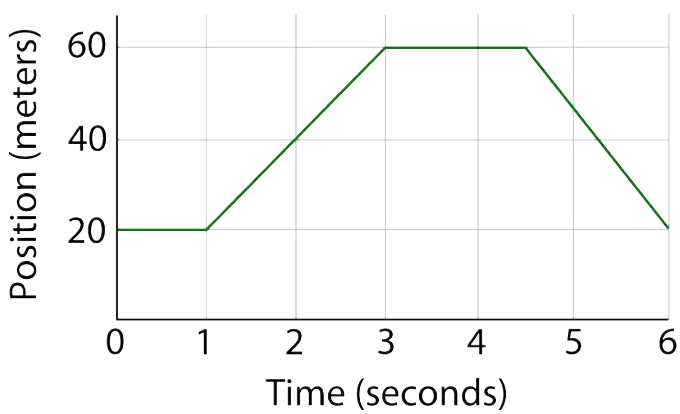
http://www.youtube.com/watch?v=sujsb5ZlM8o

MEDIA

Click image to the left for more content.

1. In the graph on the video, what is graphed on the vertical axis?
2. What is graphed on the horizontal axis.
3. What does the slope of this graph represent?

Review



1. For the motion graphed in the position versus time graph shown above, what is the average velocity in the time interval 1 to 3 seconds?
2. For the motion graphed in the position versus time graph shown above, what is the average velocity in the time interval 3 to 4 seconds?
3. For the motion graphed in the position versus time graph shown above, what is the average velocity in the time interval 5 to 6 seconds?

- **instantaneous velocity:** The velocity of an object at any given instant.

References

1. Image copyright Denis Kuvaev, 2013. http://www.shutterstock.com. Used under license from Shutterstock.com
2. CK-12 Foundation - Richard Parsons. . CC-BY-NC-SA 3.0
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CHAPTER 6 Average Acceleration

- Define average acceleration.
- Given initial velocity, final velocity and time, calculate acceleration.
- Given three of initial velocity, acceleration, time, and final velocity, calculate the fourth.

End of an era. The Space Shuttle Atlantis blasts off on mission STS-125, the final mission to service and upgrade the Hubble Space Telescope, one of NASA's greatest legacies and triumphs. Canceled in the wake of the Columbia tragedy and then reinstated, the only mission not to go to the international space station post-accident will see seven astronauts undertake one of the most ambitious shuttle missions in history, with five spacewalks to install new and replace old components on Hubble. It will be the closing chapter in one of the original purposes of the shuttle.

Average Acceleration

An object whose velocity is changing is said to be accelerating. **Average acceleration**, $\overline{a}$ is defined as the rate of change of velocity, or the change in velocity per unit time. A symbol with a bar over it is read as average –so a-bar is average acceleration.

Example: A car accelerates along a straight road from rest to 60. km/h in 5.0 s. What is the magnitude of its average acceleration?

Solution:

This is read as kilometers per hour per second. In general, it is undesirable to have to different units for the same quantity in a unit expression. For example, in this case, it is undesirable to have two different units for time (hours and seconds) in the same unit expression. To eliminate this problem, we would convert the hour units to seconds. If we converted the original 60. km/h to m/s, it would be 17 m/s. Then the acceleration would be 3.4 m/s^2.

Example: An automobile is moving along a straight highway in the positive direction and the driver puts on the brakes. If the initial velocity is 15.0 m/s and 5.0 s is required to slow down to 5.0 m/s, what was the car's acceleration?

Solution:

$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{-10.\ \mathbf{m/s}}{5.0\ \mathbf{s}} = -2.0\ \mathbf{m/s/s}$

Summary

- Average acceleration is the rate of change of velocity, or the change in velocity per unit time.

Practice

The following url has a lesson on the difference between average and instantaneous acceleration and practice calculating average acceleration.

http://www.brighthubeducation.com/homework-math-help/102434-definition-and-how-to-calculate-acceleration/

Review

1. The velocity of a car increases from 2.0 m/s to 16.0 m/s in a time period of 3.5 s. What was the average acceleration?
2. If an automobile slows from 26 m/s to 18 m/s in a period of 4.0 s, what was the average acceleration?
3. If a runner increases his velocity from 0 m/s to 20 m/s in 2.0 s, what was his average acceleration?
4. If a runner decreases his velocity from 20 m/s to 10 m/s in 2.0 s, what was his average acceleration?

- **average acceleration:** The change in velocity over the change in time.

References

1. Courtesy of NASA. http://spaceflight.nasa.gov/gallery/images/shuttle/sts-125/html/sts125-s-025.html. Public Domain

Chapter 7 Uniform Acceleration

- Define uniform acceleration.
- Given initial velocity, acceleration, and time, calculate final velocity.

Wingtip vortices are often thought to be a type of contrail but are actually produced from a different process. During very specific weather conditions you may see vapor trails form at the rear of the wingtips of jet aircraft on takeoff or landing. This phenomenon occurs due to a decrease in pressure and temperature as the wing generates lift.

The image is an F-35 departing from Elgin Air Force Base in Florida.

Uniform Acceleration

Acceleration that does not change in time is called **uniform or constant acceleration.** The velocity at the beginning of the time interval is called initial velocity, v_i, and the velocity at the end of the time interval is called final velocity, v_f. In a velocity versus time graph for uniform acceleration, the slope of the line is the acceleration. The equation that describes the curve is $v_f = v_i + at$.

Example: If an automobile with a velocity of 4.0 m/s accelerates at a rate of 4.0 m/s^2 for 2.5 s, what is the final velocity?

Solution:

$v_f = v_i + at = 4.0 \text{ m/s} + (4.0 \text{ m/s}^2)(2.5 \text{ s}) = 4.0 \text{ m/s} + 10. \text{ m/s} = 14 \text{ m/s}$

Example: If a cart slows from 22.0 m/s to 4.0 m/s with an acceleration of -2.0 m/s^2, how long does it require?

Solution:

$t = \frac{v_f - v_i}{a} = \frac{-18 \ \mathbf{m/s}}{-2.0 \ \mathbf{m/s^2}} = 9.0 \ \mathbf{s}$

Summary

- Acceleration that does not change in time is uniform or constant acceleration.
- The equation relating initial velocity, final velocity, time, and acceleration is $v_f = v_i + at$.

Practice

The following url has instruction in one dimensional uniformly accelerated motion and it also has a series of practice problems.

http://dallaswinwin.com/Motion_in_One_Dimension/uniform_accelerated_motion.htm

Review

1. If an object has zero acceleration, does that mean it has zero velocity? Give an example.
2. If an object has zero velocity, does that mean it has zero acceleration? Give an example.
3. If the acceleration of a motorboat is 4.0 m/s^2, and the motorboat starts from rest, what is its velocity after 6.0 s?
4. The friction of the water on a boat produces an acceleration of -10. m/s^2. If the boat is traveling at 30. m/s and the motor is shut off, how long it take the boat to slow down to 5.0 m/s?

- **uniform acceleration:** Acceleration that does not change in time is uniform or constant acceleration.

References

1. Courtesy of Senior Airman Julianne Showalter/U.S. Air Force. http://www.af.mil/news/story.asp?id=123261835. Public Domain

CHAPTER 8 Displacement During Constant Acceleration

- Plot and interpret a velocity vs time graph.
- Find the area under a curve on a velocity vs time graph and calculate the displacement from such a graph.
- Calculate the displacement of an object undergoing uniform acceleration when given two of the three quantities acceleration, time, velocity.

Long distance runners try to maintain constant velocity with very little acceleration or deceleration because acceleration requires more energy than simply maintaining velocity.

Displacement During Constant Acceleration

When the acceleration is constant, there are three equations that relate displacement to two of the other three quantities we use to describe motion –time, velocity, and acceleration. It is absolutely vital that you do **NOT** try to use these equations when the acceleration is **NOT** constant. Fortunately, there are quite a few cases of motion where the acceleration is constant. One of the most common, if we ignore air resistance, are objects falling due to gravity.

When an object is moving with constant velocity, the displacement can be found by multiplying the velocity by the time interval. $d = vt$

If the object is moving with constant acceleration, the velocity in that equation is replaced with the average velocity. The average velocity for a uniformly accelerated object can be found by adding the beginning and final velocities and dividing by 2. $v_{\mathbf{ave}} = \frac{1}{2}(v_f + v_i)$

The distance, then, for uniformly accelerating motion can be found by multiplying the average velocity by the time.

$d = \frac{1}{2}(v_f + v_i)(t)$ (**Equation** 1)

We know that the final velocity for constantly accelerated motion can be found by multiplying the acceleration times time and adding the result to the initial velocity, $v_f = v_i + at$**.**

The second equation that relates, displacement, time, initial velocity, and final velocity is generated by substituting into equation 1.

$$d = \frac{1}{2}(v_f + v_i)(t) = \frac{1}{2}v_f t + \frac{1}{2}v_i t$$
$$\text{but } v_f = v_i + \text{at and substituting for } v_f \text{ yields}$$
$$d = \frac{1}{2}v_i t + \frac{1}{2}(t)(v_i + at) = \frac{1}{2}v_i t + \frac{1}{2}v_i t + \frac{1}{2}at^2$$
$$d = v_i t + \frac{1}{2}at^2 \qquad \text{(Equation 2)}$$

The third equation is formed by combining $v_f = v_i + at$ and $d = \frac{1}{2}(v_f + v_i)(t)$. If we solve the first equation for t and then substitute into the second equation, we get

$d = \left(\frac{1}{2}\right)(v_f + v_i)\left(\frac{v_f - v_i}{a}\right) = \left(\frac{1}{2}\right)\left(\frac{v_f^2 - v_i^2}{a}\right)$

And solving for ${v_f}^2$ yields

${v_f}^2 = {v_i}^2 + 2ad$ (**Equation** 3)

Keep in mind that these three equations are only valid when acceleration is constant. In many cases, the initial velocity can be set to zero and that simplifies the three equations considerably.

With both constant acceleration and initial velocity of zero,

$$d = \frac{1}{2}\, v_f\, t$$
$$d = \frac{1}{2}\, at^2 \text{ and}$$
$${v_f}^2 = 2ad.$$

Example: Suppose a planner is designing an airport for small airplanes. Such planes must reach a speed of 56 m/s before takeoff and can accelerate at 12.0 m/s^2. What is the minimum length for the runway of this airport?

Solution: The acceleration in this problem is constant and the initial velocity of the airplane is zero, therefore, we can use the equation ${v_f}^2 = 2ad$ and solve for d.

$d = \frac{{v_f}^2}{2a} = \frac{(56\text{ m/s})^2}{(2)(12.0\text{ m/s}^2)} = 130\text{ m}$

Example: How long does it take a car to travel 30.0 m if it accelerates from rest at a rate of 2.00 m/s^2?

Solution: The acceleration in this problem is constant and the initial velocity is zero, therefore, we can use $d = \frac{1}{2}at^2$ solved for t.

$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{(2)(30.0\text{ m})}{2.00\text{ m/s}^2}} = 5.48\text{ s}$

Example: A baseball pitcher throws a fastball with a speed of 30.0 m/s. Assuming the acceleration is uniform and the distance through which the ball is accelerated is 3.50 m. What is the acceleration?

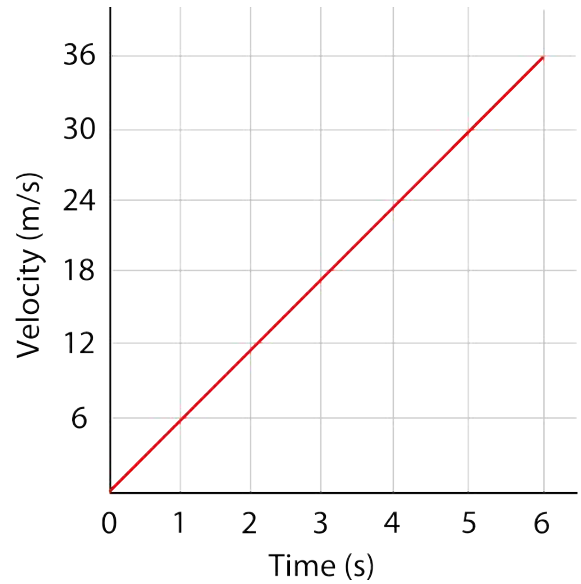
Solution: Since the acceleration is uniform and the initial velocity is zero, we can use ${v_f}^2 = 2ad$ solved for a.

$a = \frac{v_f^2}{2d} = \frac{(30.0 \text{ m/s})^2}{(2)(3.50 \text{ m})} = \frac{900. \text{ m}^2/\text{s}^2}{7.00 \text{ m}} = 129 \text{ m/s}^2$

Suppose we plot the velocity versus time graph for an object undergoing uniform acceleration. In this first case, we will assume the object started from rest.

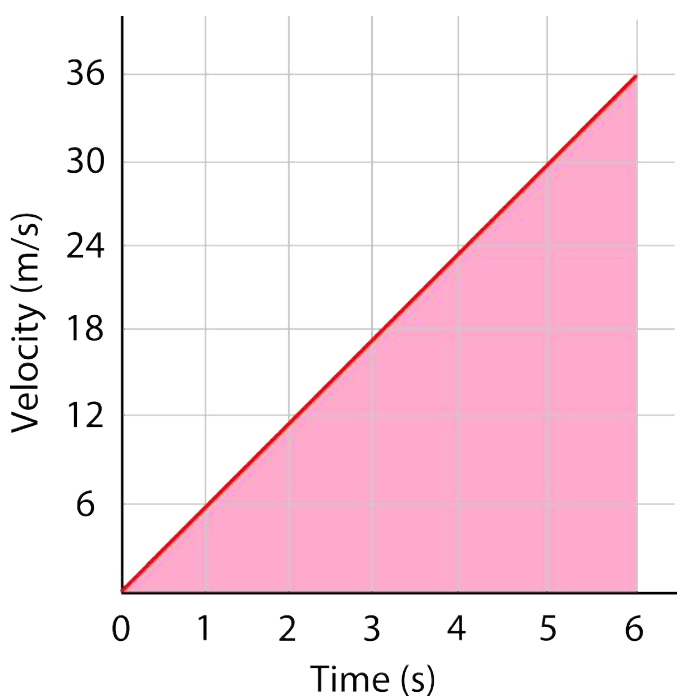
If the object has a uniform acceleration of 6.0 m/s^2 and started from rest, then each succeeding second, the velocity will increase by 6.0 m/s. Here is the table of values and the graph.

Velocity vs Time	
Time (s)	Velocity (m/s)
0	0
1	6
2	12
3	18
4	24
5	30
6	36



In displacement versus time graphs, the slope of the line is the velocity of the object and in this case of velocity versus time graph, the slope of the line is the acceleration. If you take any segment of this line and determine the Δy to Δx ratio, you will get 6.0 m/s^2 which we know to be the constant acceleration of this object.

The area of a triangle is calculated by multiplying one-half the base times the height. The area under the curve in the image above is the area of the triangle shown below.



The area of this triangle would be calculated by area $= \left(\frac{1}{2}\right)(6.0 \text{ s})(36 \text{ m/s}) = 108 \text{ m}$.

The distance traveled by an object accelerating uniformly from rest at 6.0 m/s^2 would be displacement $= \frac{1}{2}at^2$.

Therefore, the displacement of this object in the first 6 seconds of travel would be

displacement $= \left(\frac{1}{2}\right)(6.0 \text{ m/s}^2)(6.0 \text{ s})^2 = 108 \text{ m}$.

In fact, the area underneath the curve in a velocity versus time graph is always equal to the displacement that occurs during that time interval.

Summary

- There are three equations relating displacement to two of the other three quantities we use to describe motion –time, velocity, and acceleration:
  - $d = \left(\frac{1}{2}\right)(v_f + v_i)(t)$ (Equation 1)
  - $d = v_i t + \frac{1}{2}at^2$ (Equation 2)
  - ${v_f}^2 = {v_i}^2 + 2ad$ (Equation 3)
- When the initial velocity of the object is zero, these three equations become:
  - $d = \left(\frac{1}{2}\right)(v_f)(t)$ (Equation 1')
  - $d = \frac{1}{2}at^2$ (Equation 2')
  - ${v_f}^2 = 2ad$ (Equation 3')
- The slope of a velocity versus time graph is the acceleration of the object.
- The area under the curve of a velocity versus time graph is the displacement that occurs during the given time interval.

Practice

Use this resource to answer the questions that follow.

MEDIA

Click image to the left for more content.

1. For the example in the video, what acceleration is used?
2. What time period is used in the example?
3. What does the slope of the line in the graph represent?
4. What does the area under the curve of the line represent?

Review

1. An airplane accelerates with a constant rate of 3.0 m/s^2 starting at a velocity of 21 m/s. If the distance traveled during this acceleration was 535 m, what is the final velocity?
2. An car is brought to rest in a distance of 484 m using a constant acceleration of -8.0 m/s^2. What was the velocity of the car when the acceleration first began?
3. An airplane starts from rest and accelerates at a constant 3.00 m/s^2 for 20.0 s. What is its displacement in this time?
4. A driver brings a car to a full stop in 2.0 s.

(a) If the car was initially traveling at 22 m/s, what was the acceleration?
(b) How far did the car travel during braking?

References

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CHAPTER 9 Acceleration Due to Gravity

- Solve problems of the motion of objects uniformly accelerated by gravity.

In the absence of air resistance, all objects fall toward the earth with the same acceleration. Man, however, make maximum use of air resistance in the construction of parachutes for both entertainment and military use.

The image at left was taken during a 2008 Graduation demonstration jump by the U.S. Army Parachute Team. The 2008 team contained the first amputee member and the largest number of females in history.

Acceleration Due to Gravity

One of the most common examples of uniformly accelerated motion is that an object allowed to fall vertically to the earth. In treating falling objects as uniformly accelerated motion, we must ignore air resistance. Galileo's original statement about the motion of falling objects is:

At a given location on the earth and in the absence of air resistance, all objects fall with the same uniform acceleration.

We call this **acceleration due to gravity** on the earth and we give it the symbol g. The value of g is 9.80 m/s^2. All of the equations involving constant acceleration can be used for falling bodies but we insert g wherever "a" appeared and the value of g is always 9.80 m/s^2.

Example: A rock is dropped from a tower 70.0 m high. How far will the rock have fallen after 1.00 s, 2.00 s, and 3.00 s? Assume the distance is positive downward.

Solution: We are looking for displacement and we have time and acceleration. Therefore, we can use $d = \frac{1}{2}at^2$.

Displacement after 1.00 s $= \left(\frac{1}{2}\right)(9.80\text{ m/s}^2)(1.00\text{ s})^2 = 4.90\text{ m}$

Displacement after 2.00 s $= \left(\frac{1}{2}\right)(9.80\text{ m/s}^2)(2.00\text{ s})^2 = 19.6\text{ m}$

Displacement after 3.00 s $= \left(\frac{1}{2}\right)(9.80\text{ m/s}^2)(3.00\text{ s})^2 = 44.1\text{ m}$

Example: (a) A person throws a ball upward into the air with an initial velocity of 15.0 m/s. How high will it go before it comes to rest? (b) How long will the ball be in the air before it returns to the person's hand?

Solution: In part (a), we know the initial velocity (15.0 m/s), the final velocity (0 m/s), and the acceleration -9.80 m/s^2. We wish to solve for the displacement, so we can use ${v_f}^2 = {v_i}^2 + 2ad$ and solve for d.

$d = \frac{{v_f}^2 - {v_i}^2}{2a} = \frac{(0\text{ m/s})^2 - (15.0\text{ m/s})^2}{(2)(9.80\text{ m/s}^2)} = 11.5\text{ m}$

There are a number of methods by which we can solve part (b). Probably the easiest is to divide the distance traveled by the average velocity to get the time going up and then double this number since the motion is symmetrical –that is, time going up equals the time going down.

The average velocity is half of 15.0 m/s or 7.5 m/s and dividing this into the distance of 11.5 m yields 1.53 seconds. This is the time required for the ball to go up and the time for the ball to come down will also be 1.53 s, so the total time for the trip up and down is 3.06 seconds.

Example: A car accelerates with uniform acceleration from 11.1 m/s to 22.2 m/s in 5.0 s. (a) What was the acceleration and (b) how far did it travel during the acceleration?

Solution:

(a) $a = \frac{\Delta v}{\Delta t} = \frac{22.2\text{ m/s} - 11.1\text{ m/s}}{5.0\text{ S}} = 2.22\text{ m/s}^2$

(b) We can find the distance traveled by $d = v_i t + \frac{1}{2}at^2$ and we can also find the distance traveled by determining the average velocity and multiply it by the time.

$$\begin{aligned}
d &= v_i t + \frac{1}{2}at^2 \\
&= (11.1\text{ m/s})(5.0\text{ s}) + \left(\frac{1}{2}\right)(2.22\text{ m/s}^2)(5.0\text{ s})^2 \\
&= 55.5\text{ m} + 27.8\text{ m} \\
&= 83\text{ m}
\end{aligned}$$

$d = (v_{\text{ave}})(t) = (16.6\text{ m/s})(5.0\text{ s}) = 83\text{ m}$

Example: A stone is dropped from the top of a cliff. It is seen to hit the ground after 5.5 s. How high is the cliff?

Solution:

$d = v_i t + \frac{1}{2}at^2 = (0\text{ m/s})(5.5\text{ s}) + \left(\frac{1}{2}\right)(9.80\text{ m/s}^2)(5.5\text{ s})^2 = 150\text{ m}$

Summary

- At a given location on the earth and in the absence of air resistance, all objects fall with the same uniform acceleration.
- We call this acceleration the acceleration due to gravity on the earth and we give it the symbol g.
- The value of g is 9.80 m/s^2.

Practice

This url shows a video of a discussion and demonstration of the acceleration due to gravity.

http://www.youtube.com/watch?v=izXGpivLvgY

MEDIA

Click image to the left for more content.

Review

1. A baseball is thrown vertically into the air with a speed of 24.7 m/s.
   (a) How high does it go?
   (b) How long does the round trip up and down require?
2. A salmon jumps up a waterfall 2.4 m high. With what minimum speed did the salmon leave the water below to reach the top?
3. A kangaroo jumps to a vertical height of 2.8 m. How long will it be in the air before returning to earth?

- **acceleration due to gravity:** The acceleration experienced by a body in free fall in a gravitational field.

References

1. Courtesy of Donna Dixon/U.S. Military. http://commons.wikimedia.org/wiki/File:Flickr_-_The_U.S._Army_-_U.S._Army_Parachute_Team_graduates_first_wounded_warrior_and_largest_female_class_%282%29.jpg. Public Domain

CHAPTER 10 Graphing Motion

Students will learn how to graph motion vs time. Specifically students will learn how to take the slope of a graph and relate that to the instantaneous velocity or acceleration for position or velocity graphs, respectively. Finally students will learn how to take the area of a velocity vs time graph in order to calculate the displacement.

Students will learn how to graph motion vs time. Specifically students will learn how to take the slope of a graph and relate that to the instantaneous velocity or acceleration for position or velocity graphs, respectively. Finally students will learn how to take the area of a velocity vs time graph in order to calculate the displacement.

Key Equations

For a graph of position vs. time. The slope is the rise over the run, where the rise is the displacement and the run is the time. thus,

Slope = $v_{avg} = \frac{\Delta x}{\Delta t}$

Note: Slope of the tangent line for a particular point in time = the instantaneous velocity

For a graph of velocity vs. time. The slope is the rise over the run, where the rise is the change in velocity and the run is the time. thus,

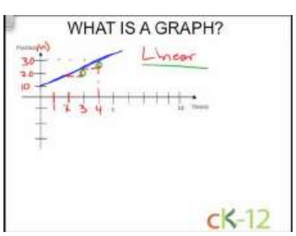
Slope = $a_{avg} = \frac{\Delta v}{\Delta t}$

Note: Slope of the tangent line for a particular point in time = the instantaneous acceleration

Guidance

- One must first read a graph correctly. For example on a position vs. time graph (thus the position is graphed on the y-axis and the time on the x-axis) for a given a data point, go straight down from it to get the time and straight across to get the position.
- If there is constant acceleration the graph x vs. t produces a parabola. The slope of the x vs. t graph equals the instantaneous velocity. The slope of a v vs. t graph equals the acceleration.
- The **slope** of the graph v vs. t can be used to find **acceleration;** the **area** of the graph v vs. t can be used to find **displacement.** Welcome to calculus!

What is a Graph



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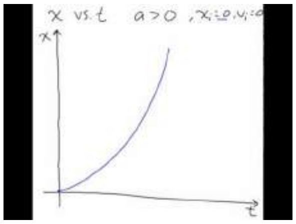
Watch this Explanation



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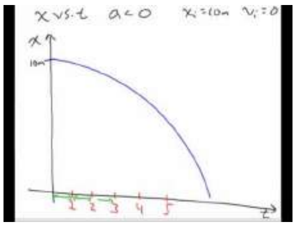

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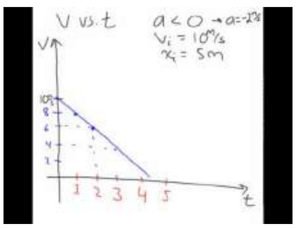

MEDIA

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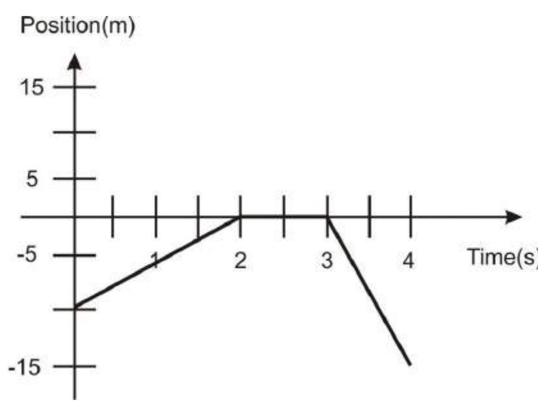
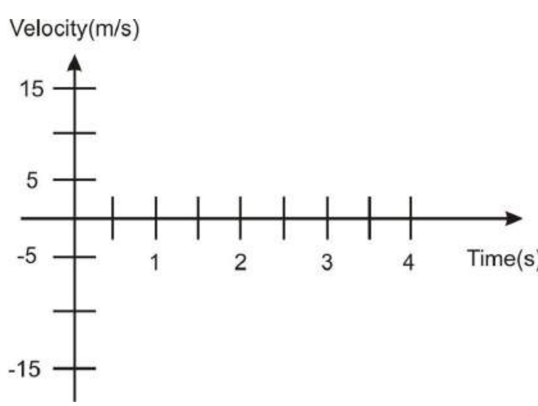
Time for Practice

1. The position graph below is of the movement of a fast turtle who can turn on a dime.

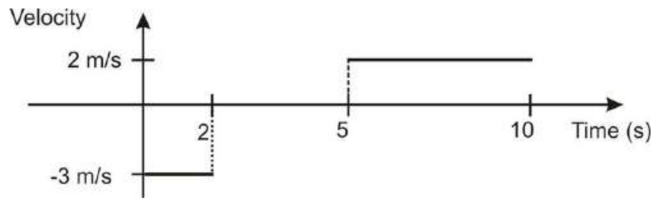


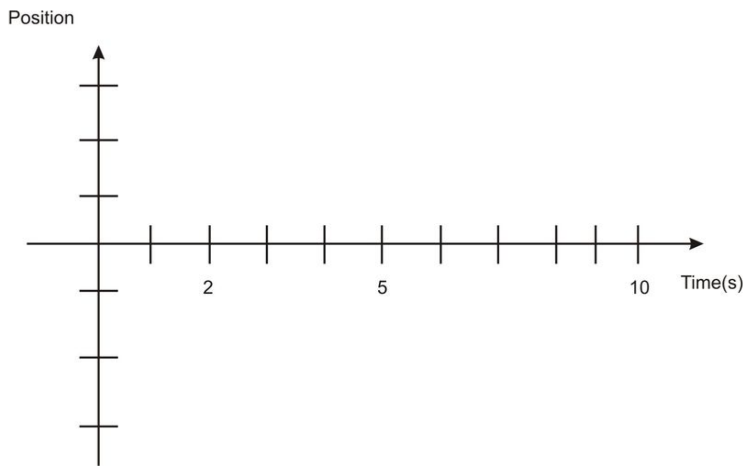
a. Sketch the velocity vs. time graph of the turtle below.



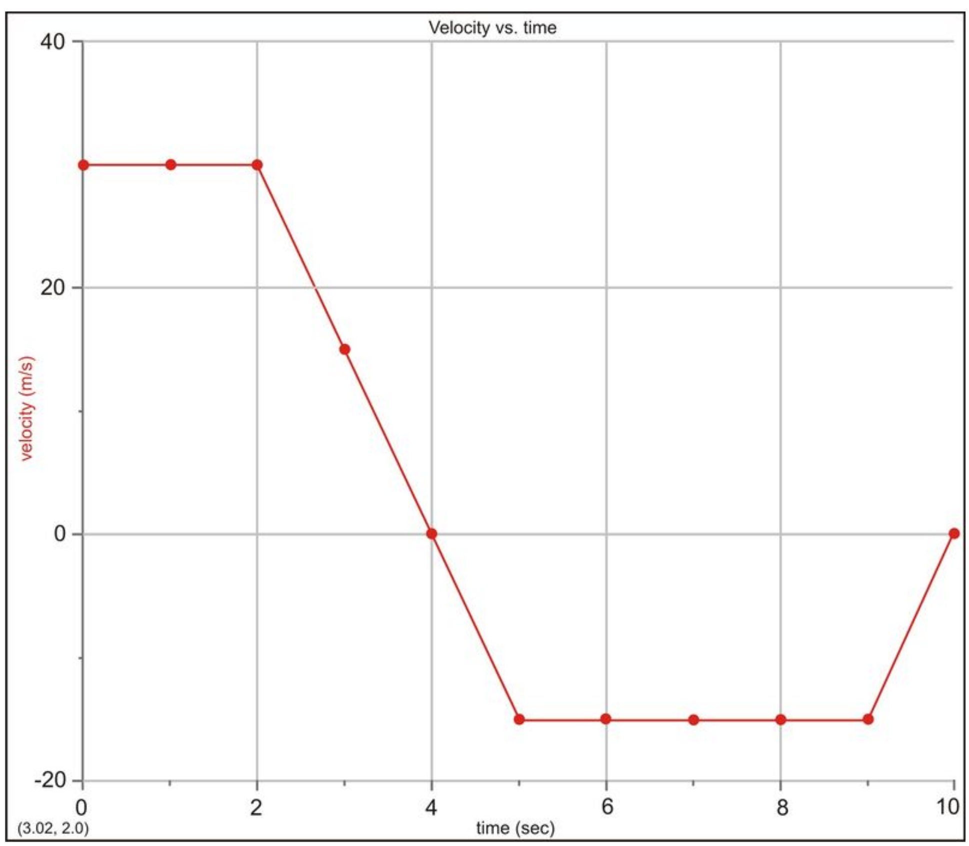
b. Explain what the turtle is doing (including both *speed* and *direction*) from: i) 0-2s. ii) 2-3s. iii) 3-4s. c. How much distance has the turtle covered after 4s? d. What is the turtle's displacement after 4s?

2. Draw the position vs. time graph that corresponds to the velocity vs. time graph below. You may assume a starting position $x_0 = 0$. Label the $y-$axis with appropriate values.





3. The following velocity-time graph represents 10 seconds of actress Halle Berry's drive to work (it's a rough morning).

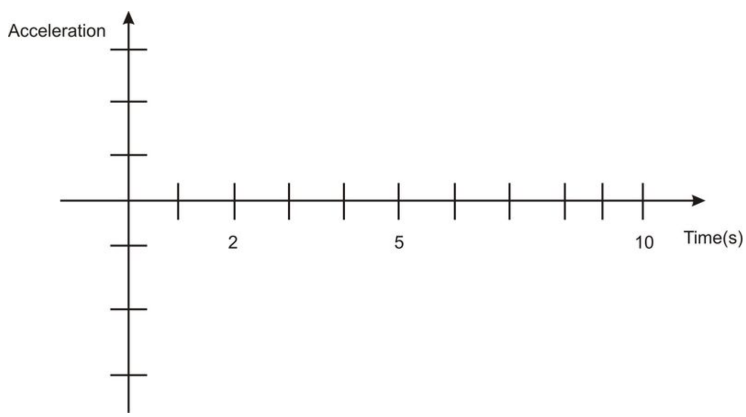


a. Fill in the tables below –remember that *displacement* and *position* are not the same thing!

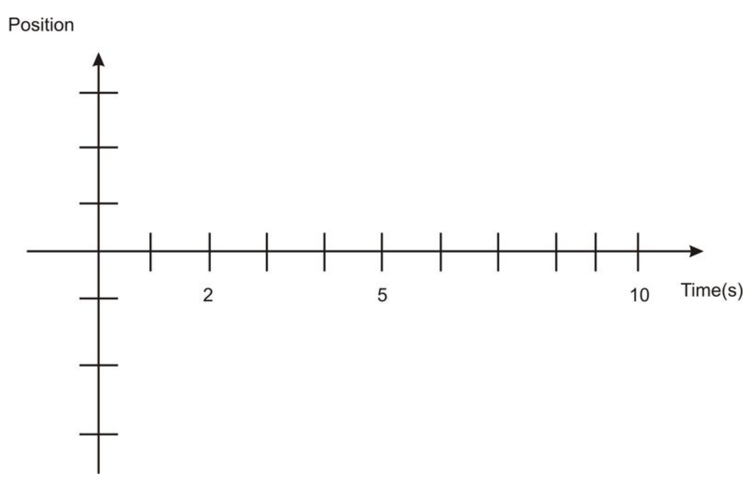
TABLE 10.1:

			Instantaneous Time (s)	**Position (m)**
Interval (s)	Displacement (m)	Acceleration(m/s^2)	0 sec	0 m
0-2 sec				
			2 sec	
2-4 sec				
			4 sec	
4-5 sec				
			5 sec	
5-9 sec				
			9 sec	
9-10 sec				
			10 sec	

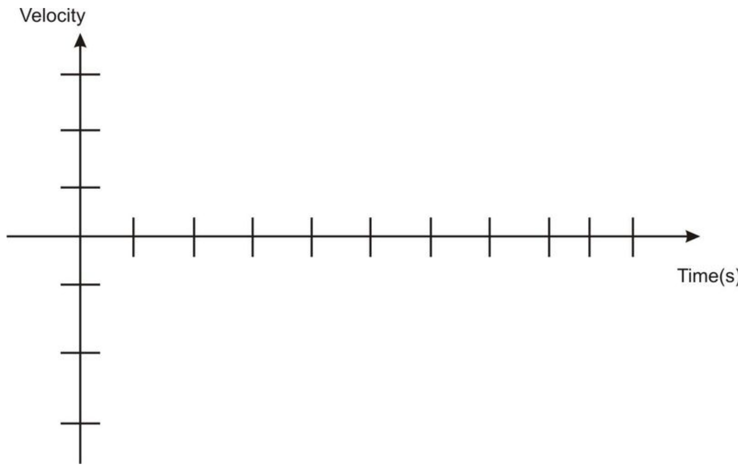
b. On the axes below, draw an *acceleration-time* graph for the car trip. Include numbers on your acceleration axis.



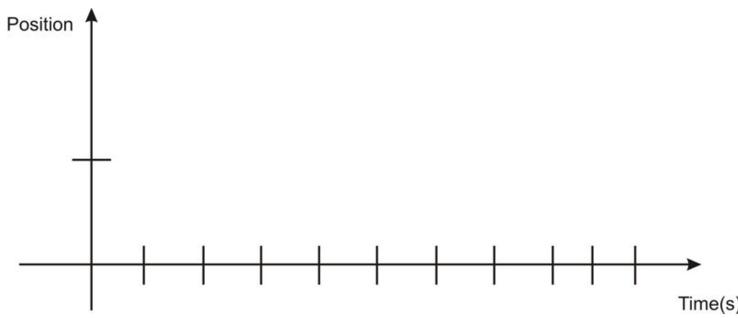
c. On the axes below, draw a *position-time* graph for the car trip. Include numbers on your position axis. Be sure to note that some sections of this graph are linear and some curve –why?



4. Two cars are drag racing down El Camino. At time $t = 0$, the yellow Maserati starts from rest and accelerates at 10 m/s^2. As it starts to move it's passed by a '63 Chevy Nova (cherry red) traveling at a constant velocity of 30 m/s. a. On the axes below, show a line for each car representing its speed as a function of time. Label each line.



b. At what time will the two cars have the same speed (use your graph)? c. On the axes below, draw a line (or curve) for each car representing its *position* as a function of time. Label each curve.



d. At what time would the two cars meet (other than at the start)?

Answers:

1c. 25 m

1d. -5 m

2. discuss in class

3. discuss in class

4b. 3 sec

4d. 6 sec

CK-12 FlexBook

Physics Unit 3 (Vectors)

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Ck12 Science

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Ck12 Science

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CHAPTER 1 Graphical Methods of Vector Addition

- Differentiate between scalars and vectors.
- Graphically add vectors in one dimension by placing the vectors head to toe on a number line.
- Define resultant.
- Graphically add vectors in two dimensions by placing them head to toe on a two-dimensional coordinate system.

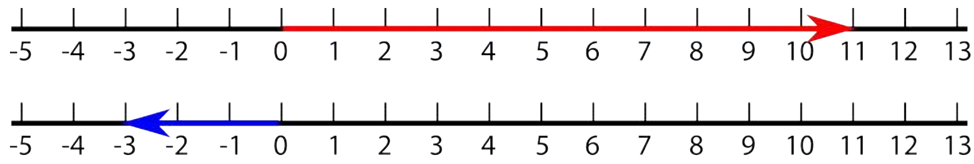
Successfully shooting a basketball requires a subconscious understanding of the vectors involved in how the basketball moves through the air. The vertical and horizontal vectors must be perfectly organized if the ball is to pass through the basket.

Graphical Methods Vector Addition

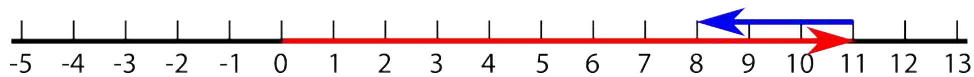
In physics, a quantity, such as mass, length, or speed, that is completely specified by its magnitude and has no direction is called a **scalar**. A **vector**, on the other hand, is a quantity possessing both magnitude and direction. A vector quantity can be represented by an arrow-tipped line segment. The length of the line, drawn to scale, represents the magnitude of the quantity. The direction of the arrow indicates the direction of the vector. Not only can vectors be represented graphically, but they can also be added graphically.

For one dimensional **vector addition**, the first vector is placed on a number line with the tail of the vector on the origin. The second vector is placed with its tail exactly on the arrow head of the first vector. The sum of the two vectors is the vector that begins at the origin and ends at the arrow head of the final added vector.

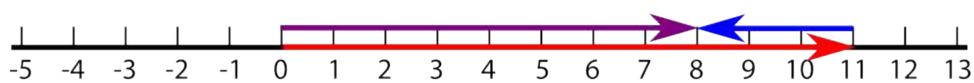
Consider the following two vectors.



The red vector has a magnitude of 11 in the positive direction on the number line. The blue vector has a magnitude of -3 in the negative direction on the number line. In order to add these two vectors, we place one of the vectors on a number line and then the second vector is placed on the same number line such that its origin is on the arrow head of the first vector.



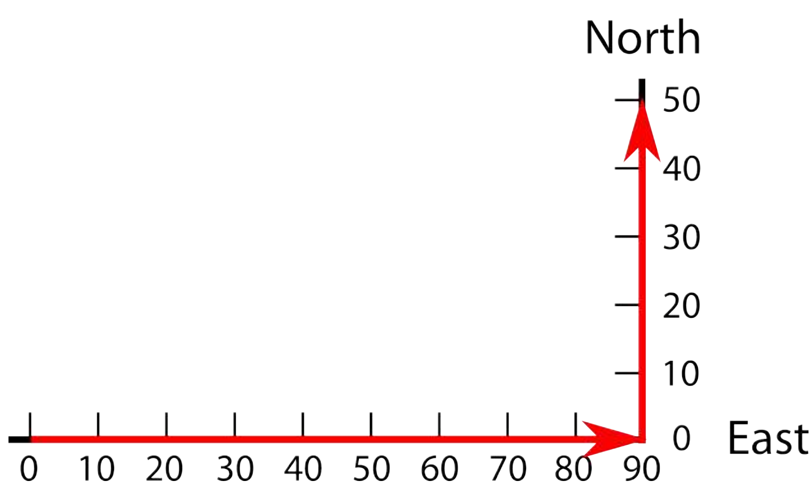
The sum of these two vectors is the vector that begins at the origin of the first vector (the red one) and ends at the arrow head of the blue vector. So the sum of these two vectors is the purple vector as shown below.



The vector sum of the first two vectors is a vector that begins at the origin and has a magnitude of 8 units in the positive direction. If we were adding three or four vectors all in one dimension, we would continue to place them head to toe in sequence on the number line. The sum would be the vector that begins at the beginning of the first vector and goes to the ending of the final vector.

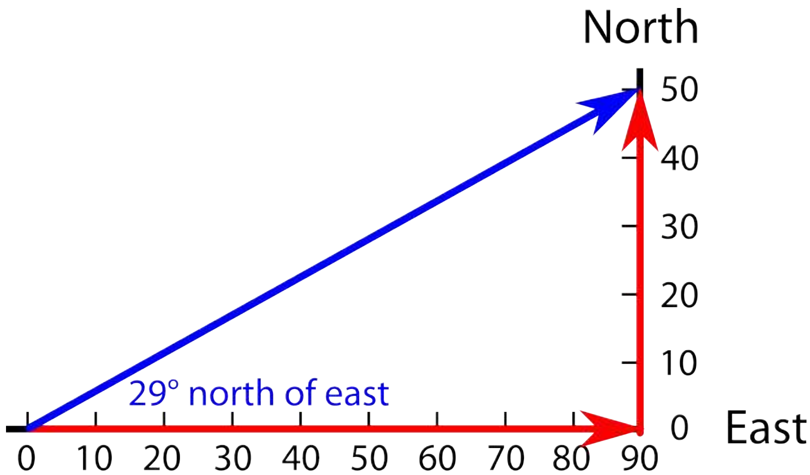
Adding Vectors in Two Dimensions

In the following image, vectors *A* and *B* represent the two displacements of a person who walked 90. m east and then 50. m north. We want to add these two vectors to get the vector sum of the two movements.



The graphical process for adding vectors in two dimensions is to place the tail of the second vector on the arrow head of the first vector as shown above.

The sum of the two vectors is the vector that begins at the origin of the first vector and goes to the ending of the second vector, as shown below.



If we are using totally graphic means of adding these vectors, the magnitude of the sum would be determined by measuring the length of the sum vector and comparing it to the original standard. We would also use a compass to measure the angle of the summation vector.

If we are using calculation means, we can determine the inverse tangent of 50 units divided by 90 units and get the angle of 29° north of east. The length of the sum vector can also be determined mathematically by the Pythagorean theorem, $a^2+b^2=c^2$. In this case, the length of the hypotenuse would be the square root of $(8100 + 2500)$ or 103 units.

If three or four vectors are to be added by graphical means, we would continue to place each new vector head to toe with the vectors to be added until all the vectors were in the coordinate system and then the sum vector would be the vector goes from the origin of the first vector to the arrowhead of the last vector. The magnitude and direction of the sum vector would be measured.

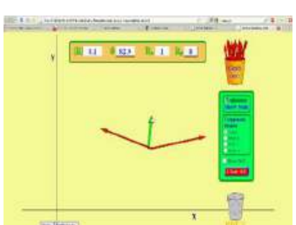
Summary

- Scalars are quantities, such as mass, length, or speed, that are completely specified by magnitude and has no direction.
- Vectors are quantities possessing both magnitude and direction and can be represented by an arrow; the direction of the arrow indicates the direction of the quantity and the length of the arrow is proportional to the magnitude.
- Vectors that are in one dimension can be added arithmetically.
- Vectors that are in two dimensions are added geometrically.
- When vectors are added graphically, graphs must be done to scale and answers are only as accurate as the graphing.

Practice

Video on the graphical method of adding vectors.

http://www.youtube.com/watch?v=_Vppxdho6JU



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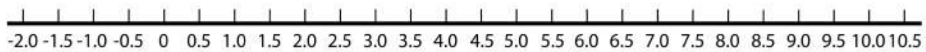
Review

1. On the following number line, add the vector 7.5 m/s and the vector -2.0 m/s.



2. On a sheet of graph paper, add a vector that is 4.0 N due east and a vector that is 3.0 N due north.

- **scalar:** A quantity, such as mass, length, or speed, that is completely specified by its magnitude and has no direction.
- **vector:** A quantity possessing both magnitude and direction, represented by an arrow the direction ofwhich indicates the direction of the quantity and the length of which is proportional to the magnitude.
- **vector addition:** The process of finding one vector that is equivalent to the result of the successive application of two or more given vectors.

References

1. Official White House photo by Pete Souza. http://commons.wikimedia.org/wiki/File:Barack_Obama_playing_basketball.jpg. Public Domain
2. CK-12 Foundation - Richard Parsons. . CC-BY-NC-SA 3.0
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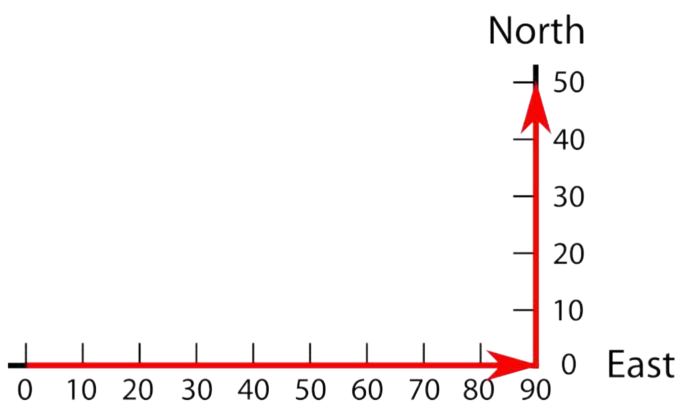
CHAPTER 2 Vector Addition

- Describe the independence of perpendicular vectors.
- Resolve vectors into axial components.
- Define resultant.
- Add vectors using geometric and trigonometric methods.

Vector Addition

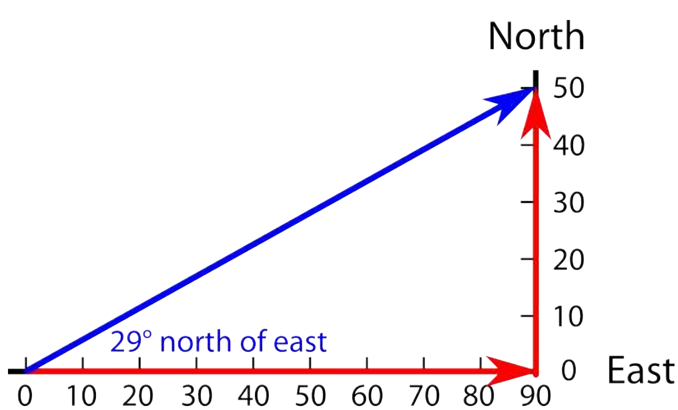
Adding Vectors in Two Dimensions

In the following image, vectors A and B represent the two displacements of a person who walked 90. m east and then 50. m north. We want to add these two vectors to get the vector sum of the two movements.



The graphical process for adding vectors in two dimensions is to place the tail of the second vector on the arrow head of the first vector as shown above.

The sum of the two vectors is the vector that begins at the origin of the first vector and goes to the ending of the second vector, as shown below.



If we are using totally graphic means of adding these vectors, the magnitude of the sum would be determined by measuring the length of the sum vector and comparing it to the original standard. We would also use a compass to measure the angle of the summation vector.

If we are using calculation means, we can determine the inverse tangent of 50 units divided by 90 units and get the angle of 29° north of east. The length of the sum vector can also be determined mathematically by the Pythagorean theorem, $a^2+b^2=c^2$. In this case, the length of the hypotenuse would be the square root of (8100 + 2500) or 103 units.

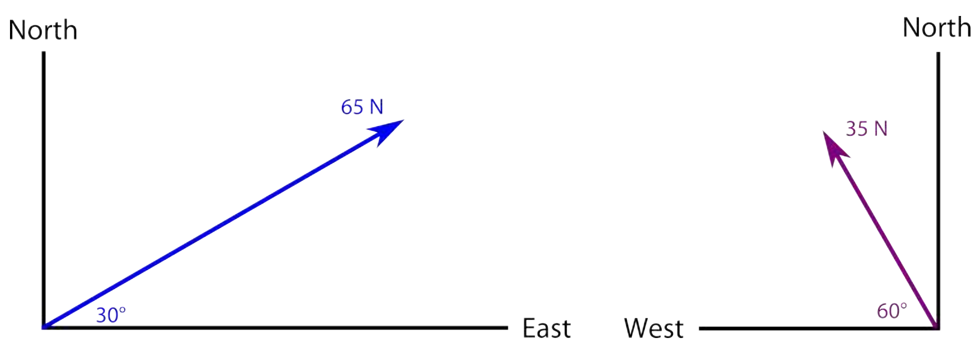
If three or four vectors are to be added by graphical means, we would continue to place each new vector head to toe with the vectors to be added until all the vectors were in the coordinate system and then the sum vector would be the vector goes from the origin of the first vector to the arrowhead of the last vector. The magnitude and direction of the sum vector would be measured.

Mathematical Methods of Vector Addition

We can add vectors mathematically using trig functions, the law of cosines, or the Pythagorean theorem.

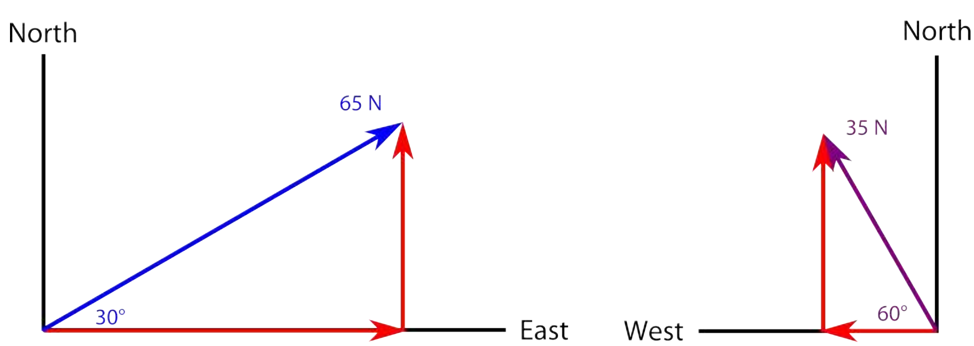
If the vectors to be added are at right angles to each other, we would assign them to the sides of a right triangle and calculate the sum as the hypotenuse of the right triangle. We would also calculate the direction of the sum vector by using an inverse sin or some other trig function.

Suppose, however, that we wish to add two vectors that are not at right angles to each other. Let's consider the vectors in the following images.



The two vectors we are to add is a force of 65 N at 30° north of east and a force of 35 N at 60° north of west.

We know that vectors in the same dimension can be added by regular arithmetic. Therefore, we can resolve each of these vectors into components that lay on the axes –pictured below.



We can resolve each of the vectors into two components. The components are on the axes lines. The r**esolution of vectors** reduces each vector to a component on the north-south axis and a component on the east-west axis.

We can now mathematically determine the magnitude of the components and add then arithmetically because they are in the same dimension. Once we have added the components, we will once again have only two vectors that are perpendicular to each other and can be the legs of a right triangle.

The east-west component of the first vector is (65 N)(cos 30°) = (65 N)(0.866) = 56.3 N north

The north-south component of the first vector is (65 N)(sin 30°) = (65 N)(0.500) = 32.5 N north

The east-west component of the second vector is (35 N)(cos 60°) = (35 N)(0.500) = 17.5 N west

The north-south component of the second vector is (35 N)(sin 60°) = (35 N)(0.866) = 30.3 N north

The sum of the two east-west components is 56.3 N - 17.5 N = 38.8 N east

The sum of the two north-south components is 32.5 N + 30.3 N = 62.8 N north

We can now consider those two vectors to be the sides of a right triangle and use the Pythagorean Theorem to find the length of the hypotenuse and use a trig function to find its direction.

$c = \sqrt{38.8^2 + 62.8^2} = 74 \text{ N}$

$\sin x = \frac{62.8}{74}$ so $x = \sin^{-1} 0.84$ so $x = 58°$

The direction of the sum vector is 74 N at 58° north of east.

Perpendicular vectors have no components in the other direction. For example, if a boat is floating down a river due south, and you are paddling the boat due east, the eastward vector has no component in the north-south direction and therefore, has no effect on the north-south motion. If the boat is floating down the river at 5 miles/hour south and you paddle the boat eastward at 5 miles/hour, the boat continues to float southward at 5 miles/hour. The eastward motion has absolutely no effect on the southward motion. Perpendicular vectors have NO effect on each other.

Example Problem: A motorboat heads due east at 16 m/s across a river that flows due north at 9.0 m/s.

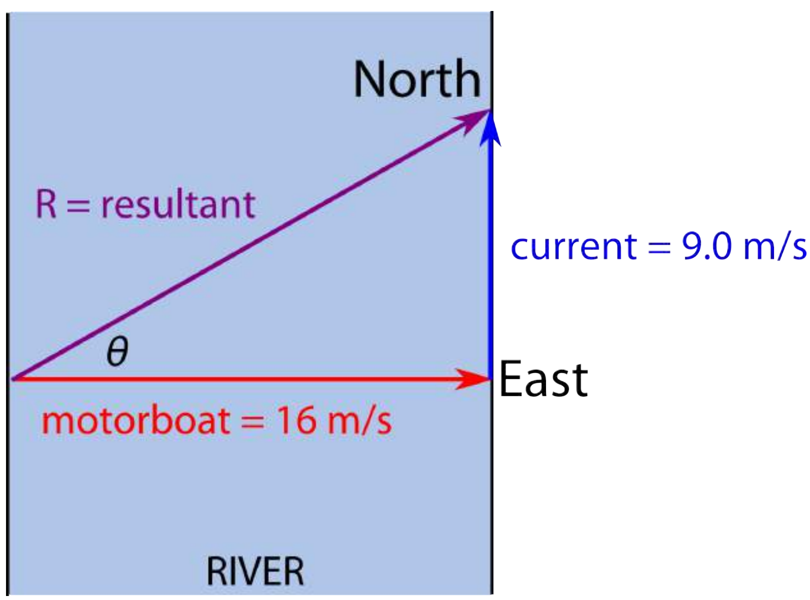
(a) What is the resultant velocity of the boat?

(b) If the river is 135 m wide, how long does it take the boat to reach the other side?

(c) When the boat reaches the other side, how far downstream will it be?

Solution:

Sketch:



(a) Since the two motions are perpendicular to each other, they can be assigned to the legs of a right triangle and the hypotenuse (resultant) calculated.

$c = \sqrt{a^2 + b^2} = \sqrt{(16 \text{ m/s})^2 + (9.0 \text{ m/s})^2} = 18 \text{ m/s}$

$\sin\theta = \frac{9.0}{18} = 0.500$ and therefore $\theta = 30°$

The resultant is 18 m/s at 30° north of east.

(b) The boat is traveling across the river at 16 m/s due to the motor. The current is perpendicular and therefore has no effect on the speed across the river. The time required for the trip can be determined by dividing the distance by the velocity.

$t = \frac{d}{v} = \frac{135 \text{ m}}{16 \text{ m/s}} = 8.4 \text{ s}$

(c) The boat is traveling across the river for 8.4 seconds and therefore, it is also traveling downstream for 8.4 seconds. We can determine the distance downstream the boat will travel by multiplying the speed downstream by the time of the trip.

$d_{\text{downstream}} = (v_{\text{downstream}})(t) = (9.0 \text{ m/s})(8.4 \text{ s}) = 76 \text{ m}$

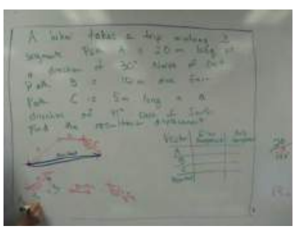
Summary

- Vectors can be added mathematically using geometry and trigonometry.
- Vectors that are perpendicular to each other have no effect on each other.
- Vector addition can be accomplished by resolving into **axial components** those vectors that are to be added, adding up the axial components, and then combining the two axial components.

Practice

A video demonstrating the component method of vector addition.

http://www.youtube.com/watch?v=nFDzRWw08Ew



MEDIA

Click image to the left for more content.

Review

1. A hiker walks 11 km due north from camp and then turns and walks 11 km due east.
   (a) What is the total distance walked by the hiker?
   (b) What is the displacement (on a straight line) of the hiker from the camp?
2. While flying due east at 33 m/s, an airplane is also being carried due north at 12 m/s by the wind. What is the plane's resultant velocity?
3. Two students push a heavy crate across the floor. John pushes with a force of 185 N due east and Joan pushes with a force of 165 N at 30° north of east. What is the resultant force on the crate?
4. An airplane flying due north at 90. km/h is being blown due west at 50. km/h. What is the resultant velocity of the plane?
5. A golf ball is struck with a golf club and travels in a parabolic curve. The horizontal distance traveled by the golf ball is 240 meters and the time of flight is 4.00 seconds. What was the initial velocity magnitude and direction?

- **axial component:** A component situated in or on an axis.
- **resolution of vectors:** Any vector directed at an angle to the horizontal (or the vertical) can be thought of as having two parts (or components) that lie on the axes (one horizontal and one vertical). The process of identifying these two components is known as the resolution of the vector.

References

1. CK-12 Foundation - Richard Parsons. . CC-BY-NC-SA 3.0
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Physics Unit 4 (Two Dimensional Motion)

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James Dann, Ph.D.

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4	**Projectile Motion Problem Solving**	**11**

CHAPTER 1 Projectile Motion

- Describe projectile motion and state when it occurs.
- Give examples of projectile motion.

The archer in the opening image is aiming his arrow a little bit above the bull's eye of the target, rather than directly at it. Why doesn't he aim at the bull's eye instead? The answer is projectile motion.

Combining Forces

When the archer releases the bowstring, the arrow will be flung forward toward the top of the target where she's aiming. But another force will also act on the arrow in a different direction. The other force is gravity, and it will pull the arrow down toward Earth. The two forces combined will cause the arrow to move in the curved path shown in the **Figure** 1.1. This type of motion is called **projectile motion**. It occurs whenever an object curves down toward the ground because it has both a horizontal force and the downward force of gravity acting on it.

Because of projectile motion, to hit the bull's eye of a target with an arrow, you actually have to aim for a spot above the bull's eye. You can see in the**Figure** 1.2 what happens if you aim at the bull's eye instead of above it.

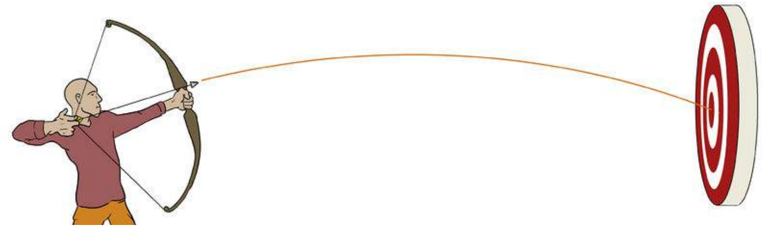


FIGURE 1.1

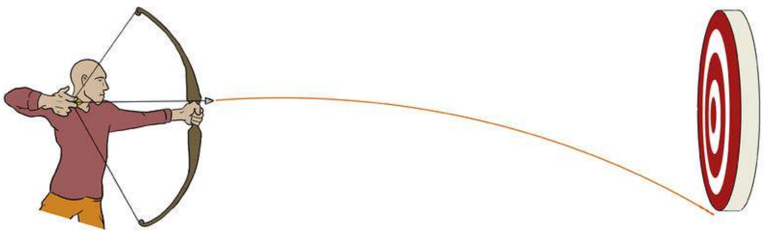


FIGURE 1.2

Another Example of Projectile Motion

You can probably think of other examples of projectile motion. One is shown in the **Figure** 1.3. The cannon shoots a ball straight ahead, giving it horizontal motion. At the same time, gravity pulls the ball down toward the ground.

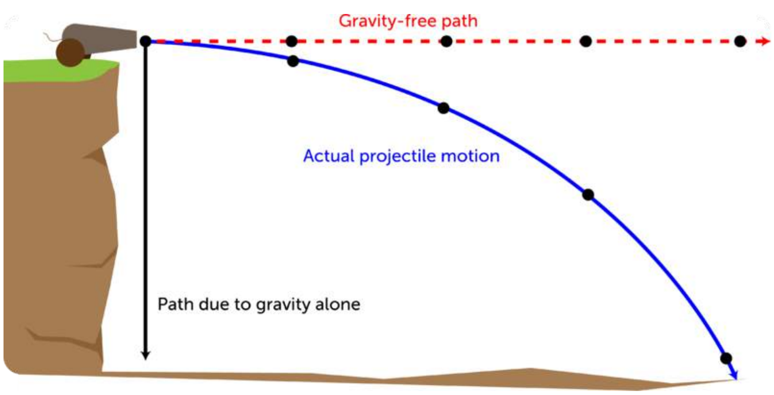


FIGURE 1.3

Q: How would you show the force of gravity on the cannon ball in the **Figure** 1.3?

A: You would add a line pointing straight down from the cannon to the ground.

To get a better feel for projectile motion, try these interactive animations:

- http://phet.colorado.edu/en/simulation/projectile-motion
- http://jersey.uoregon.edu/vlab/ (Click on the applet "Cannon.")

Summary

- Projectile motion is movement of an object in a curved path toward the ground because it has both a horizontal force and the downward force of gravity acting on it.

- Examples of objects that have projectile motion include arrows and cannon balls.

Vocabulary

- **projectile motion**: Motion of an object that has initial horizontal velocity but is also pulled down toward Earth by gravity.

Practice

Play the game at the following URL by shooting the cannon at a stationary target. Experiment with three variablespower, height of barrel, and angle of barrelyou find at least three different combinations of variables that allow the cannon ball to hit the target. Record the values for the three combinations of variables. Then summarize what you learned by doing the activity.

http://www.science-animations.com/support-files/projektielbeweging.swf

Review

1. What is projectile motion? When does it occur?
2. How might knowledge of projectile motion help you shoot baskets in basketball?

References

1. Laura Guerin. . CC BY-NC 3.0
2. Laura Guerin. . CC BY-NC 3.0
3. Christopher Auyeung. . CC-BY-NC-SA 3.0

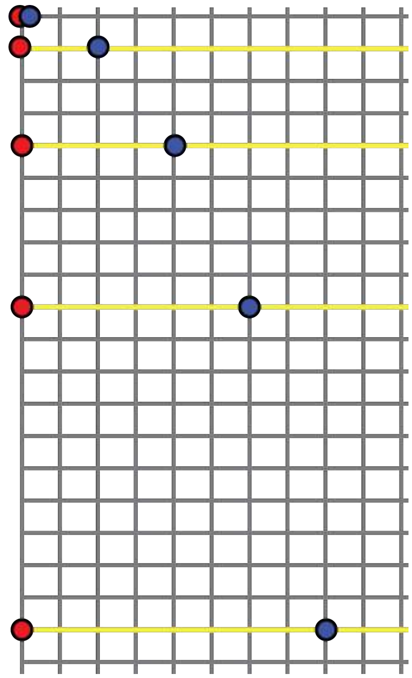
CHAPTER 2 Projectile Motion for an Object Launched Horizontally

- State the relationship between the vertical and horizontal velocities of a projectile launched horizontally.
- Find the time for a horizontally launched projectile to strike the ground.
- Calculate the range of a horizontally launched projectile.

The activity of bike jumping, like other sports that involve vector motions in perpendicular directions, requires more physical practice than mathematical analysis. The laws of physics apply to the activity, however, whether the biker is aware of them or not.

Projectile Motion for an Object Launched Horizontally

Objects that are launched into the air are called projectiles. The path followed by an object in **projectile motion** is called a **trajectory**. The motion of a projectile is described in terms of its position, velocity, and acceleration. Our knowledge that perpendicular components of vectors do not affect each other allow us to analyze the motion of projectiles.



In the diagram, two balls (one red and one blue) are dropped at the same time. The red ball is released with no horizontal motion and the blue ball is dropped but also given a horizontal velocity of 10 m/s. As the balls fall to the floor, a photograph is taken every second so that in 5 seconds, we have 5 images of the two balls. Each vertical line on the diagram represents 5 m. Since the blue ball has a horizontal velocity of 10 m/s, you will see that for every second, the blue ball has moved horizontally 10 m. That is, in each second, the blue ball has increased its horizontal distance by 10 m. This horizontal motion is constant velocity motion.

The red ball was dropped straight down with no horizontal velocity and therefore, in each succeeding second, the red ball falls straight down with no horizontal motion. The succeeding distances between seconds with the red ball motion indicates this motion is accelerated.

A very important point here is that, the vertical motion of these two balls is identical. That is, they each fall exactly the same distance vertically in each succeeding second. The constant horizontal velocity of the blue ball has no effect on its accelerated vertical motion!

Therefore, the vertical motion of the blue ball (the projectile) can be analyzed exactly the same as the vertical motion of the red ball.

Example Problem: If an arrow if fired from a bow with a perfectly horizontal velocity of 60.0 m/s and the arrow was 2.00 m above the ground when the it was released, how far will the arrow fly horizontally before it strikes the ground?

Solution: This problem is solved by determining how long it takes the arrow to fall to the ground in exactly the same manner as if the arrow was dropped with no horizontal velocity. Then the time required for the fall is multiplied by the horizontal velocity to get the horizontal distance.

$d = \frac{1}{2}at^2$ solved for $t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{(2)(2.00\text{ m})}{9.80\text{ m/s}^2}} = 0.639\text{ s}$

The time required for the arrow to fall to the ground is the same time that the arrow flies horizontally at 60.0 m/s, so

$d_{\text{horizontal}} = (v_{\text{horizontal}})(\text{time}) = (60.0\text{ m/s})(0.639\text{ s}) = 38.3\text{ m}$

Example Problem: A rock was thrown horizontally from a 100.0 m high cliff. It strikes the ground 90.0 m from the base of the cliff. At what speed was it thrown?

Solution: We can calculate how long it takes for a rock to free fall 100.0 m and then divide this time into the horizontal distance to get the horizontal velocity.

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{(2)(100.0 \text{ m})}{9.80 \text{ m/s}^2}} = 4.52 \text{ s}$$
$$v = \frac{d}{t} = \frac{90.0 \text{ m}}{4.52 \text{ s}} = 19.9 \text{ m/s}$$

Summary

- Perpendicular components of vectors do not influence each other.
- The horizontal motion of a projectile does not influence its free fall.

Practice

The following video discusses projectile motion for projectiles launched horizontally.

http://www.youtube.com/watch?v=-uUsUaPJUc0

1. Why does speedy need to drive a convertible?
2. What is speedy doing in this video?
3. How does the horizontal velocity change during the fall?
4. How does the vertical velocity change during the fall?

Review

1. If a bullet is fired from a high powered rifle at the exact time a duplicate bullet is dropped by hand near the barrel of the rifle, which bullet will hit the ground first?
   (a) the one dropped straight down
   (b) the one fired horizontally
   (c) both will hit the ground at the same time
2. A cannon is fired from the edge of a small cliff. The height of the cliff is 80.0 m. The cannon ball is fired with a perfectly horizontal velocity of 80.0 m/s. How far will the cannon ball fly horizontally before it strikes the ground?
3. A cliff diver running 3.60 m/s dives out horizontally from the edge of a vertical cliff and reaches the water below 2.00 s later. How high is the cliff and how far from the base of the cliff did the diver hit the water?

- **projectile motion:** A form of motion where a particle (called a projectile) is thrown obliquely near the earth's surface, it moves along a curved path under the action of gravity. The path followed by a projectile motion is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning of the trajectory after which there is no interference apart from gravity.
- **trajectory:** The path followed by an object in projectile motion.

References

1. Courtesy of PDPhoto.org. http://www.pdphoto.org/PictureDetail.php?mat=&pg=7672. Public Domain
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

CHAPTER 3

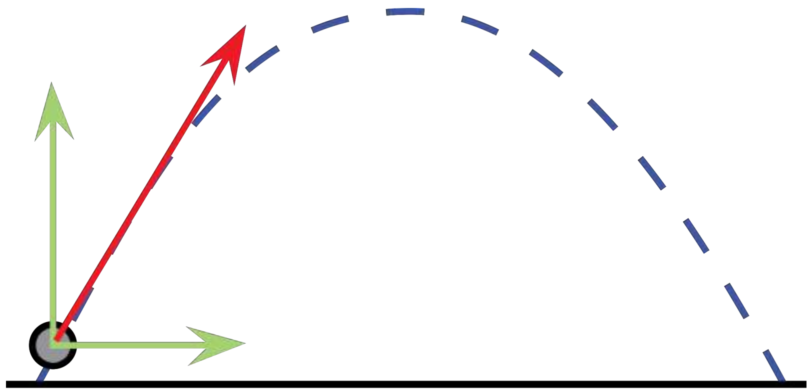
Projectile Motion for an Object Launched at an Angle

- The student will calculate the maximum height and range of projectiles launched at an angle given the initial velocity and angle.

In the case of the human cannonball shown, all the vector and gravitational calculations must be worked out perfectly before the first practice session. With this activity, you cannot afford trial and error –the first miss might be the last trial.

Projectile Motion for an Object Launched at an Angle

When an object is projected from rest at an upward angle, its initial velocity can be resolved into two components. These two components operate independently of each other. The upward velocity undergoes constant downward acceleration which will result in it rising to a highest point and then falling backward to the ground. The horizontal motion is constant velocity motion and undergoes no changes due to gravity. As usual, the analysis of the motion involves dealing with the two motions independently.



Example Problem: A cannon ball is fired with an initial velocity of 100. m/s at an angle of 45° above the horizontal. What maximum height will it reach and how far will it fly horizontally?

Solution: The first step in the analysis of this motion is to resolve the initial velocity into its vertical and horizontal components.

$v_{\text{i-up}} = (100.\text{ m/s})(\sin 45°) = (100.\text{ m/s})(0.707) = 70.7\text{ m/s}$

$v_{\text{i-horizontal}} = (100.\text{ m/s})(\cos 45°) = (100.\text{ m/s})(0.707) = 70.7\text{ m/s}$

We will deal with the vertical motion first. The vertical motion is symmetrical. The object will rise up to its highest point and then fall back. The distance it travels up will be the same as the distance it falls down. The time it takes to reach the top will be the same time it takes to fall back to its initial point. The initial velocity upward will be the same magnitude (opposite in direction) as the final velocity when it returns to its original height. There are several ways we could approach the upward motion. We could calculate the time it would take gravity to bring the initial velocity to rest. Or, we could calculate the time it would take gravity to change the initial velocity from +70.7 m/s to -70.0 m/s. Yet another way would be to calculate the time for the height of the object to return to zero.

$v_f = v_i + at \quad \text{so} \quad t = \frac{v_f - v_i}{a}$

If we calculate the time required for the ball to rise up to its highest point and come to rest, the initial velocity is 70.7 m/s and the final velocity is 0 m/s. Since we have called the upward velocity positive, then the acceleration must be negative or -9.80 m/s^2.

$t = \frac{v_f - v_i}{a} = \frac{0\text{ m/s} - 70.7\text{ m/s}}{-9.80\text{ m/s}^2} = 7.21\text{ s}$

The maximum height reached can be calculated by multiplying the time for the upward trip by the average vertical velocity. The average upward velocity during the trip up is one-half the initial velocity.

$v_{\text{up-ave}} = \left(\frac{1}{2}\right)(70.7\text{ m/s}) = 35.3\text{ m/s}$

$\text{height} = (v_{\text{up-ave}})(t_{\text{up}}) = (35.3\text{ m/s})(7.21\text{ s}) = 255\text{ m}$

Since this is the time required for the cannon ball to rise up to its highest point and come to rest, then the time required for the entire trip up and down would be double this value, or 14.42 s. The horizontal distance traveled during the flight is calculated by multiplying the total time times the constant horizontal velocity.

$d_{\text{horizontal}} = (14.42\text{ s})(70.7\text{ m/s}) = 1020\text{ m}$

Example Problem: A golf ball was knocked into the with an initial velocity of 4.47 m/s at an angle of 66° with the horizontal. How high did the ball go and how far did it fly horizontally?

Solution:

$v_{\text{i-up}} = (4.47\text{ m/s})(\sin 66°) = (4.47\text{ m/s})(0.913) = 4.08\text{ m/s}$

$v_{\text{i-hor}} = (4.47\text{ m/s})(\cos 66°) = (4.47\text{ m/s})(0.407) = 1.82\text{ m/s}$

$t_{\text{up}} = \frac{v_f - v_i}{a} = \frac{0 \text{ m/s} - 4.08 \text{ m/s}}{-9.80 \text{ m/s}^2} = 0.416 \text{ s}$

$v_{\text{up-ave}} = \left(\frac{1}{2}\right)(4.08 \text{ m/s}) = 2.04 \text{ m/s}$

$\text{height} = (v_{\text{up-ave}})(t_{\text{up}}) = (2.04 \text{ m/s})(0.416 \text{ s}) = 0.849 \text{ m}$

$t_{\text{total trip}} = (2)(0.416 \text{ s}) = 0.832 \text{ s}$

$d_{\text{horizontal}} = (0.832 \text{ s})(1.82 \text{ m/s}) = 1.51 \text{ m}$

Example Problem: Suppose a cannon ball is fired downward from a 50.0 m high cliff at an angle of 45° with an initial velocity of 80.0 m/s. How far horizontally will it land from the base of the cliff?

Solution: In this case, the initial vertical velocity is downward and the acceleration due to gravity will increase this downward velocity.

$v_{\text{i-down}} = (80.0 \text{ m/s})(\sin 45°) = (80.0 \text{ m/s})(0.707) = 56.6 \text{ m/s}$

$v_{\text{i-hor}} = (80.0 \text{ m/s})(\cos 45°) = (80.0 \text{ m/s})(0.707) = 56.6 \text{ m/s}$

$d = v_{\text{i-down}}t + \frac{1}{2}at^2$

$50.0 = 56.6t + 4.9t^2$

Changing to standard quadratic form yields $4.9t^2 + 56.6t - 50.0 = 0$

This equation can be solved with the quadratic formula. The quadratic formula will produce two possible solutions for t:

$t = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $t = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-56.6 + \sqrt{(56.6)^2 - (4)(4.9)(-50)}}{(2)(4.9)} = 0.816 \text{ s}$

The other solution to the quadratic formula yields a negative number which is clearly not a reasonable solution for this problem.

$d_{\text{horizontal}} = (0.816 \text{ s})(56.6 \text{ m/s}) = 46.2 \text{ m}$

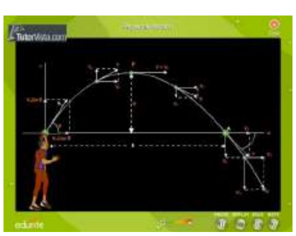
Summary

- When an object is projected from rest at an upward angle, its initial velocity can be resolved into two components. These two components operate independently of each other.
- The upward velocity undergoes constant downward acceleration which will result in it rising to a highest point and then falling backward to the ground.
- The horizontal motion is constant velocity motion and undergoes no changes due to gravity.
- The analysis of the motion involves dealing with the two motions independently.

Practice

The following video shows a motion analysis for projectile motion launched upward.

http://www.youtube.com/watch?v=rMVBc8cE5GU



MEDIA

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The following video shows the famous "shoot the monkey" demonstration. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=cxvsHNRXLjw

In this demonstration, a stuffed toy is hung from a high support and is attached to the support by an electric switch. A golf ball cannon is aimed up at the "monkey" while it is hanging on the support. The cannon is designed such that when the golf ball projectile leaves the barrel, it triggers the switch and releases the toy monkey from its perch. It would seem (to the non-physicist) that the projectile will miss the monkey because the monkey will fall under the line of fire. The physicist knows, however, that the projectile falls from its line of fire by exactly the same amount that the monkey falls and therefore, the projectile will hit the monkey every time . . . in fact, it cannot miss.

1. What is the cannon ball in this video?
2. What is used as the monkey in this video?

Review

1. A player kicks a football from ground level with a velocity of magnitude 27.0 m/s at an angle of 30.0° above the horizontal.
   (a) Find the time the ball is in the air.
   (b) Find the maximum height of the ball.
   (c) Find the horizontal distance the ball travels.
2. A person standing on top of a 30.0 m high building throws a ball with an initial velocity of 20. m/s at an angle of 20.0° below horizontal. How far from the base of the building will the ball land?
3. An arrow is fired downward at an angle of 45 degrees from the top of a 200 m cliff with a velocity of 60.0 m/s.
   (a) How long will it take the arrow to hit the ground?
   (b) How far from the base of the cliff will the arrow land?

- **trajectory:** The ballistic trajectory of a projectile is the path that a thrown or launched projectile will take under the action of gravity, neglecting all other forces, such as air resistance, without propulsion.

References

1. Flickr: JoshBerglund19. http://www.flickr.com/photos/tyrian123/1539636464/. CC-BY 2.0
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

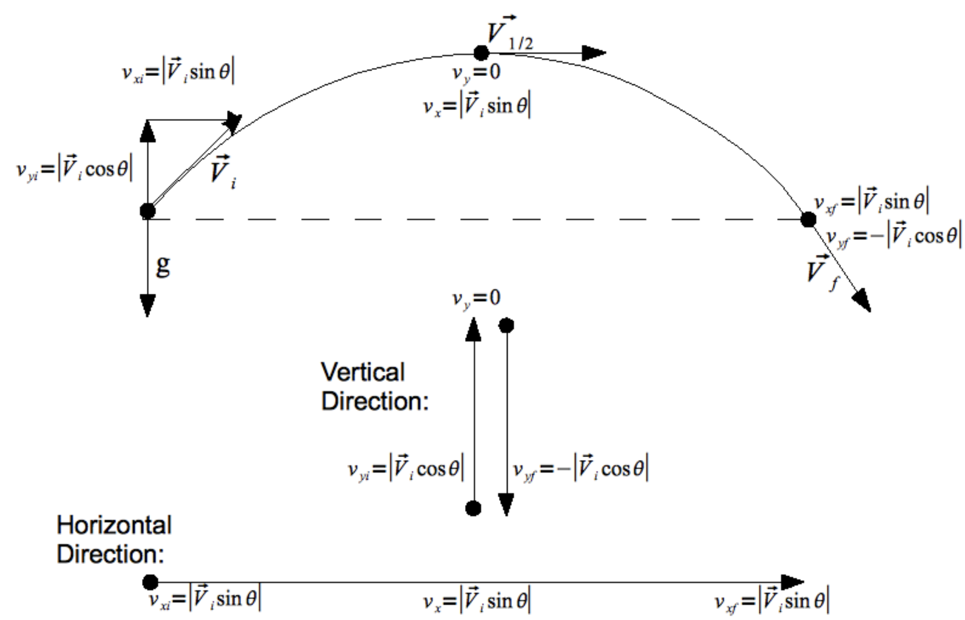
CHAPTER 4 Projectile Motion Problem Solving

Students will learn how to use the equations of motion in two dimensions in order to solve problems for projectiles. It is necessary to understand how to break a vector into its x and y components.

Students will learn how to use the equations of motion in two dimensions in order to solve problems for projectiles. It is necessary to understand how to break a vector into its x and y components.

Key Equations

Break the Initial Velocity Vector into its Components



Apply the Kinematics Equations

Vertical Direction	Horizontal Direction
$y(t) = y_i + v_{iy}t - \frac{1}{2}gt^2$	$x(t) = x_i + v_{ix}t$
$v_y(t) = v_{iy} - gt$	$v_x(t) = v_{ix}$
${v_y}^2 = {v_{0y}}^2 - 2g(\Delta y)$	
$a_y = -g = -9.8\text{m/s}^2 \approx -10\text{m/s}^2$	$a_x = 0$

Guidance

- To work these problems, separate the "Big Three" equations into two sets: one for the vertical direction, and one for the horizontal. Keep them separate.
- The only variable that can go into both sets of equations is time; use time to communicate between the x and y components of the object's motion.

Example 1

CSI discovers a car at the bottom of a 72 m cliff. How fast was the car going if it landed 22m horizontally from the cliff's edge? (Note that the cliff is flat, i.e. the car came off the cliff horizontally).

Question: $v = ?\ [m/s]$

Given: $h = \Delta y = 72\ m$

$d = \Delta x = 22\ m$

$g = 10.0\ m/s^2$

Equation: $h = v_{iy}t + \frac{1}{2}gt^2$ and $d = v_{ix}t$

Plug n' Chug: Step 1: Calculate the time required for the car to freefall from a height of 72 m.

$h = v_{iy}t + \frac{1}{2}gt^2$ but since $v_{iy} = 0$, the equation simplifies to $h = \frac{1}{2}gt^2$ rearranging for the unknown variable, t, yields

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(72\ m)}{10.0\ m/s^2}} = 3.79\ s$

Step 2: Solve for initial velocity:

$v_{ix} = \frac{d}{t} = \frac{22\ m}{3.79\ s} = 5.80\ m/s$

Answer:

$$\boxed{\mathbf{5.80\ m/s}}$$

Example 2

Question: A ball of mass m is moving horizontally with a speed of v_i off a cliff of height h. How much time does it take the ball to travel from the edge of the cliff to the ground? Express your answer in terms of g (acceleration due to gravity) and h (height of the cliff).

Solution: Since we are solving or how long it takes for the ball to reach ground, any motion in the x direction is not pertinent. To make this problem a little simpler, we will define down as the positive direction and the top of the cliff to be

$$y = 0$$

. In this solution we will use the equation

$$y(t) = y_o + v_{oy}t + \frac{1}{2}gt^2$$

.

$y(t) = y_o + v_{oy}t + \frac{1}{2}gt^2$ start with the equation

$h = y_o + v_{oy}t + \frac{1}{2}gt^2$ substitute h for $y(t)$ because that's the position of the ball when it hits the ground after time t

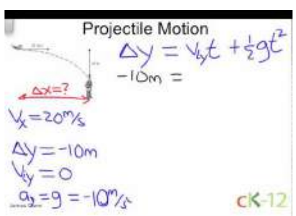
$h = 0 + v_{oy} + \frac{1}{2}gt^2$ substitute 0 for $y_o$because the ball starts at the top of the cliff

$h = 0 + 0 + \frac{1}{2}gt^2$ substitute 0 for v_{oy} becauese the ball starts with no vertical component to it's velocity

$h = \frac{1}{2}gt^2$ simplify the equation

$t = \sqrt{\frac{2h}{g}}$ solve for t

Watch this Explanation



MEDIA

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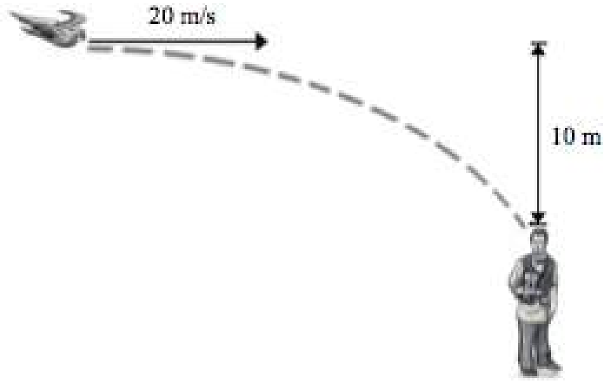
Time for Practice

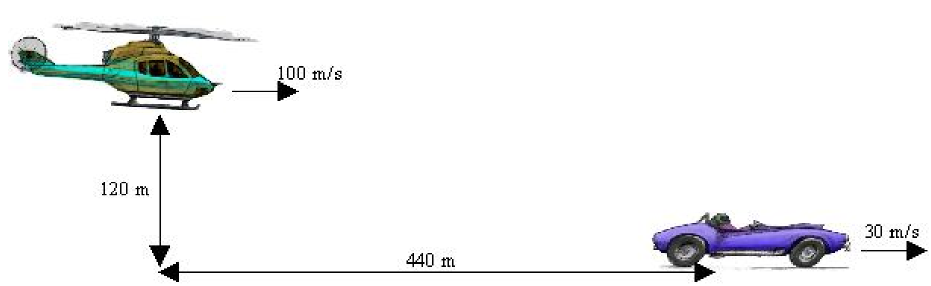
1. A stone is thrown horizontally at a speed of 8.0 m/s from the edge of a cliff 80 m in height. How far from the base of the cliff will the stone strike the ground?
2. A toy truck moves off the edge of a table that is 1.25 m high and lands 0.40 m from the base of the table.

   a. How much time passed between the moment the car left the table and the moment it hit the floor?
   b. What was the horizontal velocity of the car when it hit the ground?

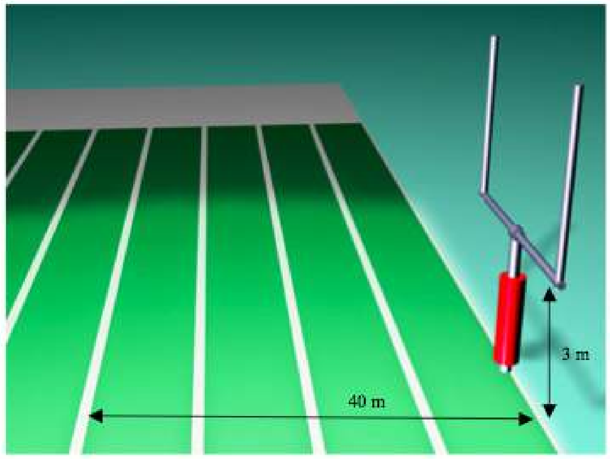
3. A hawk in level flight 135 m above the ground drops the fish it caught. If the hawk's horizontal speed is 20.0 m/s, how far ahead of the drop point will the fish land?
4. A pistol is fired horizontally toward a target 120 m away, but at the same height. The bullet's velocity is 200 m/s. How long does it take the bullet to get to the target? How far below the target does the bullet hit?
5. A bird, traveling at 20 m/s, wants to hit a waiter 10 m below with his dropping (see image). In order to hit the waiter, the bird must release his dropping some distance before he is directly overhead. What is this distance?



6. Joe Nedney of the *San Francisco 49ers* kicked a field goal with an initial velocity of 20 m/s at an angle of 60°.
   a. How long is the ball in the air? *Hint:* you may assume that the ball lands at same height as it starts at.
   b. What are the range and maximum height of the ball?
7. A racquetball thrown from the ground at an angle of 45° and with a speed of 22.5 m/s lands exactly 2.5 s later on the top of a nearby building. Calculate the horizontal distance it traveled and the height of the building.
8. Donovan McNabb throws a football. He throws it with an initial velocity of 30 m/s at an angle of 25°. How much time passes until the ball travels 35 m horizontally? What is the height of the ball after 0.5 seconds? (Assume that, when thrown, the ball is 2 m above the ground.)
9. Pablo Sandoval throws a baseball with a horizontal component of velocity of 25 m/s. After 2 seconds, the ball is 40 m above the release point. Calculate the horizontal distance it has traveled by this time, its initial vertical component of velocity, and its initial angle of projection. Also, is the ball on the way up or the way down at this moment in time?
10. Barry Bonds hits a 125 m$(450')$ home run that lands in the stands at an altitude 30 m above its starting altitude. Assuming that the ball left the bat at an angle of 45° from the horizontal, calculate how long the ball was in the air.
11. A golfer can drive a ball with an initial speed of 40.0 m/s. If the tee and the green are separated by 100 m, but are on the same level, at what angle should the ball be driven? (*Hint:* you should use $2\cos(x)\sin(x) = \sin(2x)$ at some point.)
12. How long will it take a bullet fired from a cliff at an initial velocity of 700 m/s, at an angle 30° below the horizontal, to reach the ground 200 m below?
13. A diver in Hawaii is jumping off a cliff 45 m high, but she notices that there is an outcropping of rocks 7 m out at the base. So, she must clear a horizontal distance of 7 m during the dive in order to survive. Assuming the diver jumps horizontally, what is his/her minimum push-off speed?
14. If Monte Ellis can jump 1.0 m high on Earth, how high can he jump on the moon assuming same initial velocity that he had on Earth (where gravity is 1/6 that of Earth's gravity)?
15. James Bond is trying to jump from a helicopter into a speeding Corvette to capture the bad guy. The car is going 30.0 m/s and the helicopter is flying completely horizontally at 100 m/s. The helicopter is 120 m above the car and 440 m behind the car. How long must James Bond wait to jump in order to safely make it into the car?



16. A field goal kicker lines up to kick a 44 yard (40 m) field goal. He kicks it with an initial velocity of 22 m/s at an angle of 55°. The field goal posts are 3 meters high.



   a. Does he make the field goal?
   b. What is the ball's velocity and direction of motion just as it reaches the field goal post (*i.e.,* after it has traveled 40 m in the horizontal direction)?

17. In a football game a punter kicks the ball a horizontal distance of 43 yards (39 m). On TV, they track the hang time, which reads 3.9 seconds. From this information, calculate the angle and speed at which the ball was kicked. *(Note for non-football watchers: the projectile starts and lands at the same height. It goes* 43 *yards horizontally in a time of* 3.9 *seconds)*

Answers to Selected Problems

1. 32 m
2. a. 0.5 s b. 0.8 m/s
3. 104 m
4. $t = 0.60$ s, 1.8 m below target
5. 28 m.
6. a. 3.5 s. b. 35 m; 15 m
7. 40 m; 8.5 m
8. 1.3 seconds, 7.1 meters
9. 50 m; $v_{0y} = 30$ m/s; 50^0; on the way up
10. 4.4 s
11. 19°
12. 0.5 s
13. 2.3 m/s
14. 6 m
15. 1.4 seconds
16. a. yes b. 14 m/s @ 23 degrees from horizontal
17. 22 m/s @ 62 degrees

CK-12 FlexBook

Physics Unit 5 (Forces and Newton's Laws)

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CHAPTER 1 Newton's First and Second Laws of Motion

- Define force.
- State the fundamental units for the Newton.
- State Newton's First Law of Motion.
- Given two of the three values in $F = ma$, calculate the third.

This image is of Buzz Aldrin, one of the first men to walk on the moon. Apollo 11 was the spaceflight that landed the first humans, Neil Armstrong and Buzz Aldrin, on the Moon on July 20, 1969. Armstrong became the first to step onto the lunar surface 6 hours later on July 21.

This accomplishment could not have occurred without a thorough understanding of forces and acceleration.

Newton's First and Second Laws of Motion

What is a **force**? A force can be defined as a push or pull. When you place a book on a table, the book pushes downward on the table and the table pushes upward on the book. The two forces are equal and there is no resulting motion of the book. If, on the other hand, you drop the book, it will fall to the ground pulled by a force called gravity.

If you slide a book across the floor, it will experience a force of friction which acts in the opposite direction of the motion. This force will slow down the motion of the book and eventually bring it to rest. If the floor is smoother, the force of friction will be less and the book will slide further before coming to rest. If a perfectly smooth floor could be created, there would be no friction and the book would slide forever at constant speed.

Newton's First Law of Motion describes an object moving with constant speed in a straight line. In the absence of any force, the object will continue to move at the same constant speed and in the same straight line. If the object is at rest, in the absence of any force, it will remain at rest. Newton's First Law states that an object with no force acting on it moves with constant velocity. (The constant velocity could, of course, be 0 m/s.) A more careful expression of Newton's First Law is "an object with no net force acting on it remains at rest or moves with constant velocity in a straight line."

The statement above is equivalent to a statement that "if there is no net force on an object, there will be no acceleration." In the absence of acceleration, an object will remain at rest or will move with constant velocity in a straight line. The acceleration of an object is the result of an unbalanced force. If an object suffers two forces, the motion of the object is determined by the net force. The magnitude of the acceleration is directly proportional to the magnitude of the unbalanced force. The direction of the acceleration is the same direction as the direction of the unbalanced force. The magnitude of the acceleration is inversely proportional to the mass of the object. i.e. The more massive the object, the smaller will be the acceleration produced by the same force.

These relationships are stated in Newton's Second Law of Motion, "the acceleration of an object is directly proportional to the net force on the object and inversely proportional to the mass of the object."

Newton's Second Law can be summarized in an equation.

$a = \frac{F}{m}$ or more commonly, $F = ma$

According to Newton's second law, a new force on an object causes it to accelerate and the larger the mass, the smaller the acceleration. Sometimes, the word inertia is used to express the resistance of an object to acceleration. Therefore, we say that a more massive object has greater inertia.

The units for force are defined by the equation for Newton's second law. Suppose we wish to express the force that will give a 1.00 kg object an acceleration of 1.00 m/s^2.

$F = ma = (1.00\ \mathbf{kg})(1.00\ \mathbf{m/s}^2) = 1.00\ \mathbf{kg}\cdot\mathbf{m/s}^2$

This unit is defined as 1.00 Newton or 1.00 N.

$\frac{kg\cdot m}{s^2} = \text{Newton}$

Example Problem: What new force is required to accelerate a 2000. kg car at 2.000 m/s^2?

Solution: $F = ma = (2000.\ \text{kg})(2.000\ \text{m/s}^2) = 4000.\ \text{N}$

Example Problem: A net force of 150 N is exerted on a rock. The rock has an acceleration of 20. m/s^2 due to this force. What is the mass of the rock?

Solution: $m = \frac{F}{a} = \frac{(150\ \text{N})}{(20.\ \text{m/s}^2)} = 7.5\ \text{kg}$

Example Problem: A net force of 100. N is exerted on a ball. If the ball has a mass of 0.72 kg, what acceleration will it undergo?

Solution: $a = \frac{F}{m} = \frac{(100.\ \text{N})}{(0.72\ \text{kg})} = 140\ \text{m/s}^2$

Summary

- A force is a push or pull.
- Newton's First Law states that an object with no net force acting on it remains at rest or moves with constant velocity in a straight line.
- Newton's Second Law of Motion states that the acceleration of an object is directly proportional to the net

force on the object and inversely proportional to the mass of the object. Expressed as an equation, $F = ma$.

Practice

Professor Mac explains Newton's Second Law of Motion.

http://www.youtube.com/watch?v=-KxbIIw8hlc

MEDIA

Click image to the left for more content.

Review

1. A car of mass 1200 kg traveling westward at 30. m/s is slowed to a stop in a distance of 50. m by the car's brakes. What was the braking force?
2. Calculate the average force that must be exerted on a 0.145 kg baseball in order to give it an acceleration of 130 m/s^2.
3. After a rocket ship going from the Earth to the Moon leaves the gravitational pull of the Earth, it can shut off its engine and the ship will continue on to the Moon due to the gravitational pull of the Moon.
   (a) True
   (b) False
4. If a space ship traveling at 1000 miles per hour enters an area free of gravitational forces, its engine must run at some minimum level in order to maintain the ships velocity.
   (a) True
   (b) False
5. Suppose a space ship traveling at 1000 miles per hour enters an area free of gravitational forces and free of air resistance. If the pilot wishes to slow the ship down, he can accomplish that by shutting off the engine for a while.
   (a) True
   (b) False

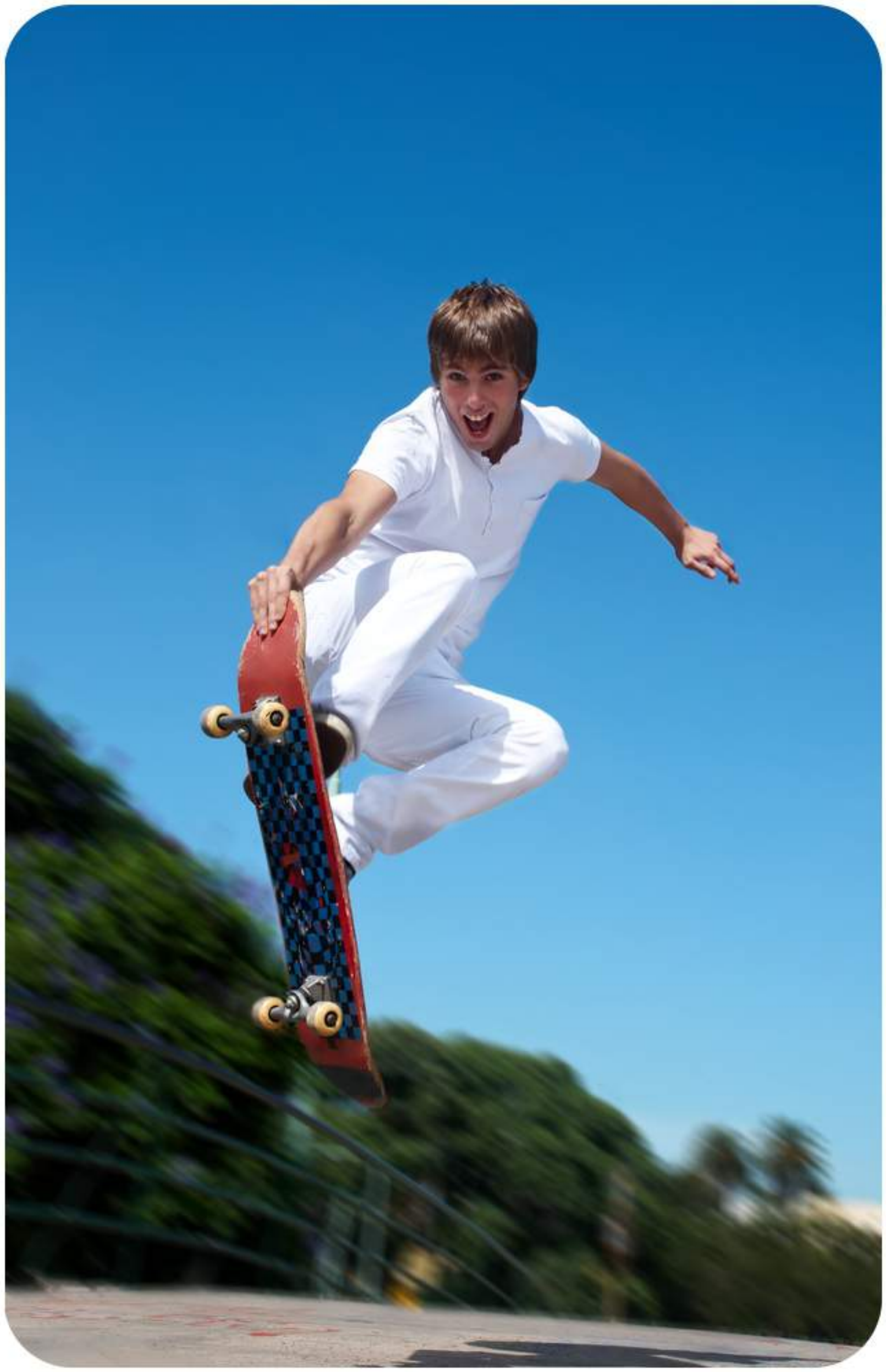
- **force:** A push or pull.

References

1. Courtesy of Neil A. Armstrong, NASA. http://spaceflight.nasa.gov/gallery/images/apollo/apollo11/html/as11-40-5873.html. Public Domain

Chapter 2 Newton's First Law

- Use skateboarding to explain Newton's first law of motion.



There's no doubt from Corey's face that he loves skateboarding! Corey and his friends visit Newton's Skate Park every chance they get. They may not know it, but while they're having fun on their skateboards, they're actually applying science concepts such as forces and motion.

Starting and Stopping

Did you ever ride a skateboard? Even if you didn't, you probably know that to start a skateboard rolling over a level surface, you need to push off with one foot against the ground. That's what Corey's friend Nina is doing in this picture 2.1.

FIGURE 2.1

Do you know how to stop a skateboard once it starts rolling? Look how Nina's friend Laura does it in the **Figure** 2.2. She steps down on the back of the skateboard so it scrapes on the pavement. This creates friction, which stops the skateboard.

Even if Laura didn't try to stop the skateboard, it would stop sooner or later. That's because there's also friction between the wheels and the pavement. Friction is a force that counters all kinds of motion. It occurs whenever two surfaces come into contact.

Video Break

Laura learned how to use forces to start and stop her skateboard by watching the videos below. Watch the video to see how the forces are applied. You can pick up some skateboarding tips at the same time!

Starting: http://www.youtube.com/watch?v=OpZIVjbMAOU

MEDIA

Click image to the left for more content.

Stopping: http://www.youtube.com/watch?v=6fuOwhx91zM

FIGURE 2.2

MEDIA
Click image to the left for more content.

Laws of the Park: Newton's First Law

If you understand how a skateboard starts and stops, then you already know something about **Newton's first law of motion**. This law was developed by English scientist Isaac Newton around 1700. Newton was one of the greatest scientists of all time. He developed three laws of motion and the law of gravity, among many other contributions.

Newton's first law of motion states that an object at rest will remain at rest and an object in motion will stay in motion unless it is acted on by an unbalanced force. Without an unbalanced force, a moving object will not only keep moving, but its speed and direction will also remain the same. Newton's first law of motion is often called the law of inertia because inertia is the tendency of an object to resist a change in its motion. If an object is already at rest, inertia will keep it at rest. If an object is already in motion, inertia will keep it moving.

Do You Get It?

Q: How does Nina use Newton's first law to start her skateboard rolling?

A: The skateboard won't move unless Nina pushes off from the pavement with one foot. The force she applies when she pushes off is stronger than the force of friction that opposes the skateboard's motion. As a result, the force on the skateboard is unbalanced, and the skateboard moves forward.

Q: How does Nina use Newton's first law to stop her skateboard?

A: Once the skateboard starts moving, it would keep moving at the same speed and in the same direction if not for another unbalanced force. That force is friction between the skateboard and the pavement. The force of friction is unbalanced because Nina is no longer pushing with her foot to keep the skateboard moving. That's why the skateboard stops.

Changing Direction

Corey's friend Jerod likes to skate on the flat banks at Newton's Skate Park. That's Jerod in the picture above. As he reaches the top of a bank, he turns his skateboard to go back down. To change direction, he presses down with his heels on one edge of the skateboard. This causes the skateboard to turn in the opposite direction.

Video Break

Can you turn a skateboard like Jerod? To see how to apply forces to change the direction of a skateboard, watch this video:

http://www.youtube.com/watch?v=iOnlcEk50CM

MEDIA

Click image to the left for more content.

Do You Get It?

Q: How does Jerod use Newton's first law of motion to change the direction of his skateboard?

A: Pressing down on just one side of a skateboard creates an unbalanced force. The unbalanced force causes the skateboard to turn toward the other side. In the picture, Jerod is pressing down with his heels, so the skateboard turns toward his toes.

Summary

- Newton's first law of motion states that an object at rest will remain at rest and an object in motion will remain in motion unless it is acted on by an unbalanced force.

FIGURE 2.3

- Using unbalanced forces to control the motion of a skateboard demonstrates Newton's first law of motion.

Vocabulary

- **Newton's first law of motion**: Law stating that an object's motion will not change unless an unbalanced force acts on the object.

Practice

Do you think you understand Newton's first law? Go to the URL below to find out. Review Newton's law and watch what happens to the skateboarder in the animation. Then answer the questions at the bottom of the Web page.

http://teachertech.rice.edu/Participants/louviere/Newton/law1.html

Review

1. State Newton's first law of motion.
2. You don't need to push off with a foot against the ground to start a skateboard rolling down a bank. Does this violate Newton's first law of motion? Why or why not?

FIGURE 2.4

3. Nina ran into a rough patch of pavement, but she thought she could ride right over it. Instead, the skateboard stopped suddenly and Nina ended up on the ground (see **Figure** above). Explain what happened.
4. Now that you know about Newton's first law of motion, how might you use it to ride a skateboard more safely?

References

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2. Image copyright DenisNata, 2012. . Used under license from Shutterstock.com
3. Image copyright Nikola Bilic, 2012. . Used under license from Shutterstock.com
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CHAPTER 3 Newton's Second Law

- State Newton's second law of motion.
- Compare and contrast the effects of force and mass on acceleration.

These boys are racing around the track at Newton's Skate Park. The boy who can increase his speed the most will win the race. Tony, who is closest to the camera in this picture, is bigger and stronger than the other two boys, so he can apply greater force to his skates.

Q: Does this mean that Tony will win the race?

A: Not necessarily, because force isn't the only factor that affects acceleration.

Force, Mass, and Acceleration

Whenever an object speeds up, slows down, or changes direction, it accelerates. Acceleration occurs whenever an unbalanced force acts on an object. Two factors affect the acceleration of an object: the net force acting on the object and the object's mass. **Newton's second law of motion** describes how force and mass affect acceleration. The law states that the acceleration of an object equals the net force acting on the object divided by the object's mass. This can be represented by the equation:

$\text{Acceleration} = \frac{\text{Net force}}{\text{Mass}}$

or $a = \frac{F}{m}$

Q: While Tony races along on his rollerblades, what net force is acting on the skates?

A: Tony exerts a backward force against the ground, as you can see in the **Figure** 3.1, first with one skate and then with the other. This force pushes him forward. Although friction partly counters the forward motion of the skates, it is weaker than the force Tony exerts. Therefore, there is a net forward force on the skates.

FIGURE 3.1

Direct and Inverse Relationships

Newton's second law shows that there is a direct relationship between force and acceleration. The greater the force that is applied to an object of a given mass, the more the object will accelerate. For example, doubling the force on the object doubles its acceleration.

The relationship between mass and acceleration is different. It is an inverse relationship. In an inverse relationship, when one variable increases, the other variable decreases. The greater the mass of an object, the less it will accelerate when a given force is applied. For example, doubling the mass of an object results in only half as much acceleration for the same amount of force.

Q: Tony has greater mass than the other two boys he is racing above. How will this affect his acceleration around the track?

A: Tony's greater mass will result in less acceleration for the same amount of force.

Summary

- Newton's second law of motion states that the acceleration of an object equals the net force acting on the object divided by the object's mass.
- According to the second law, there is a direct relationship between force and acceleration and an inverse relationship between mass and acceleration.

Vocabulary

- **Newton's second law of motion**: Law stating that the acceleration of an object equals the net force acting on the object divided by the object's mass.

Practice

At the following URL, use the simulator to experiment with force, mass, and acceleration. First keep force constant at 1 N, and vary mass from 1–5 kg. Next keep mass constant at 1 kg, and vary force from 1–5 N. In each simulation, record the values you tested and the resulting acceleration. Finally, make two line graphs to plot your results. On one graph, show acceleration when force is constant and mass changes. On the other graph, show acceleration when mass is constant and force changes. Describe in words what the two graphs show.

http://janggeng.com/newtons-second-law-of-motion/

Review

1. State Newton's second law of motion.
2. How can Newton's second law of motion be represented with an equation?
3. If the net force acting on an object doubles, how will the object's acceleration be affected?
4. Tony has a mass of 50 kg, and his friend Sam has a mass of 45 kg. Assume that both friends push off on their rollerblades with the same force. Explain which boy will have greater acceleration.

References

1. Uploaded by User:Shizhao/Wikimedia Commons. . CC BY 2.5

CHAPTER 4 Newton's Third Law of Motion

- Define force.
- State the fundamental units for the Newton.
- State Newton's First Law of Motion.
- Given two of the three values in $F = ma$, calculate the third

The image at above is a NASA artist's concept of a space elevator. It was imagined as a geo-stationary transfer station for passengers and cargo between earth and space. This idea was not pursued but it began where all great ideas begin . . . in someone's mind.

Newton's Third Law of Motion

Where do forces come from? Observations suggest that a force applied to an object is always applied by another object. A hammer strikes a nail, a car pulls a trailer, and a person pushes a grocery cart. Newton realized that forces are not so one sided. When the hammer exerts a force on the nail, the nail also exerts a force on the hammer –after all, the hammer comes to rest after the interaction. This led to **Newton's Third Law of Motion**, which states that whenever one object exerts a force on a second object, the second object also exerts a force on the first object, equal in magnitude and opposite in direction.

This law is sometimes paraphrased as "for every action, there is an equal and opposite reaction." A very important point to remember is that the two forces are on different objects –never on the same object. It is frequently the case that one of the objects moves as a result of the force applied but the motion of the other object in the opposite direction is not apparent. Consider the situation where an ice skater is standing at the edge of the skating rink holding on to the side rail. If the skater exerts a force on the rail, the rail is held in place with tremendous friction and therefore, will not move in any noticeable way. The skater, on the other hand, had little friction with the ice, and therefore will be accelerated in the direction opposite of his/her original push. This is the process people use to jump up into the air. The person's feet exert force on the ground and the ground exerts an equal and opposite force on the person's feet. The force on the feet is sufficient to raise the person off the ground. The force on the ground has little effect because the earth is so large. One of the accelerations is visible but the other is not visible.

A case where the reaction motion due to the reaction force is visible is the case of a person throwing a heavy object out of a boat. The object is accelerated in one direction and the boat is accelerated in the opposite direction. In this case, both the motion of the object is visible and the motion of the boat in the opposite direction is also visible.

Rockets also work in this manner. It is a misconception that the rocket moves forward because the escaping gas pushes on the ground or the surrounding air to make the rocket go forward. Rockets work in outer space where there is no ground or surrounding air. The rocket exerts a force on the gases causing them to be expelled and the gases exert a force on the rocket causing it to be accelerated forward.

Summary

- A force applied to an object is always applied by another object.
- Newton's Third Law of Motion says, "whenever one object exerts a force on a second object, the second object also exerts a force on the first object, equal in magnitude and opposite in direction."

Practice

The following video contains a discussion and an example of Newton's Third Law of Motion.

http://www.youtube.com/watch?v=fKJDpPi-UN0

MEDIA

Click image to the left for more content.

Review

1. What is wrong with the following statement: When you exert a force on a baseball, the equal and opposite force on the ball balances the original force and therefore, the ball will not accelerate in any direction.
2. When a bat strikes a ball, the force exerted can send the ball deep into the outfield. Where is the equal and opposite force in this case?
3. Suppose you wish to jump horizontally and in order for you to jump a distance of 4 feet horizontally, you must exert a force of 200 N. When you are standing on the ground, you have no trouble jumping 4 feet horizontally. If you are standing in a canoe, however, and you need to jump 4 feet to reach the pier, you will surely fall into the lake. Why is it that you cannot jump 4 feet out of a canoe when you can easily do this when on land?

- **Newton's Third Law of Motion:** Whenever one object exerts a force on a second object, the second object also exerts a force on the first object, equal in magnitude and opposite in direction.

References

1. Courtesy of Pat Rawling, NASA. http://commons.wikimedia.org/wiki/File:Nasa_space_elev.jpg. Public Domain

CHAPTER 5 Types of Forces

The various types of common forces are discussed and analyzed.

The various types of common forces are discussed and analyzed.

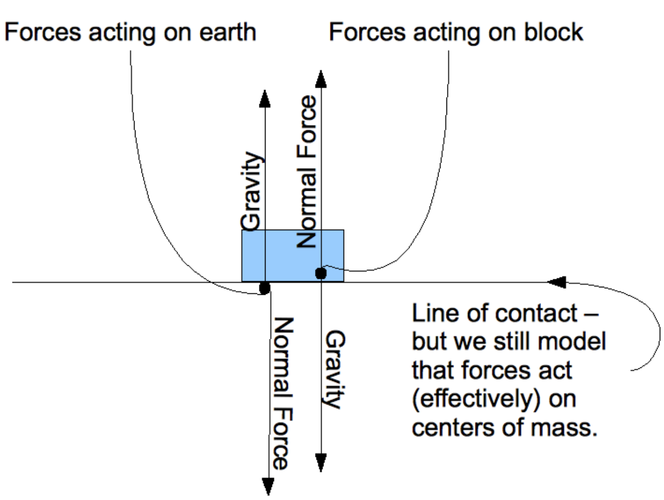
Key Equations

$$\text{Common Forces}\begin{cases} F_g = mg & \text{Gravity} \\ F_N & \text{Normal force: acts perpendicular to surfaces} \\ F_T & \text{Force of tension in strings and wires} \\ F_{sp} = -k\Delta x & \text{Force of spring stretched a distance } \Delta x \text{ from equilibrium} \end{cases}$$

Guidance

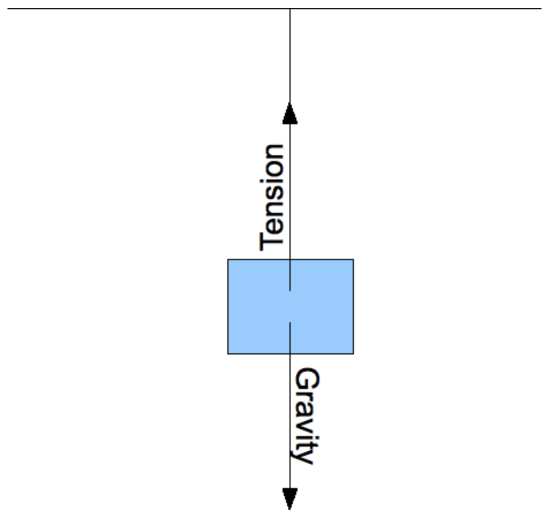
Normal Force

Often, objects experience a force that pushes them into another object, but once the objects are in contact they do not any move closer together. For instance, when you stand on the surface of the earth you are obviously not accelerating toward its center. According to Newton's Laws, there must be a force opposing the earth's gravity acting on you, so that the net force on you is zero. The same also applies for your gravity acting on the earth. We call such a force the **Normal Force**. The normal force acts between any two surfaces in contact, balancing what ever force is pushing the objects together. It is actually electromagnetic in nature (like other contact forces), and arises due to the repulsion of atoms in the two objects. Here is an illustration of the Normal force on a block sitting on earth:



Tension

Another force that often opposes gravity is known as **tension**. This force is provided by wires and strings when they hold objects above the earth. Like the Normal Force, it is electromagnetic in nature and arises due to the intermolecular bonds in the wire or string:



If the object is in equilibrium, tension must be equal in magnitude and opposite in direction to gravity. This force transfers the gravity acting on the object to whatever the wire or string is attached to; in the end it is usually a Normal Force — between the earth and whatever the wire is attached to — that ends up balancing out the force of gravity on the object.

Friction

Friction is a force that opposes motion. Any two objects in contact have what is called a mutual coefficient of friction. To find the force of friction between them, we multiply the normal force by this coefficient. Like the forces above, it arises due to electromagnetic interactions of atoms in two objects. There are actually two coefficients of friction: static and kinetic. Static friction will oppose *initial* motion of two objects relative to each other. Once the objects are moving, however, kinetic friction will oppose their continuing motion. Kinetic friction is lower than static friction, so it is easier to keep an object in motion than to set it in motion.

$$f_s \leq \mu_s|\vec{F}_N| \qquad [5] \text{ Static friction opposes potential motion of surfaces in contact}$$

$$f_k = \mu_k|\vec{F}_N| \qquad [6] \text{ Kinetic frictions opposes motion of surfaces in contact}$$

There are some things about friction that are not very intuitive:

- The magnitude of the friction force does not depend on the surface areas in contact.
- The magnitude of kinetic friction does not depend on the relative velocity or acceleration of the two objects.
- Friction always points in the direction opposing motion. If the net force (not counting friction) on an object is lower than the maximum possible value of static friction, friction will be equal to the net force in magnitude and opposite in direction.

Spring Force

Any spring has some equilibrium length, and if stretched in either direction it will push or pull with a force equal to:

$$\vec{F}_{sp} = -k\vec{\Delta x} \qquad [7] \text{ Force of spring } \vec{\Delta x} \text{ from equilibrium}$$

Example 1

Question: A woman of mass 70.0 kg weighs herself in an elevator.

a) If she wants to weigh less, should she weigh herself when accelerating upward or downward?
b) When the elevator is not accelerating, what does the scale read (i.e., what is the normal force that the scale exerts on the woman)?
c) When the elevator is accelerating upward at 2.00 m/s^2, what does the scale read?

Answer a) If she wants to weigh less, she has to decrease her force (her weight is the force) on the scale. We will use the equation

$$F = ma$$

to determine in which situation she exerts less force on the scale.

If the elevator is accelerating upward then the acceleration would be greater. She would be pushed toward the floor of the elevator making her weight increase. Therefore, she should weigh herself when the elevator is going down.

b) When the elevator is not accelerating, the scale would read 70.0kg.

c) If the elevator was accelerating upward at a speed of 2.00m/s^2, then the scale would read

$$F = ma = 70\text{kg} \times (9.8\text{m/s}^2 + 2\text{m/s}^2) = 826\text{N}$$

which is 82.6kg.

Example 2

Question: A spring with a spring constant of $k = 400\text{N/m}$ has an uncompressed length of .23m and a fully compressed length of .15m. What is the force required to fully compress the spring?

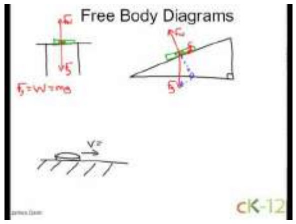
Solution: We will use the equation

$$F = kx$$

to solve this. We simply have to plug in the known value for the spring and the distance to solve for the force.

$$F = kx = (400\text{N/m})(.23\text{m} - .15\text{m}) = 32\text{N}$$

Watch this Explanation



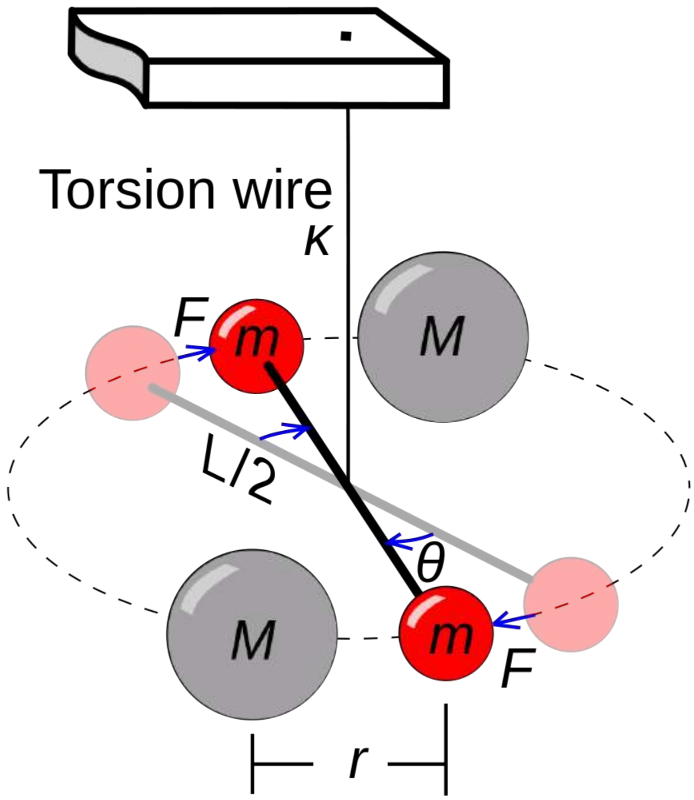
MEDIA
Click image to the left for more content.

Time for Practice

No Problems for this section. See Newton Law Problem Solving Concept.

CHAPTER 6 Universal Law of Gravity

- Describe and give the formula for Newton's universal law of gravity.
- Using Newton's law of gravity



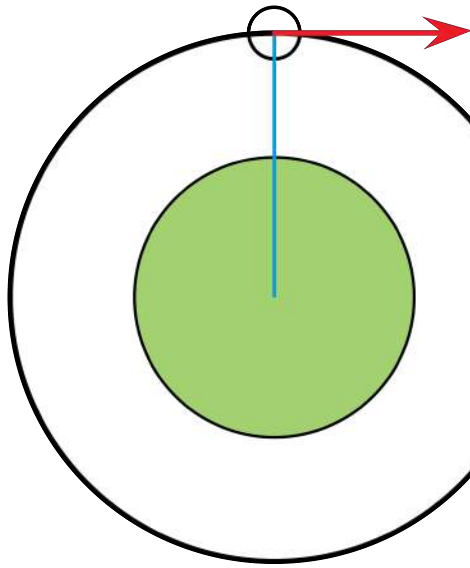
Cavendish's apparatus for experimentally determining the value of *G* involved a light, rigid rod about 2-feet long. Two small lead spheres were attached to the ends of the rod and the rod was suspended by a thin wire. When the rod becomes twisted, the torsion of the wire begins to exert a torsional force that is proportional to the angle of rotation of the rod. The more twist of the wire, the more the system pushes *backwards* to restore itself towards the original position. Cavendish had calibrated his instrument to determine the relationship between the angle of rotation and the amount of torsional force.

Force of Gravity

In the mid-1600's, Newton wrote that the sight of a falling apple made him think of the problem of the motion of the planets. He recognized that the apple fell straight down because the earth attracted it and thought this same force of attraction might apply to the moon and that motion of the planets might be controlled by the **gravity** of the sun. He eventually proposed the universal law of gravitational attraction as

$$F = G\frac{m_1 m_2}{d^2}$$

where m_1 and m_2 are the masses being attracted, d is the distance between the centers of the masses, G is the universal gravitational constant, and F is the force of attraction. The formula for gravitational attraction applies equally to two rocks resting near each other on the earth and to the planets and the sun. The value for the universal gravitational constant, G, was determined by Cavendish to be 6.67×10^{-11} N·m^2/kg^2.



The moon is being pulled toward the earth and the earth toward the moon with the same force but in the opposite direction. The force of attraction between the two bodies produces a greater acceleration of the moon than the earth because the moon has smaller mass. Even though the moon is constantly falling toward the earth, it never gets any closer. This is because the velocity of the moon is perpendicular to the radius of the earth (as shown in the image above) and therefore the moon is moving away from the earth. The distance the moon moves away from the orbit line is exactly the same distance that the moon falls in the time period. This is true of all satellites and is the reason objects remain in orbit.

Example Problem: Since we know the force of gravity on a 1.00 kg ball resting on the surface of the earth is 9.80 N and we know the radius of the earth is 6380 km, we can use the equation for gravitational force to calculate the mass of the earth.

Solution: $m_e = \frac{Fd^2}{Gm_1} = \frac{(9.80\ \mathbf{m/s^2})(6.38\times10^6\ \mathbf{m})^2}{(6.67\times10^{-11}\ \mathbf{N{\cdot}m^2/kg^2})(1.00\ \mathbf{kg})} = 5.98 \times 10^{24}\ \mathbf{kg}$

Sample Problem: John and Jane step onto the dance floor about 20. M apart at the Junior Prom and they feel an attraction to each other. If John's mass is 70. kg and Jane's mass is 50. kg, assume the attraction is gravity and calculate its magnitude.

Solution: $F_g = \frac{Gm_1m_2}{d^2} = \frac{(6.67\times10^{-11}\ \mathbf{N{\cdot}m^2/kg^2})(70.\ \mathbf{kg})(50.\ \mathbf{kg})}{(20.\ \mathbf{m})^2} = 1.2 \times 10^{-8}\ \mathbf{N}$

This is such an extremely weak force, it is probably not the force of attraction John and Jane felt.

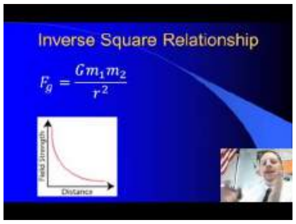
Summary

- Newton proposed the universal law of gravitational attraction as $F = G\frac{m_1m_2}{d^2}$.
- The universal gravitational constant, G, was determined by Cavendish to be 6.67×10^{-11} N·m^2/kg^2.
- Even though satellites are constantly falling toward the object they circle, they do not get closer because their straight line motion moves them away from the center at the same rate they fall.

Practice

The following video is a lecture on universal gravitation. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=OZZGJfFf8XI



MEDIA

Click image to the left for more content.

1. What factors determine the strength of the force of gravity?
2. Between which two points do we measure the distance between the earth and moon?

The following video is The Mass vs. Weight Song. Use this resource to answer the questions that follow. http://www.youtube.com/watch?v=1whMAIGNq7E

MEDIA

Click image to the left for more content.

1. What is used to measure mass?
2. What is used to measure weight?
3. What units are used to measure mass?
4. What units are used to measure weight?

The following website contains a series of solved practice problems on gravity.

http://physics.info/gravitation/practice.shtml

Review

1. The earth is attracted to the sun by the force of gravity. Why doesn't the earth fall into the sun?
2. If the mass of the earth remained the same but the radius of the earth shrank to one-half its present distance, what would happen to the force of gravity on an object that was resting on the surface of the earth?
3. Lifting an object on the moon requires one-sixth the force that would be required to lift the same object on the earth because gravity on the moon is one-sixth that on earth. What about horizontal acceleration? If you threw a rock with enough force to accelerate it at 1.0 m/s^2 horizontally on the moon, how would the required force compare to the force necessary to acceleration the rock in the same way on the earth?
4. The mass of the earth is 5.98×10^{24} kg and the mass of the moon is 7.35×10^{22} kg. If the distance between the earth and the moon is 384,000 km, what is the gravitational force on the moon?

- **gravity:** A natural phenomenon by which physical bodies appear to attract each other with a force proportional to their masses and inversely proportional to the distance separating them.

References

1. Chris Burks (Wikimedia: Chetvorno). http://commons.wikimedia.org/wiki/File:Cavendish_Torsion_Balance_Diagram.svg. Public Domain
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

Chapter 7 Mass versus Weight

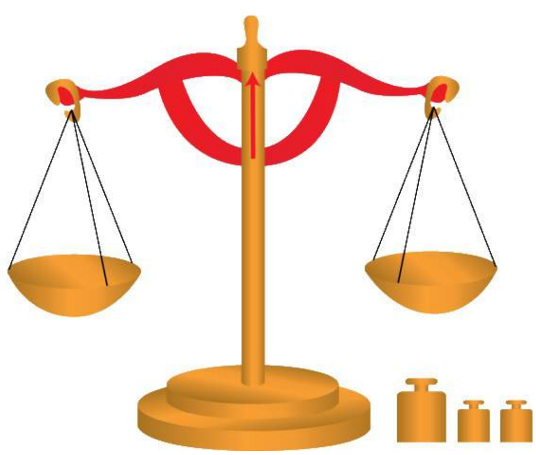
- Distinguish between mass and weight.
- Given the acceleration due to gravity and either the mass or the weight of an object, calculate the other one.

"Space exploration is an international endeavor."

Three Japan Aerospace Exploration Agency astronauts – Akihiko Hoshide, Satoshi Furukawa and Naoko Yamazaki – are training with the 11-member astronaut candidate class of 2004. JAXA astronauts Satoshi Furukawa, Akihiko Hoshide and Naoko Yamazaki experience near-weightlessness on the KC-135 training airplane.

Mass and Weight

The **mass** of an object is defined as the amount of matter in the object. The amount of mass in an object is measured by comparing the object to known masses on an instrument called a balance.



Using the balance shown above, the object would be placed in one pan and known masses would be placed in the other pan until the pans were exactly balanced. When balanced, the mass of the object would be equal to the sum of the known masses in the other pan. The unit of measurement for mass is the kilogram. The mass of an object would be the same regardless of whether the object was on the earth or on the moon. The balance and known masses work exactly the same both places and would indicate the same mass for the same object as long as some gravitational force is present.

The **weight** of an object is defined as the force pulling the object downward. On the earth, this would be the gravitational force of the earth on the object. On the moon, this would be the gravitational force of the moon on the object. Weight is measured by a calibrated spring scale as shown here.

Weight is measured in force units which is Newtons in the SI system. The weights measured for an object would not be the same on the earth and moon because the gravitational field on the surface of the moon is one-sixth of the magnitude of the gravitational field on the surface of the earth.

The force of gravity is given by Newton's Second Law, $F = ma$, when F is the force of gravity in Newtons, m is the mass of the object in kilograms, and a is the acceleration due to gravity, 9.80 m/s^2. When the formula is used specifically for finding weight from mass or vice versa, it may appear as $W = mg$.

Example Problem: What is the weight of an object sitting on the earth's surface if the mass of the object is 43.7 kg?

Solution: $W = mg = (43.7 \text{ kg})(9.80 \text{ m/s}^2) = 428 \text{ N}$

Example Problem: What is the mass of an object whose weight sitting on the earth is 2570 N?

$m = \frac{W}{a} = \frac{2570 \text{ N}}{9.80 \text{ m/s}^2} = 262 \text{ kg}$

Summary

- The mass of an object is measured in kilograms and is defined as the amount of matter in an object.
- The mass of an object is determined by comparing the mass to known masses on a balance.
- The weight of an object on the earth is defined as the force acting on the object by the earth's gravity.
- Weight is measured by a calibrated spring scale.
- The formula relating mass and weight is $W = mg$.

Practice

A song about the difference between mass and weight sung by Mr. Edmunds to the tune of Sweet Caroline. Remember to make allowances for the fact that he is a teacher, not a professional singer. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=1whMAIGNq7E

MEDIA
Click image to the left for more content.

1. What is used to measure mass?
2. What is used to measure weight?
3. What units are used to measure mass?
4. What units are used to measure weight?

This video shows what appears to be a magic trick but is actually a center of gravity demonstration.

http://www.darktube.org/watch/simple-trick-magic-no-physics

Review

1. The mass of an object on the earth is 100. kg.
   (a) What is the weight of the object on the earth?
   (b) What is the mass of the object on the moon?
   (c) Assuming the acceleration due to gravity on the moon is EXACTLY one-sixth of the acceleration due to gravity on earth, what is the weight of the object on the moon?
2. A man standing on the Earth can exert the same force with his legs as when he is standing on the moon. We know that the mass of the man is the same on the Earth and the Moon. We also know that $F = ma$ is true on both the Earth and the Moon. Will the man be able to jump higher on the Moon than the Earth? Why or why not?

- **mass:** The mass of an object is measured in kilograms and is defined as the amount of matter in an object.
- **weight:** The weight of an object on the earth is defined as the force acting on the object by the earth's gravity.

References

1. Courtesy of NASA. http://spaceflight.nasa.gov/gallery/images/behindthescenes/training/html/jsc2004e45082.html. Public Domain
2. CK-12 Foundation - Christopher Auyeung. . CC BY-NC-SA 3.0
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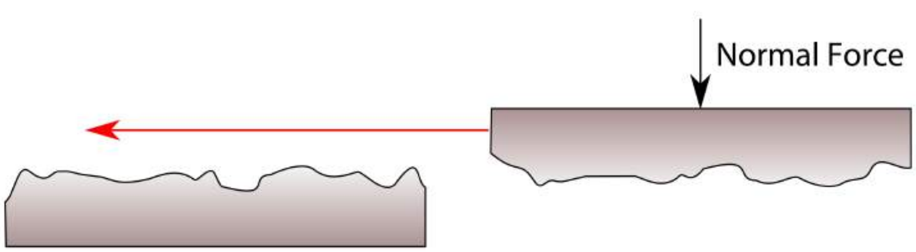
CHAPTER 8 Friction

- Define both static and sliding friction.
- Explain what causes surface friction.
- Define the coefficient of friction.
- Calculate frictional forces.
- Calculate net forces when friction is involved.

Dealing with friction and a lack of friction becomes a very important part of the game in tennis played on a clay court. It's necessary for this player to learn how to keep her shoes from sliding when she wants to run but also necessary to know how her shoes will slide when coming to a stop.

Friction

Most of the time, in beginning physics classes, **friction** is ignored. Concepts can be understood and calculations made assuming friction to be non-existent. Whenever physics intersects with the real world, however, friction must be taken into account. Friction exists between two touching surfaces because even the smoothest looking surface is quite rough on a microscopic scale.



With the bumps, lumps, and imperfections emphasized as in the image above, it becomes more apparent that if we try to slide the top block over the lower block, there will be collisions involved when bumps impact on bumps. The forward motion causes the collisions with bumps which then exert a force in opposite way the block is moving. The force of friction always opposes whatever motion is causing the friction.

The force of friction between these two blocks is related to two factors. The first factor is the roughness of the surfaces that are interacting which is called the **coefficient of friction**, μ (Greek letter mu). The second factor is the magnitude of the force pushing the top block down onto the lower block. It is reasonable that the more forcefully the blocks are pushed together, the more difficult it will be for one to slide over the other. A very long time ago, when physics was young, the word "normal" was used in the same way that we use the word "perpendicular" today. The force pushing these blocks together is the perpendicular force pushing the top block down on the lower block and this force is called the **normal force**. Much of the time, this normal force is simply the weight of the top block but on some occasions, the weight of the top block has some added or reduced force so the normal force is not always the weight. The force of friction then, can be calculated by

$F_{\text{friction}} = \mu \times F_{\text{normal}}$

This is an approximate but reasonably useful and accurate relationship. It is not exact because μ depends on whether the surfaces are wet or dry and so forth.

The frictional force we have been discussing is referred to as sliding friction because it is involved when one surface is sliding over another. If you have ever tried to slide a heavy object across a rough surface, you may be aware that it is a great deal easier to keep an object sliding than it is to start the object sliding in the first place. When the object to slide is resting on a surface with no movement, the force of friction is called static friction and it is somewhat greater than sliding friction. Surfaces that are to move against one another will have both a coefficient of static friction and a coefficient of sliding friction and the two values will NOT be the same. For example, the coefficient of sliding friction for ice on ice is 0.03 whereas the coefficient of static friction for ice on ice is 0.10 –more than three times as great.

Example Problem: A box weighing 2000. N is sliding across a cement floor. The force pushing the box is 500. N and the coefficient of sliding friction between the box and the floor is 0.20. What is the acceleration of the box.

Solution: In this case, the normal force for the box is its weight. Using the normal force and the coefficient of friction, we can find the frictional force. We can also find the mass of the box from its weight since we know the acceleration due to gravity. Then we can find the net force and the acceleration.

$F_F = \mu F_N = (0.20)(2000.\text{ N}) = 400.\text{ N}$

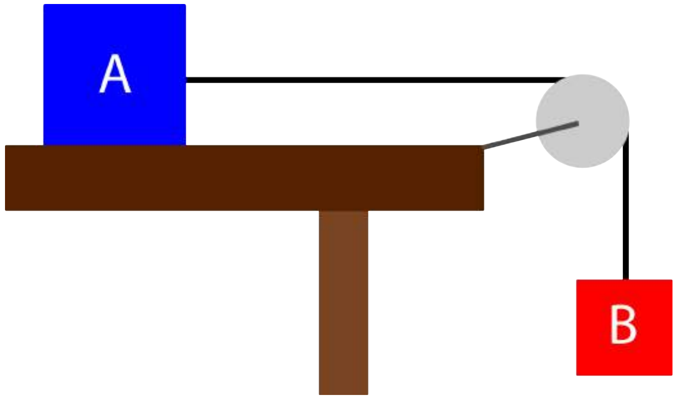
$\text{mass of box} = \frac{\text{weight}}{g} = \frac{2000.\text{ N}}{9.8\text{ m/s}^2} = 204\text{ g}$

$F_{\text{NET}} = \text{Pushing force} - \text{frictional force} = 500.\text{ N} - 400.\text{ N} = 100.\text{ N}$

$a = \frac{F_N}{m} = \frac{100.\text{ N}}{204\text{ kg}} = 0.49\text{ m/s}^2$

Example Problem: Two boxes are connected by a rope running over a pulley (see image). The coefficient of sliding friction between box A and the table is 0.20. (Ignore the masses of the rope and the pulley and any friction in the pulley.) The mass of box A is 5.0 kg and the mass of box B is 2.0 kg. The entire system (both boxes) will move together with the same acceleration and velocity.

Find the acceleration of the system.



Solution: The force tending to move the system is the weight of box B and the force resisting the movement is the force of friction between the table and box A. The mass of the system would be the sum of the masses of both boxes. The acceleration of the system will be found by dividing the net force by the total mass.

$F_N(\text{box A}) = mg = (5.0 \text{ kg})(9.8 \text{ m/s}^2) = 49 \text{ N}$

$F_{\text{friction}} = \mu F_N = (0.20)(49 \text{ N}) = 9.8 \text{ N}$

Weight of box B $= mg = (2.0 \text{ kg})(9.8 \text{ m/s}^2) = 19.6 \text{ N}$

$F_{\text{NET}} = 19.6 \text{ N} - 9.8 \text{ N} = 9.8 \text{ N}$

$a = \frac{F_{NET}}{\text{mass}} = \frac{9.8 \text{ N}}{7.0 \text{ kg}} = 1.4 \text{ m/s}^2$

Summary

- Friction is caused by bodies sliding over rough surfaces.
- The degree of surface roughness is indicated by the coefficient of friction, μ.
- The force of friction is calculated by multiplying the coefficient of friction by the normal force.
- The frictional force always opposes motion.
- Acceleration is caused by the net force which is found by subtracting the frictional force from the applied force.

Practice

A video explaining friction.

http://www.youtube.com/watch?v=CkTCp7SZdYQ

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Click image to the left for more content.

Review

1. A 52 N sled is pulled across a cement sidewalk at constant speed. A horizontal force of 36 N is exerted. What is the coefficient of sliding friction between the sidewalk and the metal runners of the sled?
2. If the coefficient of sliding friction between a 25 kg crate and the floor is 0.45, how much force is required to move the crate at a constant velocity across the floor?

3. A smooth wooden 40.0 N block is placed on a smooth wooden table. A force of 14.0 N is required to keep the block moving at constant velocity.
   (a) What is the coefficient of sliding friction between the block and the table top?
   (b) If a 20.0 N brick is placed on top of the wooden block, what force will be required to keep the block and brick moving at constant velocity?

- **friction:** A force that resists the relative motion or tendency to such motion of two bodies or substances in contact.
- **coefficient of friction:** The ratio of the force that maintains contact between an object and a surface (i.e. the normal force) and the frictional force that resists the motion of the object.
- **normal force:** The perpendicular force one surface exerts on another surface when the surfaces are in contact.

References

1. Courtesy of Eric Harris, U.S. Air Force. http://commons.wikimedia.org/wiki/File:Maria_Sharapova,_2008_Family_Circle_Cup.JPG. Public Domain
2. CK-12 Foundation - Joy Sheng. . CC-BY-NC-SA 3.0
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CHAPTER 9 Free Body Diagrams

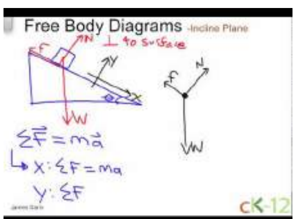
Students will learn how to draw a free-body diagram and apply it to the real world.

Students will learn how to draw a free-body diagram and apply it to the real world.

Guidance

For every problem involving forces it is essential to draw a free body diagram (FBD) before proceeding to the problem solving stage. The FBD allows one to visualize the situation and also to make sure all the forces are accounted. In addition, a very solid understanding of the physics is gleaned and many questions can be answered solely from the FBD.

Example 1



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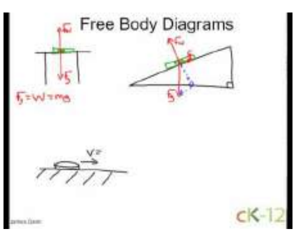

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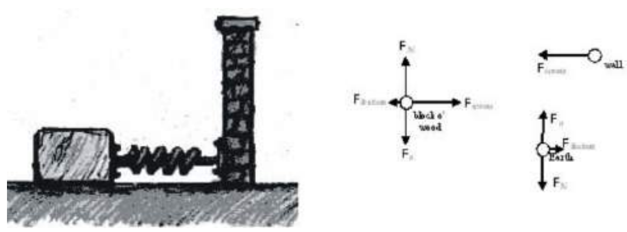
Time for Practice

1. Draw free body diagrams (FBDs) for all of the following objects involved (in **bold**) and label all the forces appropriately. Make sure the lengths of the vectors in your FBDs are proportional to the strength of the force: smaller forces get shorter arrows!
   a. A **man** stands in an elevator that is accelerating upward at 2 m/s^2.
   a. A boy is dragging a **sled** at a constant speed. The boy is pulling the sled with a rope at a 30° angle.
   a. The **picture** shown here is attached to the ceiling by three wires.

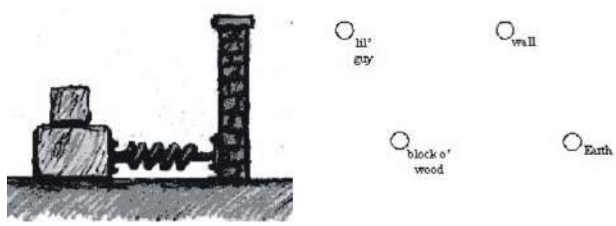
a. A **bowling ball** rolls down a lane at a constant velocity.

a. A **car** accelerates down the road. There is friction f between the tires and the road.

2. For the following situation, identify the 3^{rd} law force pairs on the associated free body diagrams. Label each member of one pair "A,″ each member of the next pair "B,″ and so on. The spring is stretched so that it is pulling the block of wood to the right.



Draw free body diagrams for the situation below. Notice that we are pulling the bottom block *out from beneath* the top block. There is friction between the blocks! After you have drawn your FBDs, identify the 3^{rd} law force pairs, as above.



Answers

Discuss in class

CHAPTER 10 Problem Solving 1

In this lesson, students will learn how to solve difficult problems using Newton's 2nd law.

In this lesson, students will learn how to solve difficult problems using Newton's 2nd law.

Key Equations

$$\text{Common Forces}\begin{cases} F_g = mg & \text{Gravity} \\ F_N & \text{Normal force: acts perpendicular to surfaces} \\ F_T & \text{Force of tension in strings and wires} \\ F_{sp} = -k\Delta x = & \text{Force of spring}\Delta x\text{from equilibrium} \end{cases}$$

$$\text{Force Sums}\begin{cases} F_{\text{net}} = \sum_i F_i = ma & \text{Net force is the vector sum of all the forces} \\ F_{\text{net},x} = \sum_i F_{ix} = ma_x & \text{Horizontal components add also} \\ F_{\text{net},y} = \sum_i F_{iy} = ma_y & \text{As do vertical ones} \end{cases}$$

$$\text{Static and Kinetic Friction'}\begin{cases} f_s \leq \mu_s |F_N| & \text{Opposes potential motion of surfaces in contact} \\ f_k = \mu_k |F_N| & \text{Opposes motion of surfaces in contact} \end{cases}$$

Ultimately, many of these "contact" forces are due to attractive and repulsive electromagnetic forces between atoms in materials.

Guidance

Problem Solving for Newton's Laws, Step-By-Step

1. *Figure out which object is "of interest."*
   a. If you're looking for the motion of a rolling cart, the cart is the object of interest.
   b. If the object of interest is not moving, that's OK, don't panic yet.
   c. Draw a sketch! This may help you sort out which object is which in your problem.
2. *Identify all the forces acting on the object and draw them on object. (This is a free-body diagram –FBD)*
   a. If the object has mass and is near the Earth, the easiest (and therefore, first) force to write down is the force of gravity, pointing downward, with value *mg*.
   b. If the object is in contact with a flat surface, it means there is a normal force acting on the object. This normal force points away from and is perpendicular to the surface.
   c. There may be more than one normal force acting on an object. For instance, if you have a bologna sandwich, remember that the slice of bologna feels normal forces from both the slices of bread!
   d. If a rope, wire, or cord is pulling on the object in question, you've found yourself a tension force. The direction of this force is in the same direction that the rope is pulling.
   e. Don't worry about any forces acting on other objects. For instance, if you have a bologna sandwich as your object of interest, and you're thinking about the forces acting on the slice of bologna, don't worry about the force of gravity acting on either piece of bread.

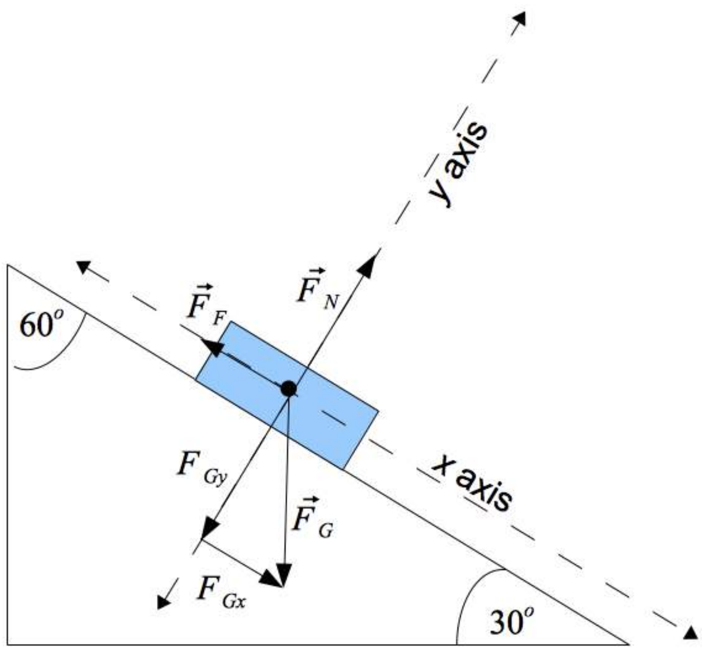
f. Remember that Newton's 3^{rd} Law, calling for "equal and opposite forces," does not apply to a single object. None of your forces should be "equal and opposite" on the same object in the sense of Newton's 3^{rd} Law. Third law pairs act on two different objects.
g. Recall that scales (like a bathroom scale you weigh yourself on) read out the normal force acting on you, not your weight. If you are at rest on the scale, the normal force equals your weight. If you are accelerating up or down, the normal force had better be higher or lower than your weight, or you won't have an unbalanced force to accelerate you.
h. Never include "m**a**" as a force acting on an object. "m**a**" is the *result* of the net force F_{net} which is found by summing all the forces acting on your object of interest.

3. *Determine how to orient your axes*
   a. A good rule to generally follow is that you want one axis (usually the x-axis) to be parallel to the surface your object of interest is sitting on.
   b. If your object is on a ramp, tilt your axes so that the x-axis is parallel to the incline and the y-axis is perpendicular. In this case, this will force you to break the force of gravity on the object into its components. But by tilting your axes, you will generally have to break up fewer vectors, making the whole problem simpler.
4. *Identify which forces are in the $x-$ direction, which are in the $y-$ direction, and which are at an angle.*
   a. If a force is upward, make it in the $y-$direction and give it a positive sign. If it is downward, make it in the $y-$direction and give it a negative sign.
   b. Same thing applies for right vs. left in the $x-$direction. Make rightward forces positive.
   c. If forces are at an angle, draw them at an angle. A great example is that when a dog on a leash runs ahead, pulling you along, it's pulling both forward *and* down on your hand.
   d. Draw the free body diagram (FBD).
   e. Remember that the FBD is supposed to be helping you with your problem. For instance, if you forget a force, it'll be really obvious on your FBD.
5. Break the forces that are at angles into their x and y components
   a. Use right triangle trigonometry
   b. Remember that these components aren't *new* forces, but are just what makes up the forces you've already identified.
   c. Consider making a second FBD to do this component work, so that your first FBD doesn't get too messy.
6. *Add up all the $x-$ forces and $x-$ components.*
   a. Remember that all the rightward forces add with a plus $(+)$ sign, and that all the leftward forces add with a minus $(-)$ sign.
   b. Don't forget about the $x-$components of any forces that are at an angle!
   c. When you've added them all up, call this "the sum of all x forces" or "the net force in the $x-$direction."
7. *Add up all the $y-$ forces and $y-$ components.*
   a. Remember that all the upward forces add with a $(+)$ sign, all the downward forces add with a $(-)$ sign.
   b. Don't forget about the $y-$components of any forces that are at an angle!
   c. When you've added them all up, call this "the sum of all y forces" or "net force in the $y-$direction."
8. *Use Newton's Laws twice.*
   a. The sum of all $x-$forces, divided by the mass, is the object's acceleration in the $x-$direction.
   b. The sum of all $y-$forces, divided by the mass, is the object's acceleration in the $y-$direction.
   c. If you happen to know that the acceleration in the $x-$direction or $y-$direction is zero (say the object is just sitting on a table), then you can plug this in to Newton's 2^{nd} Law directly.
   d. If you happen to know the acceleration, you can plug this in directly too.
9. *Each body should have a FBD.*
   a. Draw a separate FBD for each body.

b. Set up a sum of forces equation based on the FBD for each body.
c. Newton's Third Law will tell you which forces on different bodies are the same in magnitude.
d. Your equations should equal your unknown variables at this point.

Example 1

Question: Using the diagram below, find the net force on the block. The block weighs 3kg and the inclined plane has a coefficient of friction of .6.



Answer:

The first step to solving a Newton's Laws problem is to identify the object in question. In our case, the block on the slope is the object of interest.

Next, we need to draw a free-body diagram. To do this, we need to identify all of the forces acting on the block and their direction. The forces are friction, which acts in the negative x direction, the normal force, which acts in the positive y direction, and gravity, which acts in a combination of the negative y direction and the positive x direction. Notice that we have rotated the picture so that the majority of the forces acting on the block are along the y or x axis. This does not change the answer to the problem because the direction of the forces is still the same relative to each other. When we have determined our answer, we can simply rotate it back to the original position.

Now we need to break down gravity (the only force not along one of the axises) into its component vectors (which do follow the axises).

$$\text{The x component of gravity}: 9.8\text{m/s}^2 \times cos60 = 4.9\text{m/s}^2$$
$$\text{The y component of gravity}: 9.8\text{m/s}^2 \times sin60 = 8.5\text{m/s}^2$$

Yet these are only the acceleration of gravity so we need to multiply them by the weight of the block to get the force.

$$F = ma = 3\text{kg} \times 4.9\text{m/s}^2 = 14.7\text{N} F = ma = 3\text{kg} \times 8.5\text{m/s}^2 = 25.5\text{N}$$

Now that we have solved for the force of the y-component of gravity we know the normal force (they are equal). Therefore the normal force is 25.5N. Now that we have the normal force and the coefficient of static friction, we can find the force of friction.

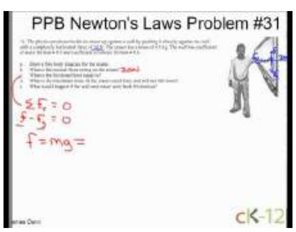
$$F_s = \mu_s F_N = .6 \times 25.5\text{N} = 15.3\text{N}$$

The force of static friction is greater than the component of gravity that is forcing the block down the inclined plane. Therefore the force of friction will match the force of the x-component of gravity. So the net force on the block is

$$\begin{aligned} \text{net force in the x} - \text{direction :} \quad & \overbrace{14.7\text{N}}^{x-component\ of\ gravity} - \overbrace{14.7\text{N}}^{force\ of\ friction} = 0\text{N} \\ \text{net force in the y} - \text{direction :} \quad & \underbrace{25.5\text{N}}_{Normal\ Force} - \underbrace{25.5\text{N}}_{y-component\ of\ gravity} = 0\text{N} \end{aligned}$$

Therefore the net force on the block is 0N.

Example 2



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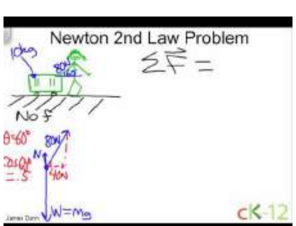

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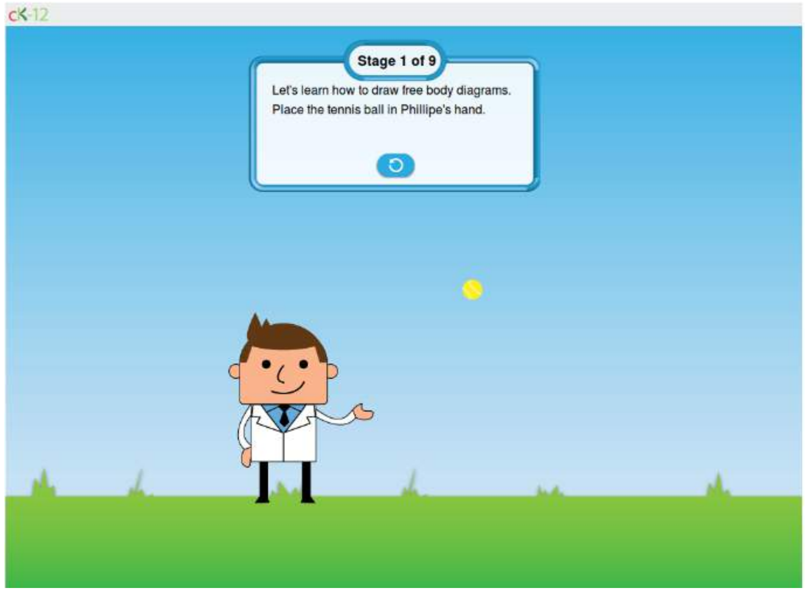
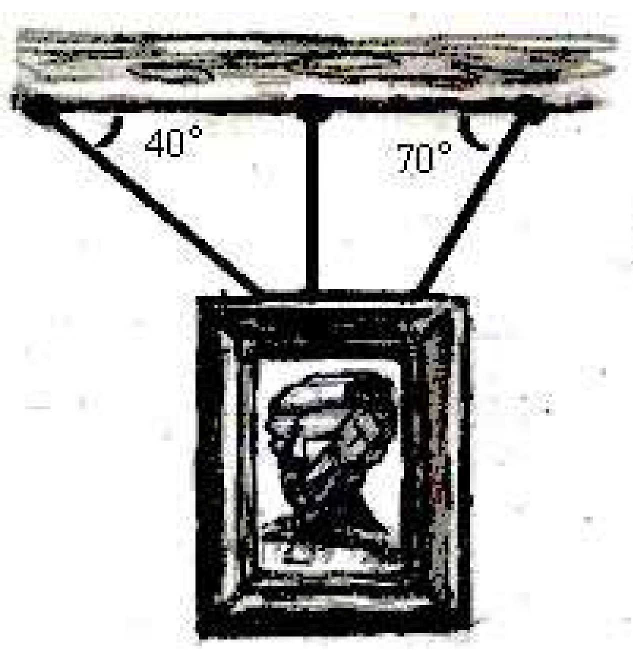
Simulation



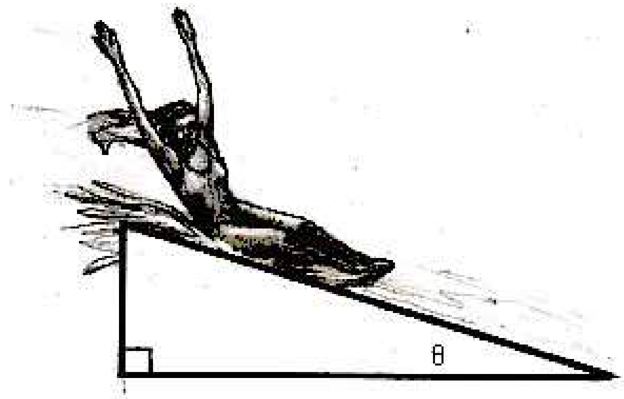
- http://simulations.ck12.org/FreeBody/

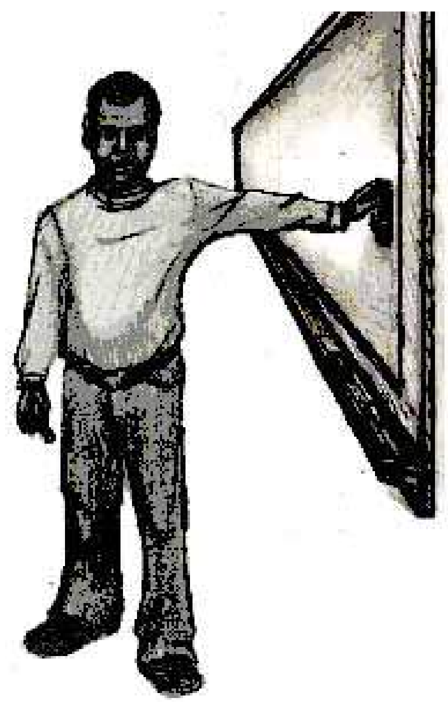
Time for Practice

1. Find the mass of the painting. The tension in the leftmost rope is 7.2 N, in the middle rope it is 16 N, and in the rightmost rope it is 16 N.



2. Find Brittany's acceleration down the frictionless waterslide in terms of her mass m, the angle θ of the incline, and the acceleration of gravity g.



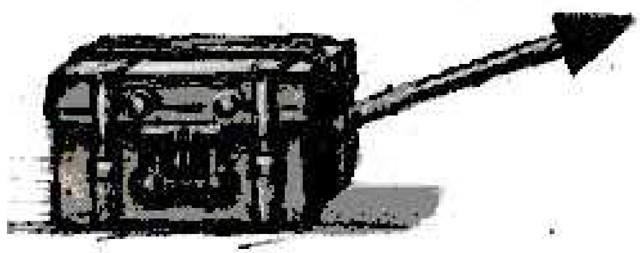


3. The physics professor holds an eraser up against a wall by pushing it directly against the wall with a completely horizontal force of 20 N. The eraser has a mass of 0.5 kg. The wall has coefficients of friction $\mu_S = 0.8$ and $\mu_K = 0.6$.

   a. Draw a free body diagram for the eraser.
   b. What is the normal force F_N acting on the eraser?
   c. What is the frictional force F_S equal to?
   d. What is the maximum mass m the eraser could have and still not fall down?
   e. What would happen if the wall and eraser were both frictionless?

4. A tractor of mass 580 kg accelerates up a 10° incline from rest to a speed of 10 m/s in 4 s. For all of answers below, provide a magnitude and a direction.

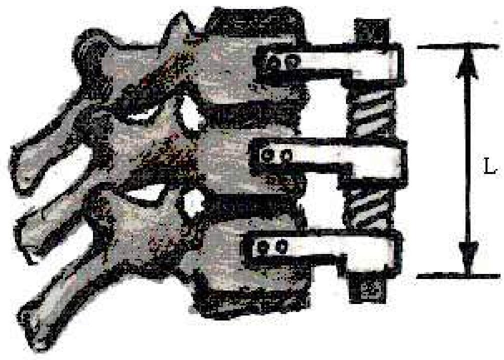
a. What net force F_{net} has been applied to the tractor?
b. What is the normal force, F_N on the tractor?
c. What is the force of gravity F_g on the tractor?
d. What force has been applied to the tractor so that it moves uphill?
e. What is the source of this force?



5. A heavy box (mass 25 kg) is dragged along the floor by a kid at a 30° angle to the horizontal with a force of 80 N (which is the maximum force the kid can apply).

a. Draw the free body diagram.
b. What is the normal force F_N?
c. Does the normal force decrease or increase as the angle of pull increases? Explain.
d. Assuming no friction, what is the acceleration of the box?
e. Assuming it begins at rest, what is its speed after ten seconds?
f. Is it possible for the kid to lift the box by pulling straight up on the rope?
g. In the absence of friction, what is the net force in the $x-$direction if the kid pulls at a 30° angle?
h. In the absence of friction, what is the net force in the $x-$direction if the kid pulls at a 45° angle?
i. In the absence of friction, what is the net force in the $x-$direction if the kid pulls at a60° angle?
j. The kid pulls the box at constant velocity at an angle of 30°. What is the coefficient of kinetic friction μ_K between the box and the floor?
k. The kid pulls the box at an angle of 30°, producing an acceleration of 2 m/s^2. What is the coefficient of kinetic friction μ_K between the box and the floor?

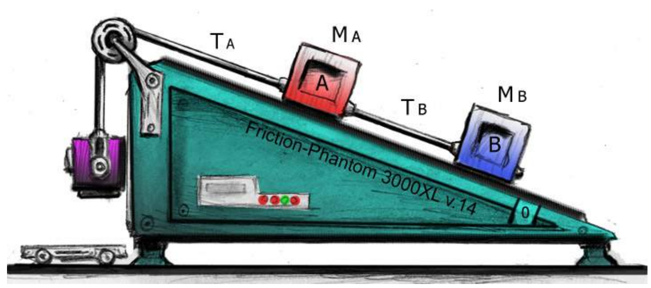
6. Spinal implant problem —this is a real life bio-med engineering problem!



Here's the situation: both springs are compressed by an amount x_o. The rod of length L is fixed to both the top plate and the bottom plate. The two springs, each with spring constant k, are wrapped around the rod on both sides of the middle plate, but are free to move because they are not attached to the rod or the plates. The middle plate has negligible mass, and is constrained in its motion by the compression forces of the top and bottom springs. The medical implementation of this device is to screw the top plate to one vertebrae and the middle plate to the vertebrae directly below. The bottom plate is suspended in space. Instead of fusing broken vertebrates together, this implant allows movement somewhat analogous to the natural movement of functioning vertebrae. Below you will do the exact calculations that an engineer did to get this device patented and available for use at hospitals.

a. Find the force, F, on the middle plate for the region of its movement $\triangle x \leq x_o$. Give your answer in terms of the constants given. (*Hint: In this region both springs are providing opposite compression forces*.)
b. Find the force, F, on the middle plate for the region of its movement $\triangle x \geq x_o$. Give your answer in terms of the constants given. (*Hint: In this region, only one spring is in contact with the middle plate.*)
c. Graph F vs. x. Label the values for force for the transition region in terms of the constants given.

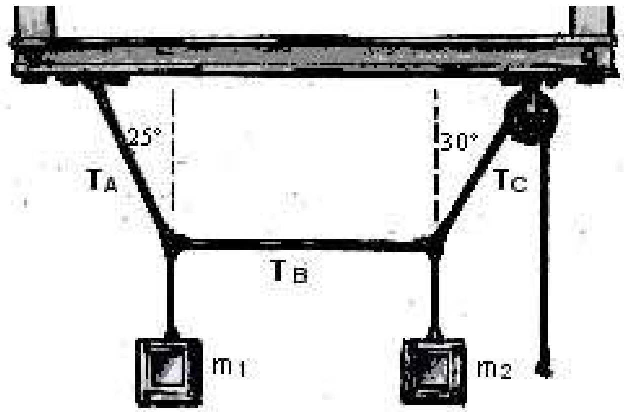
7. You design a mechanism for lifting boxes up an inclined plane by using a vertically hanging mass to pull them, as shown in the figure below.



The pulley at the top of the incline is massless and frictionless. The larger mass, M, is accelerating downward with a measured acceleration a. The smaller masses are m_A and m_B ; the angle of the incline is θ, and the coefficient of kinetic friction between each of the masses and the incline has been measured and determined to be μ_K.

a. Draw free body diagrams for each of the three masses.
b. Calculate the magnitude of the frictional force on each of the smaller masses in terms of the given quantities.
c. Calculate the net force on the hanging mass in terms of the given quantities.
d. Calculate the magnitudes of the two tension forces T_A and T_B in terms of the given quantities.
e. Design and state a strategy for solving for how long it will take the larger mass to hit the ground, assuming at this moment it is at a height h above the ground. Do not attempt to solve this: simply state the strategy for solving it.

8. You build a device for lifting objects, as shown below. A rope is attached to the ceiling and two masses are allowed to hang from it. The end of the rope passes around a pulley (right) where you can pull it downward to lift the two objects upward. The angles of the ropes, measured with respect to the vertical, are shown. Assume the bodies are at rest initially.



   a. Suppose you are able to measure the masses m_1 and m_2 of the two hanging objects as well as the tension T_C. Do you then have enough information to determine the other two tensions, T_A and T_B? Explain your reasoning.
   b. If you only knew the tensions T_A and T_C, would you have enough information to determine the masses m_1 and m_2? If so, write m_1 and m_2 in terms of T_A and T_C. If not, what further information would you require?

9. A stunt driver is approaching a cliff at very high speed. Sensors in his car have measured the acceleration and velocity of the car, as well as all forces acting on it, for various times. The driver's motion can be broken down into the following steps: Step 1: The driver, beginning at rest, accelerates his car on a horizontal road for ten seconds. Sensors show that there is a force in the direction of motion of 6000 N, but additional forces acting in the opposite direction with magnitude 1000 N. The mass of the car is 1250 kg. Step 2: Approaching the cliff, the driver takes his foot off of the gas pedal (There is no further force in the direction of motion.) and brakes, increasing the force opposing motion from 1000 N to 2500 N. This continues for five seconds until he reaches the cliff. Step 3: The driver flies off the cliff, which is 44.1 m high and begins projectile motion.
   (a) Ignoring air resistance, how long is the stunt driver in the air?
   (b) For Step 1:
      i. Draw a free body diagram, naming all the forces on the car.
      ii. Calculate the magnitude of the net force.
      iii. Find the change in velocity over the stated time period.
      iv. Make a graph of velocity in the $x-$direction vs. time over the stated time period.
      v. Calculate the distance the driver covered in the stated time period. Do this by finding the area under the curve in your graph of (iv). Then, check your result by using the equations for kinematics.
   (c) Repeat (b) for Step 2.
   (d) Calculate the distance that the stunt driver should land from the bottom of the cliff.

Answers

1. 3.6 kg

2. $g \sin \theta$
3. b.20 N c. 4.9 N d. 1.63 kg e. Eraser would slip down the wall
4. a. 1450 N b. 5600 N c. 5700 N d. Friction between the tires and the ground e. Fuel, engine, or equal and opposite reaction
5. b. 210 N c. no, the box is flat so the normal force doesn't change d. 2.8 m/s^2 e.28 m/s f. no g. 69 N h. 57 N i. 40 N j. 0.33 k. 0.09
6. a. zero b. $-kx0$
7. b. $f_1 = \mu_k m_1 g \cos \theta; f_2 = \mu_k m_2 g \cos \theta$ c. Ma d. $T_A = (m_1 + m_2)(a + \mu \cos \theta)$ and $T_B = m_2 a + \mu m_2 \cos \theta$ e. Solve by using $d = 1/2at^2$ and substituting h for d
8. a. Yes, because it is static and you know the angle and m_1 b. Yes, T_A and the angle gives you m_1 and the angle and T_C gives you $m_2, m_1 = T_A \cos 25/g$ and $m_2 = T_C \cos 30/g$
9. a. 3 seconds d. 90 m

CK-12 FlexBook

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Patrick Marshall
Ck12 Science
James H Dann, Ph.D.

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CHAPTER 1 Kinetic Energy

- Define energy.
- Define kinetic energy.
- Given the mass and speed of an object, calculate its kinetic energy.
- Solve problems involving kinetic energy.

This military jet requires a tremendous amount of work done on it to get its speed up to takeoff speed in the short distance available on the deck of an aircraft carrier. Some of the work is done by the plane's own jet engines but work from a catapult is also necessary for takeoff.

Kinetic Energy

Energy is the ability to change an object's motion or position. Energy comes in many forms and each of those forms can be converted into any other form. A moving object has the ability to change another object's motion or position simply by colliding with it and this form of energy is called **kinetic energy.** The kinetic energy of an object can be calculated by the equation

$KE = \frac{1}{2}mv^2$,

where m is the mass of the object and v is its velocity. The kinetic energy of a moving object is directly proportional to its mass and directly proportional to the square of its velocity. This means that an object with twice the mass and equal speed will have twice the kinetic energy while an object with equal mass and twice the speed will have quadruple the kinetic energy.

The SI unit for kinetic energy (and all forms of energy) is $\text{kg} \cdot \frac{\text{m}^2}{\text{s}^2}$ which is equivalent to Joules, the same unit we use for work. The kinetic energy of an object can be changed by doing work on the object. The work done on an object equals the kinetic energy gain or loss by the object. This relationship is expressed in the work-energy theorem $W_{\text{NET}} = \Delta KE$.

Example Problem: A farmer heaves a 7.56 kg bale of hay with a final velocity of 4.75 m/s.

(a) What is the kinetic energy of the bale?

(b) The bale was originally at rest. How much work was done on the bale to give it this kinetic energy?

Solution:

(a) $KE = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(7.56 \text{ kg})(4.75)^2 = 85.3$ Joules

(b) Work done $= \Delta KE = 85.3$ Joules

Example Problem: What is the kinetic energy of a 750. kg car moving at 50.0 km/h?

Solution: $\left(\frac{50.0 \textbf{ km}}{\textbf{h}}\right)\left(\frac{1000 \textbf{ m}}{\textbf{km}}\right)\left(\frac{1 \textbf{ h}}{3600 \textbf{ s}}\right) = 13.9 \textbf{ m/s}$

$KE = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(750. \text{ kg})(13.9 \text{ m/s})^2 = 72,300$ Joules

Example Problem: How much work must be done on a 750. kg car to slow it from 100. km/h to 50.0 km/h?

Solution: From the previous example problem, we know that the KE of this car when it is moving at 50.0 km/h is 72,300 Joules. If the same car is going twice as fast, its KE will be four times as great because KE is proportional to the square of the velocity. Therefore, when this same car is moving at 100. km/h, its KE is 289,200 Joules. Therefore, the work done to slow the car from 100. km/h to 50.0 km/h is $(289,200 \text{ Joules}) - (72,300 \text{ Joules}) = 217,000$ Joules.

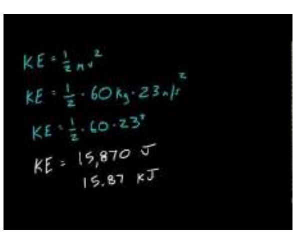
Summary

- Energy is the ability to change an object's motion or position.
- There are many forms of energy.
- The energy of motion is called kinetic energy.
- The formula for kinetic energy is $KE = \frac{1}{2}mv^2$.
- The work done on an object equals the kinetic energy gain or loss by the object, $W_{\text{NET}} = \Delta KE$.

Practice

The following video discusses kinetic energy. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=g157qwT1918



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1. Potential energy is present in objects that are _______________.
2. Kinetic energy is present in objects that are _______________.
3. What formula is given for kinetic energy?

Practice problems involving kinetic energy:

http://www.physicsclassroom.com/Class/energy/u5l1c.cfm

Review

1. A comet with a mass of 7.85×10^{11} kg is moving with a velocity of 25,000 m/s. Calculate its kinetic energy.
2. A rifle can shoot a 4.00 g bullet at a speed of 998 m/s.
   (a) Find the kinetic energy of the bullet.
   (b) What work is done on the bullet if it starts from rest?
   (c) If the work is done over a distance of 0.75 m, what is the average force on the bullet?
   (d) If the bullet comes to rest after penetrating 1.50 cm into a piece of metal, what is the magnitude of the force bringing it to rest?

- **energy:** An indirectly observed quantity that is often understood as the ability of a physical system to do work.
- **kinetic energy:** The energy an object has due to its motion.

References

1. Courtesy of Mass Communication Specialist 3rd Class Torrey W. Lee, U.S. Navy. Jet Takeoff. Public Domain

CHAPTER 2 Potential Energy

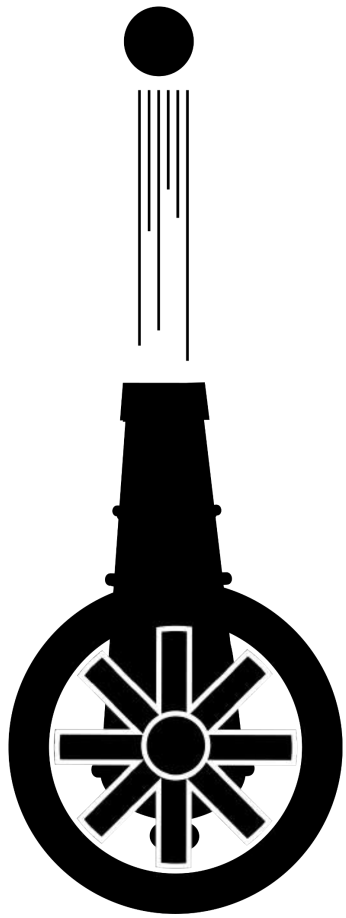
- Define potential energy.
- Solve problems involving gravitational potential energy.
- Solve problems involving the conversion of potential energy to kinetic energy and vice versa.

Shooting an arrow from a bow requires work done on the bow by the shooter's arm to bend the bow and thus produce potential energy. The release of the bow converts the potential energy of the bent bow into the kinetic energy of the flying arrow.

Potential Energy

When an object is held above the earth, it has the ability to make matter move because all you have to do is let go of the object and it will fall of its own accord. Since energy is defined as the ability to make matter move, this object has energy. This type of energy is stored energy and is called **potential energy.** An object held in a stretched rubber band also contains this stored energy. If the stretched rubber band is released, the object will move. A pebble on a flexed ruler has potential energy because if the ruler is released, the pebble will fly. If you hold two positive charges near each other, they have potential energy because they will move if you release them. Potential energy is also the energy stored in chemical bonds. If the chemical bonds are broken and allowed to form lower potential energy chemical bonds, the excess energy is released and can make matter move. This is frequently seen as increased molecular motion or heat.

If a cannon ball is fired straight up into the air, it begins with a high kinetic energy. As the cannon ball rises, it slows down due to the force of gravity pulling it toward the earth. As the ball rises, its gravitational potential energy is increasing and its kinetic energy is decreasing. When the cannon ball reaches the top of its arc, it kinetic energy is zero and its potential energy is maximum. Then gravity continues to pull the cannon ball toward the earth and the ball will fall toward the earth. As it falls, its speed increases and its height decreases. Therefore, its kinetic energy increases as it falls and its potential energy decreases. When the ball returns to its original height, its kinetic energy will be the same as when it started upward.



When work is done on an object, the work may be converted into either kinetic or potential energy. If the work results in motion, the work was converted into kinetic energy and if the work done resulted in change in position, the work was converted into potential energy. Work could also be spent overcoming friction and that work would be converted into heat but usually we will consider frictionless systems.

If we consider the potential energy of a bent stick or a stretched rubber band, the potential energy can be calculated by multiplying the force exerted by the stick or rubber band times the distance over which the force will be exerted. The formula for calculating this potential energy looks exactly like the formula for calculating work done, that is $W = Fd$. The only difference is that work is calculated when the object actually moves under the force and potential energy is calculated when the system is at rest before any motion actually occurs.

In the case of gravitational potential energy, the force exerted by the object is its weight and the distance it could travel would be its height above the earth. Since the weight of an object is calculated by $W = mg$, then gravitational potential energy can be calculate by $PE = mgh$, where m is the mass of the object, g is the acceleration due to gravity, and h is the height the object will fall.

Example Problem: A 3.00 kg object is lifted from the floor and placed on a shelf that is 2.50 m above the floor.

(a) What was the work done in lifting the object?

(b) What is the gravitational potential energy of the object sitting on the shelf?

(c) If the object falls off the shelf and free falls to the floor, what will its velocity be when it hits the floor?

Solution: weight of the object $= mg = (3.00\ kg)(9.80\ m/s^2) = 29.4\ N$

(a) $W = Fd = (29.4\ N)(2.50\ m) = 73.5\ J$

(b) $PE = mgh = (3.00\ kg)(9.80\ m/s^2)(2.50\ m) = 73.5\ J$

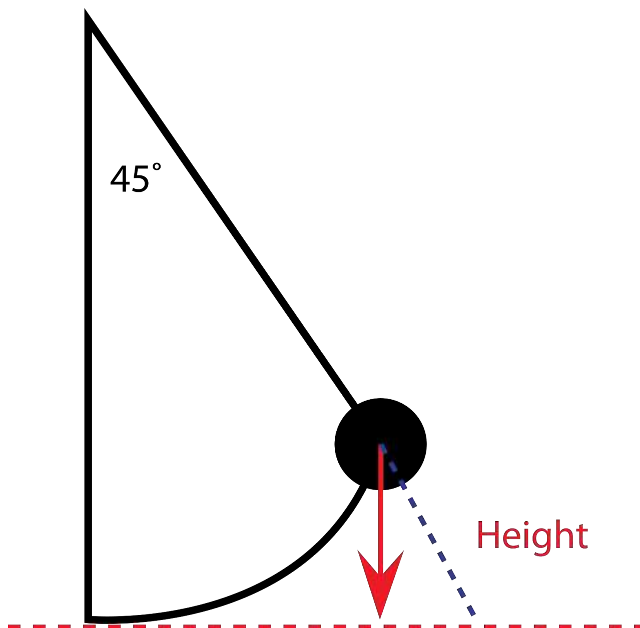
(c) $KE = PE$ so $\frac{1}{2}\ mv^2 = 73.5\ J$

$$v = \sqrt{\frac{(2)(73.5\ J)}{3.00\ kg}} = 7.00\ m/s$$

Example Problem: A pendulum is constructed from a 7.58 kg bowling ball hanging on a 3.00 m long rope. The ball is pulled back until the rope makes an angle of 45° with the vertical.

(a) What is the potential energy of the ball?

(b) If the ball is released, how fast will it be traveling at the bottom of its arc?



Solution: You can use trigonometry to find the vertical height of the ball in the pulled back position. This vertical height is found to be 0.877 m.

$PE = mgh = (7.58\ kg)(9.80\ m/s^2)(0.877\ m) = 65.1\ J$

When the ball is released, the PE will be converted into KE as the ball swings through the arc.

$KE = \frac{1}{2}\ mv^2 = 65.1\ J$

$$v = \sqrt{\frac{(2)(65.1\ kg \cdot m^2/s^2)}{7.58\ kg}} = 4.14\ m/s$$

Summary

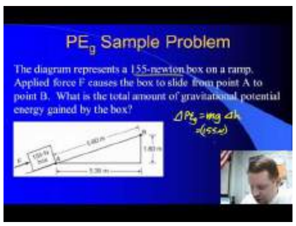
- Stored energy is called potential energy.

- Energy may be stored by holding an object elevated in a gravitational field or by holding it while a force is attempting to move it.
- Potential energy may be converted to kinetic energy.
- The formula for gravitational potential energy is $PE = mgh$.
- In the absence of friction or bending, work done on an object must become either potential energy or kinetic energy or both.

Practice

The following video discusses types of energy. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=nC6tT1wkXEc



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1. What is the definition of energy?
2. Name two types of potential energy.
3. How is energy transferred from one object to another?

Potential and kinetic energy practice problems with solutions:

http://www.physicsclassroom.com/Class/energy/U5L2bc.cfm

Review

1. A 90.0 kg man climbs hand over hand up a rope to a height of 9.47 m. How much potential energy does he have at the top?
2. A 50.0 kg shell was fired from a cannon at earth's surface to a maximum height of 400. m. What is the potential energy at maximum height?
3. If the shell in problem #3 then fell to a height of 100. m, what was the loss of PE?
4. A person weighing 645 N climbs up a ladder to a height of 4.55 m.
   (a) What work does the person do?
   (b) What is the increase in gravitational potential energy?
   (c) Where does the energy come from to cause this increase in PE?

- **potential energy:** Otherwise known as stored energy, is the ability of a system to do work due to its position or internal structure. For example, gravitational potential energy is a stored energy determined by an object's position in a gravitational field while elastic potential energy is the energy stored in a spring.

References

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CHAPTER 3 Conservation of Energy

- State the law of conservation of energy.
- Describe a closed system.
- Use the law of conservation of energy to solve problems.

There are many energy conversions between potential and kinetic energy as the cars travel around this double looping roller coaster.

Conservation of Energy

The **law of conservation of energy** states that within a closed system, energy can change form, but the total amount of energy is constant. Another way of expressing the law of conservation of energy is to say that energy can neither be created nor destroyed. An important part of using the conservation of energy is selecting the system. In the conservation of energy just as in the conservation of momentum, the system must be closed. In a **closed system**, objects may not enter or leave, and it is isolated from external forces so that no work can be done on the system.

In the analysis of the behavior of an object, you must make sure you have included everything in the system that is involved in the motion. For example, if you are considering a ball that is acted on by gravity, you must include the earth in your system. The kinetic energy of the ball considered by itself may increase and only when the earth is included in the system can you see that the increasing kinetic energy is balanced by an equivalent loss of potential energy. The sum of the kinetic energy and the potential energy of an object is often called the **mechanical energy.**

Consider a box with a weight of 20.0 N sitting at rest on a shelf that is 2.00 m above the earth. The box has zero kinetic energy but it has potential energy related to its weight and the distance to the earth's surface.

$PE = mgh = (20.0\ N)(2.00\ m) = 40.0\ J$

If the box slides off the shelf, the only force acting on the box is the force of gravity and so the box falls. We can calculate the speed of the box when it strikes the ground by several methods. We can calculate the speed directly using the formula ${v_f}^2 = 2ad$. We can also find the final velocity by setting the kinetic energy at the bottom of the fall equal to the potential energy at the top, $KE = PE$ and, $\frac{1}{2}\,mv^2 = mgh$, so ${v_f}^2 = 2gh$. You may note these formulas are essentially the same.

$v = \sqrt{(2)(9.80\ m/s^2)(2.00\ m)} = 6.26\ m/s$

Example Problem: Suppose a cannon is sitting on top of a 50.0 m high hill and a 5.00 kg cannon ball is fired with a velocity of 30.0 m/s at some unknown angle. What will be the velocity of the cannon ball when it strikes the earth?

Solution: Since the angle at which the cannon ball is fired is unknown, we cannot use the usual equations from projectile motion. However, at the moment the cannon ball is fired, it has a certain KE due to the mass of the ball and its speed and it has a certain PE due to its mass and it height above the earth. Those two quantities of energy can be calculated. When the ball returns to the earth, its PE will be zero and therefore, its KE at that point must account for the total of its original $KE + PE$. This gives us a method of solving the problem.

$E_{\text{TOTAL}} = KE + PE = \frac{1}{2}\,mv^2 + mgh = \left(\frac{1}{2}\right)(5.00\ kg)(30.0\ m/s)^2 + (5.00\ kg)(9.80\ m/s^2)(50.0\ m)$

$E_{\text{TOTAL}} = 2250\ J + 2450\ J = 4700\ J$

$\frac{1}{2}\,m{v_f}^2 = 4700\ J$ so $v_f = \sqrt{\frac{(2)(4700\ J)}{5.00\ kg}} = 43.4\ m/s$

Example Problem: A 2.00 g bullet moving at 705 m/s strikes a 0.250 kg block of wood at rest on a frictionless surface. The bullet sticks in the wood and the combined mass moves slowly down the table.

(a) What is the KE of the bullet before the collision?

(b) What is the speed of the combination after the collision?

(c) How much KE was lost in the collision?

Solution:

(a) $KE_{\text{BULLET}} = \frac{1}{2}\,mv^2 = \left(\frac{1}{2}\right)(0.00200\ kg)(705\ m/s)^2 = 497\ J$

(b) $m_Bv_B + m_Wv_W = (m_{B+W})(v_{B+W})$

$(0.00200\ kg)(705\ m/s) + (0.250\ kg)(0\ m/s) = (0.252\ kg)(V)$

$(1.41\ kg\ m/s) = (0.252\ kg)(V)$

$V = 5.60\ m/s$

(c) $KE_{\text{COMBINATION}} = \frac{1}{2}\,mv^2 = \left(\frac{1}{2}\right)(0.252\ kg)(5.60\ m/s)^2 = 3.95\ J$

$KE_{\text{LOST}} = KE_{\text{BEFORE}} - KE_{\text{AFTER}} = 497\ J - 4\ J = 493\ J$

Summary

- In a closed system, energy may change forms but the total amount of energy is constant.

Practice

The following video demonstrates Newton Ball tricks. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=JadO3RuOJGU

MEDIA

Click image to the left for more content.

1. What happens when one ball is pulled up to one side and released?
2. What happens when three balls are pulled up to one side and released?
3. What happens when two balls are pulled out from each side and released?

Practice problems with answers for the law of conservation of energy:

http://www.physicsclassroom.com/class/energy/u5l2bc.cfm

Review

1. A 15.0 kg chunk of ice falls off the top of an iceberg. If the chunk of ice falls 8.00 m to the surface of the water,
   (a) what is the kinetic energy of the chunk of ice when its hits the water, and
   (b) what is its velocity?
2. An 85.0 kg cart is rolling along a level road at 9.00 m/s. The cart encounters a hill and coasts up the hill.
   (a) Assuming the movement is frictionless, at what vertical height will the cart come to rest?
   (b) Do you need to know the mass of the cart to solve this problem?
3. A circus performer swings down from a platform on a rope tied to the top of a tent in a pendulum-like swing. The performer's feet touch the ground 9.00 m below where the rope is tied. How fast is the performer moving at the bottom of the arc?
4. A skier starts from rest at the top of a 45.0 m hill, coasts down a 30°slope into a valley, and continues up to the top of a 40.0 m hill. Both hill heights are measured from the valley floor. Assume the skier puts no effort into the motion (always coasting) and there is no friction.
   (a) How fast will the skier be moving on the valley floor?
   (b) How fast will the skier be moving on the top of the 40.0 m hill?
5. A 2.00 kg ball is thrown upward at some unknown angle from the top of a 20.0 m high building. If the initial magnitude of the velocity of the ball is 20.0 m/s, what is the magnitutde of the final velocity when it strikes the ground? Ignore air resistance.
6. If a 2.00 kg ball is thrown straight upward with a KE of 500 J, what maximum height will it reach? Neglect air resistance.

- **conservation of energy:** An empirical law of physics (meaning it cannot be derived), states that the total amount of energy within an isolated system is constant. Although energy can be transformed from one form into another, energy cannot be created or destroyed
- **closed system:** Means it cannot exchange any of heat, work, or matter with the surroundings.
- **mechanical energy:** The sum of potential energy and kinetic energy.

References

1. User:Zonk43/Wikimedia Commons. http://commons.wikimedia.org/wiki/File:Teststrecke_Roller_Coaster3.JPG. Public Domain

CHAPTER 4 Energy Problem Solving

Students will learn how to analyze and solve more complicated problems involving energy conservation.

Students will learn how to analyze and solve more complicated problems involving energy conservation.

Key Equations

$\sum E_{\text{initial}} = \sum E$

final ;$Thetotalenergydoesnotchangeinclosedsystems$

$KE = \frac{1}{2}mv^2$; Kinetic energy

$PE_g = mgh$; Potential energy of gravity

$PE_{sp} = \frac{1}{2}kx^2$; Potential energy of a spring

$W = F_x \Delta x = Fd \cos\theta$; Work is equal to the distance multiplied by the component of the force in the direction it is moving.

Guidance

The main thing to always keep prescient in your mind is that the total energy before must equal the total energy after. If some energy has transferred out of or into the system via work, you calculate that work done and include it in the energy sum equation. Generally work done by friction is listed on the 'after' side and work put into the system, via a jet pack for example, goes on the 'before' side. Another important point is that on turns or going over hills or in rollercoaster loops, one must include the centripetal motion equations -for example to insure that you have enough speed to make the loop.

Example 1

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Click image to the left for more content.

Watch this Explanation

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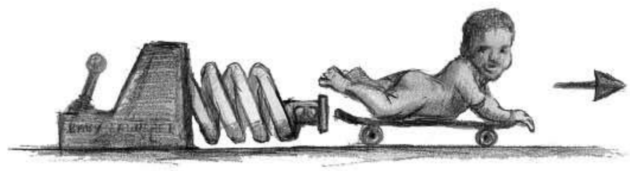
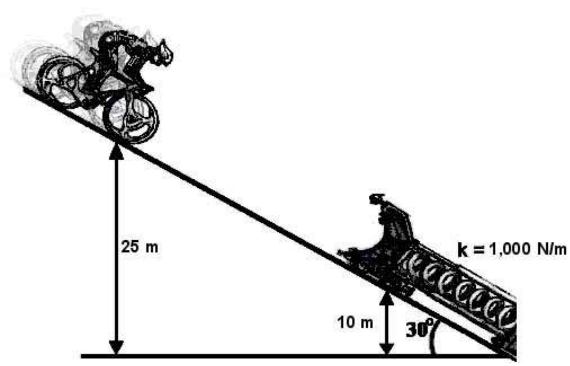
Time for Practice

1. A rock with mass m is dropped from a cliff of height h. What is its speed when it gets to the bottom of the cliff?
    a. $\sqrt{mg}$
    b. $\sqrt{\frac{2g}{h}}$
    c. $\sqrt{2gh}$
    d. gh
    e. None of the above



2. In the picture above, a 9.0 kg baby on a skateboard is about to be launched horizontally. The spring constant is 300 N/m and the spring is compressed 0.4 m. For the following questions, ignore the small energy loss due to the friction in the wheels of the skateboard and the rotational energy used up to make the wheels spin.
    a. What is the speed of the baby after the spring has reached its uncompressed length?
    b. After being launched, the baby encounters a hill 7 m high. Will the baby make it to the top? If so, what is his speed at the top? If not, how high does he make it?
    c. Are you finally convinced that your authors have lost their minds? Look at that picture!

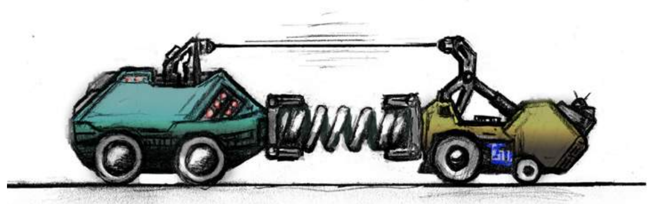


3. When the biker is at the top of the ramp shown above, he has a speed of 10 m/s and is at a height of 25 m. The bike and person have a total mass of 100 kg. He speeds into the contraption at the end of the ramp, which slows him to a stop.
    a. What is his initial total energy? (Hint: Set $U_g = 0$ at the very bottom of the ramp.)
    b. What is the length of the spring when it is maximally compressed by the biker? (Hint: The spring does *not* compress all the way to the ground so there is still some gravitational potential energy. It will help to draw some triangles.)
4. An elevator in an old apartment building in Switzerland has four huge springs at the bottom of the shaft to cushion its fall in case the cable breaks. The springs have an uncompressed height of about 1 meter. Estimate the spring constant necessary to stop this elevator, following these steps:
    a. First, guesstimate the mass of the elevator with a few passengers inside.
    b. Now, estimate the height of a five-story building.
    c. Lastly, use conservation of energy to estimate the spring constant.

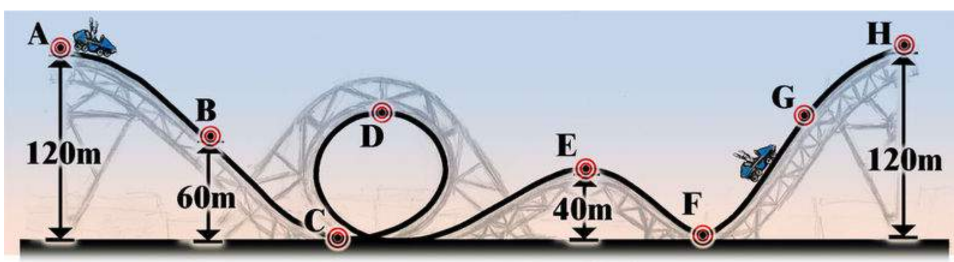
5. You are skiing down a hill. You start at rest at a height 120 m above the bottom. The slope has a 10.0° grade. Assume the total mass of skier and equipment is 75.0 kg.

   a. Ignore all energy losses due to friction. What is your speed at the bottom?
   b. If, however, you just make it to the bottom with zero speed what would be the average force of friction, including air resistance?

6. Two horrific contraptions on frictionless wheels are compressing a spring ($k = 400\ \text{N/m}$) by 0.5 m compared to its uncompressed (equilibrium) length. Each of the 500 kg vehicles is stationary and they are connected by a string. The string is cut! Find the speeds of the vehicles once they lose contact with the spring.



7. A roller coaster begins at rest 120 m above the ground, as shown. Assume no friction from the wheels and air, and that no energy is lost to heat, sound, and so on. The radius of the loop is 40 m.



   a. If the height at point G is 76 m, then how fast is the coaster going at point G?
   b. Does the coaster actually make it through the loop without falling? (Hint: You might review the material from centripetal motion lessons to answer this part.)

Answers to Selected Problems

1. .
2. a. 2.3 m/s c. No, the baby will not clear the hill.
3. a. 29,500 J b. Spring has maximum compressed length of 13 m
4. .
5. a. 48.5 m/s b. 128 N
6. 0.32 m/s each
7. a.29 m/s b. just barely, $a_C = 9.8\ \text{m/s}^2$

CHAPTER 5 Springs

Students will learn to calculate periods, frequencies, etc. of spring systems in harmonic motion.

Students will learn to calculate periods, frequencies, etc. of spring systems in harmonic motion.

Key Equations

$T = \frac{1}{f}$; Period is the inverse of frequency

$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$; Period of mass m on a spring with constant k

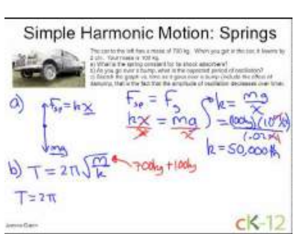
$F_{sp} = -kx$; the force of a spring equals the spring constant multiplied by the amount the spring is stretched or compressed from its equilibrium point. The negative sign indicates it is a restoring force (i.e. direction of the force is opposite its displacement from equilibrium position.

$U_{sp} = \frac{1}{2}kx^2$; the potential energy of a spring is equal to one half times the spring constant times the distance squared that it is stretched or compressed from equilibrium

Guidance

- The oscillating object does not lose any energy in SHM. Friction is assumed to be zero.
- In harmonic motion there is always a *restorative force,* which attempts to *restore* the oscillating object to its equilibrium position. The restorative force changes during an oscillation and depends on the position of the object. In a spring the force is given by Hooke's Law: $F = -kx$
- The period, T, is the amount of time needed for the harmonic motion to repeat itself, or for the object to go one full cycle. In SHM, T is the time it takes the object to return to its exact starting point and starting direction.
- The frequency, f, is the number of cycles an object goes through in 1 second. Frequency is measured in Hertz (Hz). $1\ Hz = 1$ cycle per sec.
- The amplitude, A, is the distance from the *equilibrium* (or center) *point* of motion to either its lowest or highest point (*end points*). The amplitude, therefore, is half of the total distance covered by the oscillating object. The amplitude can vary in harmonic motion, but is constant in SHM.

Example 1



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Watch this Explanation

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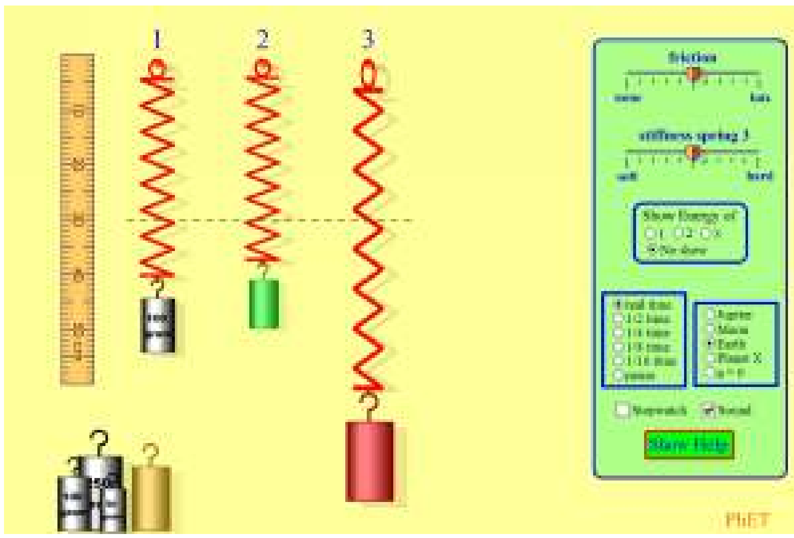
Simulation



Mass & Springs (PhET Simulation)

Time for Practice

1. A rope can be considered as a spring with a very high spring constant k, so high, in fact, that you don't notice the rope stretch at all before it "pulls back."
   a. What is the k of a rope that stretches by 1 mm when a 100 kg weight hangs from it?
   b. If a boy of 50 kg hangs from the rope, how far will it stretch?
   c. If the boy kicks himself up a bit, and then is bouncing up and down ever so slightly, what is his frequency of oscillation? Would he notice this oscillation? If so, how? If not, why not?
2. If a 5.0 kg mass attached to a spring oscillates 4.0 times every second, what is the spring constant k of the spring?
3. A horizontal spring attached to the wall is attached to a block of wood on the other end. All this is sitting on a frictionless surface. The spring is compressed 0.3 m. Due to the compression there is 5.0 J of energy stored in the spring. The spring is then released. The block of wood experiences a maximum speed of 25 m/s.
   a. Find the value of the spring constant.
   b. Find the mass of the block of wood.
   c. What is the equation that describes the position of the mass?
   d. What is the equation that describes the speed of the mass?
   e. Draw three complete cycles of the block's oscillatory motion on an x vs. t graph.

4. A spider of 0.5 g walks to the middle of her web. The web sinks by 1.0 mm due to her weight. You may assume the mass of the web is negligible.
    a. If a small burst of wind sets her in motion, with what frequency will she oscillate?
    b. How many times will she go up and down in one s? In 20 s?
    c. How long is each cycle?
    d. Draw the x vs t graph of three cycles, assuming the spider is at its highest point in the cycle at $t = 0$ s.

Answers to Selected Problems

1. a. 9.8×10^5 N/m b. 0.5 mm c. 22 Hz
2. 3.2×10^3 N/m
3. a. 110 N/m d. $v(t) = (25)\cos(83t)$
4. a. 16 Hz b. 16 complete cycles but 32 times up and down, 315 complete cycles but 630 times up and down c. 0.063 s

Investigation

1. Your task: Match the period of the circular motion system with that of the spring system. You are only allowed to change the velocity involved in the circular motion system. Consider the effective distance between the block and the pivot to be to be fixed at 1m. The spring constant(13.5N/m) is also fixed. You should view the charts to check whether you have succeeded. Instructions: To alter the velocity, simply click on the Select Tool, and select the pivot . The Position tab below will allow you to numerically adjust the rotational speed using the Motor field. To view the graphs of their respective motion in order to determine if they are in sync, click on Chart tab below.

2.

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3. Now the mass on the spring has been replaced by a mass that is twice the rotating mass. Also, the distance between the rotating mass and the pivot has been changed to 1.5 m. What velocity will keep the period the same now?

4.

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CK-12 FlexBook

Physics Unit 7 (Work, Power, Kinetic Energy Theorem)

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Ck12 Science

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To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

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2 Power **4**

3 Work-Energy Principle **7**

CONCEPT 1 Work

- Define work.
- Identify forces that are doing work.
- Given two of the three variables in the equation, $W = Fd$, calculate the third.

In order for the roller coaster to run down the incline by gravitational attraction, it first must have work done on it towing it up to the top of the hill. The work done on the coaster towing it to the top of the hill becomes potential energy stored in the coaster and that potential energy is converted to kinetic energy as the coaster runs down from the top of the hill to the bottom.

Work

The word **work** has both an everyday meaning and a specific scientific meaning. In the everyday use of the word, work would refer to anything which required a person to make an effort. In physics, however, work is defined as the force exerted on an object multiplied by the distance the object moves due to that force.

$W = Fd$

In the scientific definition of the word, if you push against an automobile with a force of 200 N for 3 minutes but the automobile does not move, then you have done NO work. Multiplying 200 N times 0 meters yields zero work. If you are holding an object in your arms, the upward force you are exerting is equal to the object's weight. If you hold the object until your arms become very tired, you have still done no work because you did not move the object in the direction of the force. When you lift an object, you exert a force equal to the object's weight and the object moves due to that lifting force. If an object weighs 200. N and you lift it 1.50 meters, then your work is, $W = Fd = (200.\text{ N})(1.50\text{ m}) = 300.\text{ N m}$.

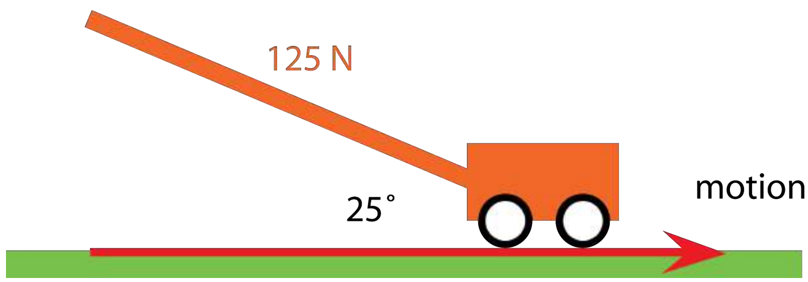
One of the units you will see for work is the Newton·meter but since a Newton is also a kilogram·m/s ^{2} , then a Newton·meter is also kg·m ^{2} /s ^{2} . This unit has also been named the ***Joule*** (pronounced Jool) in honor of James Prescott Joule, a nineteenth century English physicist.

Example Problem: A boy lifts a box of apples that weighs 185 N. The box is lifted a height of 0.800 m. How much work did the boy do?

Solution: $W = Fd = (185\text{ N})(0.800\text{ m}) = 148\text{ N m} = 148\text{ Joules}$

Work is done only if a force is exerted in the direction of motion. If the force and motion are perpendicular to each other, no work is done because there is no motion in the direction of the force. If the force is at an angle to the motion, then the component of the force in the direction of the motion is used to determine the work done.

Example Problem: Suppose a 125 N force is applied to a lawnmower handle at an angle of 25° with the ground and the lawnmower moves along the surface of the ground. If the lawnmower moves 56 m, how much work was done?



Solution: The solution requires that we determine the component of the force that was in the direction of the motion of the lawnmower. The component of the force that was pushing down on the ground does not contribute to the work done.

$F_{\text{parallel}} = (\text{Force})(\cos 25°) = (125\text{ N})(0.906) = 113\text{ N}$

$W = F_{\text{parallel}} d = (113\text{ N})(56\text{ m}) = 630\text{ J}$

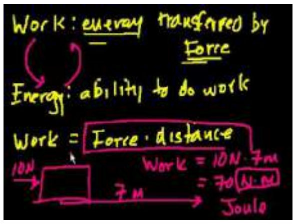
Summary

- In physics, work is defined as the force exerted on an object multiplied by the distance the object moves due to that force.
- The unit for work is called the joule.
- If the force is at an angle to the motion, then the component of the force in the direction of the motion is used to determine the work done.

Practice

The following video introduces energy and work. Use this resource to answer the questions that follow.

http://wn.com/Work_physics_#/videos



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1. What definition is given in the video for energy?
2. What is the definition given in the video for work?
3. What unit is used in the video for work?

The following website contains practice questions with answers on the topic of work.

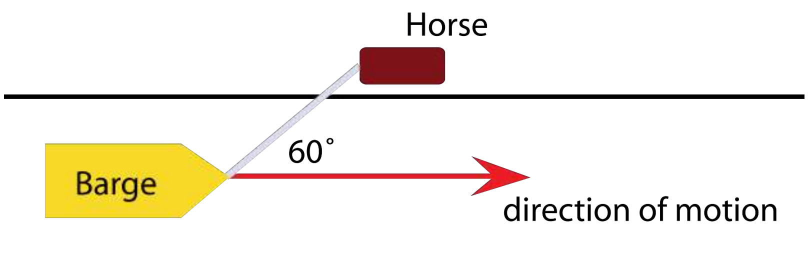
http://www.sparknotes.com/testprep/books/sat2/physics/chapter7section6.rhtml

Review

1. How much work is done by the force of gravity when a 45 N object falls to the ground from a height of 4.6 m?

2. A workman carries some lumber up a staircase. The workman moves 9.6 m vertically and 22 m horizontally. If the lumber weighs 45 N, how much work was done by the workman?

3. A barge is pulled down a canal by a horse walking beside the canal. If the angle of the rope is 60.0°, the force exerted is 400. N, and the barge is pulled 100. M, how much work did the horse do?



- **work:** A force is said to do work when it acts on a body so that there is a displacement of the point of application, however small, in the direction of the force. Thus a force does work when it results in movement. The work done by a constant force of magnitude F on a point that moves a distance d in the direction of the force is the product, $W = Fd$.
- **joule:** The SI unit of work or energy, equal to the work done by a force of one Newton when its point of application moves through a distance of one meter in the direction of the force.

References

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CONCEPT 2 Power

- Define power.
- Given two of the three variables in $P = \frac{W}{t}$, calculate the third.

Typical Pressurized Water Reactors (PWR) reactors built in the 1970's produce about 1100 megawatts, whilst the latest designs range up to around 1500 megawatts. That is 1,500,000,000 Joules/second.

A windmill farm, by comparison, using hundreds of individual windmills produces about 5 megawatts. That is 5,000,000 Joules/second (assuming the wind is blowing).

Power

Power is defined as the rate at which work is done or the rate at which energy is transformed.

$$Power = \frac{Work}{Time}$$

In SI units, power is measured in Joules per second which is given a special name, the watt **,** W **.**

1.00 watt = 1.00 J/s

Another unit for power that is fairly common is horsepower.

1.00 horsepower = 746 watts

Example Problem: A 70.0 kg man runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate the power output of the man in watts and horsepower.

Solution:

The force exerted must be equal to the weight of the man $= mg = (70.0 \text{ kg})(9.80 \text{ m/s}^2) = 686 \text{ N}$

$W = Fd = (686 \text{ N})(4.5 \text{ m}) = 3090 \text{ N m} = 3090 \text{ J}$

$P = \frac{W}{t} = \frac{3090 \text{ J}}{4.0 \text{ s}} = 770 \text{ J/s} = 770 \text{ W}$

$P = 770 \text{ W} = 1.03 \text{ hp}$

Since $P = \frac{W}{t}$ and $W = Fd$, we can use these formulas to derive a formula relating power to the speed of the object that is produced by the power.

$P = \frac{W}{t} = \frac{Fd}{t} = F\frac{d}{t} = Fv$

The velocity in this formula is the average speed of the object during the time interval.

Example Problem: Calculate the power required of a 1400 kg car if the car climbs a 10° hill at a steady 80. km/h.

Solution: 80. km/h = 22.2 m/s

In 1.00 s, the car would travel 22.2 m on the road surface but the distance traveled upward would be (22.2 m)(sin 10°) = (22.2 m)(0.174) = 3.86 m. The force in the direction of the upward motion is the weight of the car = (1400 kg)(9.80 m/s ^{2}) = 13720 N.

$W = Fd = (13720 \text{ N})(3.86 \text{ m}) = 53,000 \text{ J}$

Since this work was done in 1.00 second, the power would be 53,000 W.

If we calculated the upward component of the velocity of the car, we would divide the distance traveled in one second by one second and get an average vertical speed of 3.86 m/s. So we could use the formula relating power to average speed to calculate power.

$P = Fv = (13720 \text{ N})(3.86 \text{ m/s}) = 53,000 \text{ W}$

Summary

- Power is defined as the rate at which work is done or the rate at which energy is transformed.
- Power $= \frac{\text{Work}}{\text{Time}}$
- Power $=$ Force $\times$ velocity

Practice

In the following video, Mr. Edmond sings about work and power. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=5EsMmdaYClQ

MEDIA

Click image to the left for more content.

1. What units are used for work?
2. What units are used for power?

The following website has practice problems on work and power.

http://www.angelfire.com/scifi/dschlott/workpp.html

Review

1. If the circumference of an orbit for a toy on a string is 18 m and the centripetal force is 12 N, how much work does the centripetal force do on the toy when it follows its orbit for one cycle?
2. A 50.0 kg woman climbs a flight of stairs 6.00 m high in 15.0 s. How much power does she use?
3. (a) Assuming no friction, what is the minimum work needed to push a 1000. kg car 45.0 m up a 12.5° incline?
   (b) What power would be needed for the same problem if friction is considered and the coefficient of friction for the car is 0.30?

- **power:** The rate at which this work is performed.

References

1. Courtesy of Ryan Hagerty, U.S. Fish and Wildlife Service. http://digitalmedia.fws.gov/cdm/singleitem/collection/natdiglib/id/13455/rec/2 . Public Domain

CONCEPT 3 Work-Energy Principle

The reason the concept of work is so useful is because of a theorem, called the **work-energy principle** , which states that *the change in an object's kinetic energy is equal to the net work done on it* :

$$\Delta K_e = W_{net} \ [2]$$

Although we cannot derive this principle in general, we can do it for the case that interests us most: constant acceleration. In the following derivation, we assume that the force is along motion. This doesn't reduce the generality of the result, but makes the derivation more tractable because we don't need to worry about vectors or angles.

Recall that an object's kinetic energy is given by the formula:

$$K_e = \frac{1}{2}mv^2 \ [3]$$

Consider an object of mass m accelerated from a velocity v_i to v_f under a constant force. The change in kinetic energy, according to [2], is equal to:

$$\Delta K_e = K_{ei} - K_{ef} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2) \ [4]$$

Now let's see how much work this took. To find this, we need to find the distance such an object will travel under these conditions. We can do this by using the third of our 'Big three' equations, namely:

$$v_f{}^2 = v_i{}^2 + 2a\Delta x \ [5]$$

alternatively,

$$\Delta x = \frac{v_f{}^2 - v_i{}^2}{2a} \ [6]$$

Plugging in [6] and Newton's Third Law, $F = ma$, into [2], we find:

$$W = F\Delta x = ma \times \frac{v_f{}^2 - v_i{}^2}{2a} = \frac{1}{2}m(v_f^2 - v_i^2) \ [7],$$

which was our result in [4].

Using the Work-Energy Principle

The Work-Energy Principle can be used to derive a variety of useful results. Consider, for instance, an object dropped a height Δh under the influence of gravity. This object will experience constant acceleration. Therefore, we can again use equation [6], substituting gravity for acceleration and Δh for distance:

$$\Delta h = \frac{{v_f}^2 - {v_i}^2}{2g}$$

multiplying both sides by mg, we find:

$$mg\Delta h = m\cancel{g}\frac{{v_f}^2 - {v_i}^2}{2\cancel{g}} = \Delta K_e \text{ [8]}$$

In other words, the work performed on the object by gravity in this case is $mg\Delta h$. We refer to this quantity as gravitational potential energy; here, we have derived it as a function of height. For most forces (exceptions are friction, air resistance, and other forces that convert energy into heat), potential energy can be understood as the ability to perform work.

Spring Force

A spring with spring constant k a distance Δx from equilibrium experiences a restorative force equal to:

$$F_s = -k\Delta x \text{ [9]}$$

This is a force that can change an object's kinetic energy, and therefore do work. So, it has a potential energy associated with it as well. This quantity is given by:

$$E_{sp} = \frac{1}{2}k\Delta x^2 \qquad \text{[10] Spring Potential Energy}$$

The derivation of [10] is left to the reader. Hint: find the average force an object experiences while moving from $x = 0$ to $x = \Delta x$ while attached to a spring. The net work is then this force times the displacement. Since this quantity (work) must equal to the change in the object's kinetic energy, it is also equal to the potential energy of the spring. This derivation is very similar to the derivation of the kinematics equations — look those up.

This applet may be useful in reviewing Spring Potential Energy:

http://phet.colorado.edu/en/simulation/mass-spring-lab

CK-12 FlexBook

Physics Unit 8 Momentum

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To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

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Concept 1 Momentum

- Define momentum.
- Relate momentum to mass and velocity.
- Calculate momentum from mass and velocity.

Cody seems a little reluctant to launch himself down this ramp at Newton's Skate Park. It will be his first time down the ramp, and he knows from watching his older brother Jerod that he'll be moving fast by the time he gets to the bottom. The faster he goes, the harder it will be to stop. That's because of momentum.

What Is Momentum?

Momentum is a property of a moving object that makes it hard to stop. The more mass it has or the faster it's moving, the greater its momentum. Momentum equals mass times velocity and is represented by the equation:

Momentum = Mass x Velocity

Q : What is Cody's momentum as he stands at the top of the ramp?

A : Cody has no momentum as he stands there because he isn't moving. In other words, his velocity is zero. However, Cody will gain momentum as he starts moving down the ramp and picks up speed.

Q : Cody's older brother Jerod is pictured 1.1 . If Jerod were to travel down the ramp at the same velocity as Cody, who would have greater momentum? Who would be harder to stop?

A : Jerod obviously has greater mass than Cody, so he would have greater momentum. He would also be harder to stop.

FIGURE 1.1

You can see an animation demonstrating the role of mass and velocity in the momentum of moving objects at this URL:

http://www.science-animations.com/support-files/momentum.swf

Calculating Momentum

To calculate momentum with the equation above, mass is measured in (kg), and velocity is measured in meters per second (m/s). For example, Cody and his skateboard have a combined mass of 40 kg. If Cody is traveling at a velocity of 1.1 m/s by the time he reaches the bottom of the ramp, then his momentum is:

Momentum = 40 kg x 1.1 m/s = 44 kg • m/s

Note that the SI unit for momentum is kg • m/s.

Q : The combined mass of Jerod and his skateboard is 68 kg. If Jerod goes down the ramp at the same velocity as Cody, what is his momentum at the bottom of the ramp?

A : His momentum is:

Momentum = 68 kg x 1.1 m/s = 75 kg • m/s

Summary

- Momentum is a property of a moving object that makes it hard to stop. It equals the object's mass times its velocity.

- To calculate the momentum of a moving object, multiply its mass in kilograms (kg) by its velocity in meters per second (m/s). The SI unit of momentum is kg • m/s.

Vocabulary

- **momentum** : Property of a moving object that makes it hard to stop; equal to the object's mass times its velocity.

Practice

At the following URL, review how to calculate momentum, and then solve the problems at the bottom of the Web page.

http://www2.franciscan.edu/academic/mathsci/mathscienceintegation/MathScienceIntegation-848.htm

Review

1. Define momentum.
2. Write the equation for calculating momentum from mass and velocity.
3. What is the SI unit for momentum?
4. Which skateboarder has greater momentum?
   a. Skateboarder A: mass = 60 kg; velocity = 1.5 m/s
   b. Skateboarder B: mass = 50 kg; velocity = 2.0 m/s

References

1. . . used under license from Shutterstock

CONCEPT 2 Momentum

Students will learn what momentum is and how to calculate momentum of objects. In addition, students will learn how to use conservation of momentum to solve basic problems.

Students will learn what momentum is and how to calculate momentum of objects. In addition, students will learn how to use conservation of momentum to solve basic problems.

Key Equations

$p = mv$ Momentum is equal to the objects mass multiplied by its velocity

$\sum p_{\text{initial}} = \sum p$

final The total momentum does not change in closed systems

Example 1

A truck with mass 500 kg and originally carrying 200 kg of dirt is rolling forward with the transmission in neutral and shooting out the dirt backwards at 2 m/s (so that the dirt is at relative speed of zero compared with the ground). If the truck is originally moving at 2 m/s, how fast will it be moving after it has shot out all the dirt. You may ignore the effects of friction.

Solution

To solve this problem we will apply conservation of momentum to the truck when it is full of dirt and when it has dumped all the dirt.

$m_i v_i = m_f v_f$	start by setting the initial momentum equal to the final momentum
$(m_t + m_d) v_i = m_t v_f$	substitute the mass of the truck plus the mass of the dirt in the truck at the initial and
$v_f = \frac{(m_t + m_d) v_i}{m_t}$	solve for the final velocity
$v_f = \frac{(500 \text{ kg} + 200 \text{ kg}) * 2 \text{ m/s}}{500 \text{ kg}}$	plug in the numerical values
$v_f = 2.8 \text{ m/s}$	

Example 2

John and Bob are standing at rest in middle of a frozen lake so there is no friction between their feet and the ice. Both of them want to get to shore so they simultaneously push off each other in opposite directions. If John's mass is 50 kg and Bob's mass is 40 kg and John moving at 5 m/s after pushing off Bob, how fast is Bob moving?

Solution

For this problem, we will apply conservation of momentum to the whole system that includes both John and Bob. Since both of them are at rest to start, we know that the total momentum of the whole system must always be zero. Therefore, we know that the sum of John's and Bob's momentum after they push off each other is also zero. We can use this to solve for Bob's velocity.

$$\begin{aligned}
0 &= m_j v_j + m_b v_b \\
-m_b v_b &= m_j v_j \\
v_b &= -\frac{m_j v_j}{m_b} \\
v_b &= -\frac{50 \text{ kg} * 5 \text{ m/s}}{40 \text{ kg}} \\
v_b &= -6.25 \text{ m/s}
\end{aligned}$$

The answer is negative because Bob is traveling in the opposite direction to John.

Watch this Explanation

MEDIA

Click image to the left for more content.

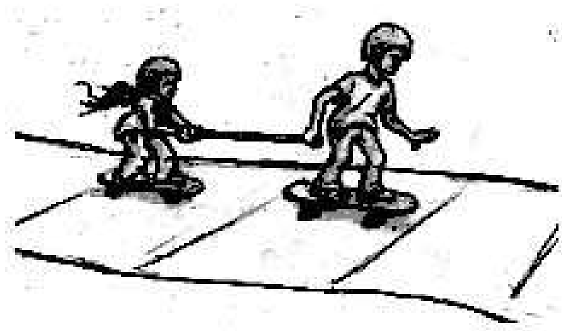
Time for Practice

1. You find yourself in the middle of a frozen lake. There is no friction between your feet and the ice of the lake. You need to get home for dinner. Which strategy will work best?

   a. Press down harder with your shoes as you walk to shore.
   b. Take off your jacket. Then, throw it in the direction opposite to the shore.
   c. Wiggle your butt until you start to move in the direction of the shore.
   d. Call for help from the great Greek god Poseidon.

2. You and your sister are riding skateboards side by side at the same speed. You are holding one end of a rope and she is holding the other. Assume there is no friction between the wheels and the ground. If your sister lets go of the rope, how does your speed change?

   a. It stays the same.
   b. It doubles.
   c. It reduces by half.



3. You and your sister are riding skateboards (see Problem 3), but now she is riding behind you. You are holding one end of a meter stick and she is holding the other. At an agreed time, you push back on the stick hard enough to get her to stop. What happens to your speed? Choose one. (For the purposes of this problem pretend you and your sister weigh the same amount.)
   a. It stays the same.
   b. It doubles.
   c. It reduces by half.
4. An astronaut is using a drill to fix the gyroscopes on the Hubble telescope. Suddenly, she loses her footing and floats away from the telescope. What should she do to save herself?
5. A 5.00 kg firecracker explodes into two parts: one part has a mass of 3.00 kg and moves at a velocity of 25.0 m/s towards the west. The other part has a mass of 2.00 kg . What is the velocity of the second piece as a result of the explosion?
6. A firecracker lying on the ground explodes, breaking into two pieces. One piece has twice the mass of the other. What is the ratio of their speeds?
7. While driving in your pickup truck down Highway 280 between San Francisco and Palo Alto, an asteroid lands in your truck bed! Despite its 220 kg mass, the asteroid does not destroy your 1200 kg truck. In fact, it landed perfectly vertically. Before the asteroid hit, you were going 25 m/s . After it hit, how fast were you going?
8. An astronaut is 100 m away from her spaceship doing repairs with a 10.0 kg wrench. The astronaut's total mass is 90.0 kg and the ship has a mass of 1.00×10^4 kg . If she throws the wrench in the opposite direction of the spaceship at 10.0 m/s how long would it take for her to reach the ship?

Answers to Selected Problems

1. .
2. .
3. .
4. .
5. 37.5 m/s
6. $v_1 = 2v_2$
7. 21 m/s
8. a. 90 sec

CONCEPT 3 Law of Conservation of Momentum

- State the law of conservation of momentum.
- Describe an example of momentum being transferred and conserved.

These skaters are racing each other at Newton's Skate Park. The first skater in line, the one on the left, is distracted by something he sees. He starts to slow down without realizing it. The skater behind him isn't paying attention and keeps skating at the same speed.

Q : Can you guess what happens next?

A : Skater 2 runs into skater 1.

Conserving Momentum

When skater 2 runs into skater 1, he's going faster than skater 1 so he has more momentum. **Momentum** is a property of a moving object that makes it hard to stop. It's a product of the object's mass and velocity. At the moment of the collision, skater 2 transfers some of his momentum to skater 1, who shoots forward when skater 2 runs into him. Whenever an action and reaction such as this occur, momentum is transferred from one object to the other. However, the combined momentum of the objects remains the same. In other words, momentum is conserved. This is the **law of conservation of momentum** .

Modeling Momentum

The **Figure** 3.1 shows how momentum is conserved in the two colliding skaters. The total momentum is the same after the collision as it was before. However, after the collision, skater 1 has more momentum and skater 2 has less momentum than before.

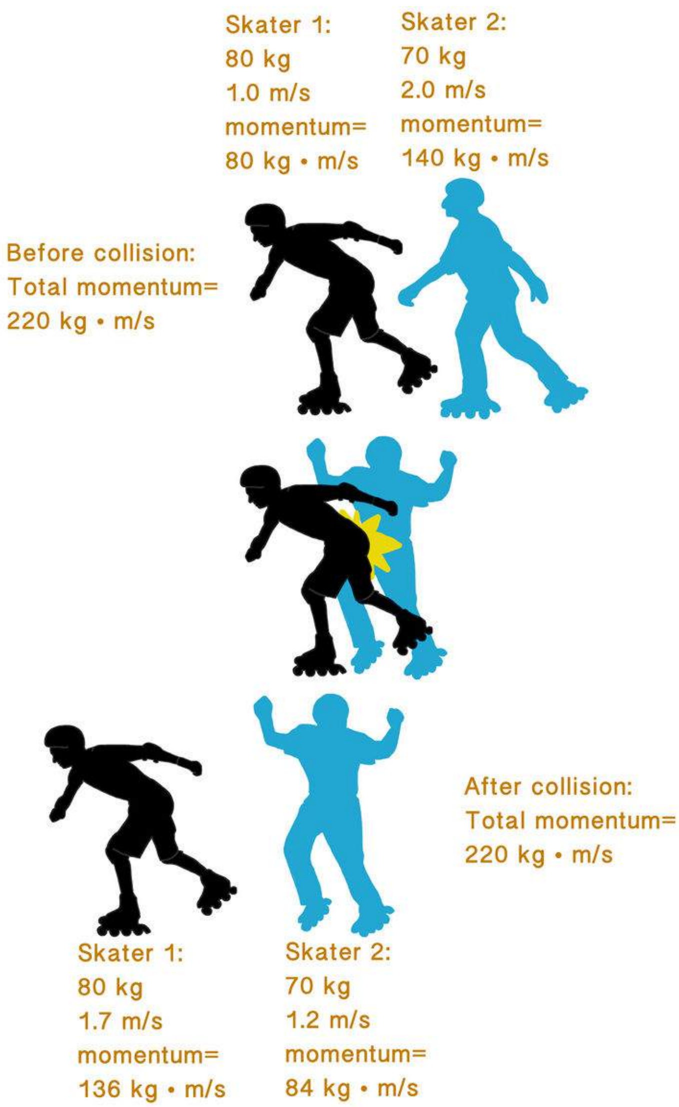


FIGURE 3.1

Q : What if two skaters have a head-on collision? Do you think momentum is conserved then?

A : As in all actions and reactions, momentum is also conserved in a head-on collision. You can see how at this URL:

http://www.physicsclassroom.com/mmedia/momentum/cthoi.cfm

Summary

- Whenever an action and reaction occur, momentum is transferred from one object to the other. However, total momentum is conserved. This is the law of conservation of momentum.

Vocabulary

- **momentum** : Property of a moving object that makes it hard to stop; equal to the object's mass times its velocity.
- **law of conservation of momentum** : Law stating that, when an action and reaction occur, the combined momentum of the objects remains the same.

Practice

Watch the astropitch animation at the following URL. Experiment with different velocities. Then take the quiz and check your answers.

http://www.phys.utb.edu/~pdukes/standard/PhysApplets/AstroPitch/TabbedastroPitch2.html

Review

1. State the law of conservation of momentum.
2. Fill in the missing velocity (x) in the diagram of a vehicle collision seen in the **Figure** 3.2 so that momentum is conserved.

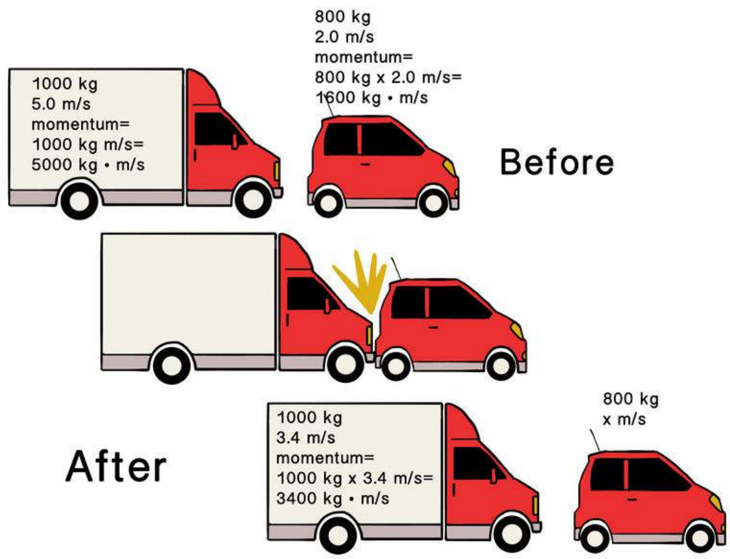


FIGURE 3.2
Solve for x.

References

1. Laura Guerin. . CC BY-NC 3.0
2. Laura Guerin. . CC BY-NC 3.0

Concept 4 Conservation of Momentum in One Dimension

- State the law of conservation of momentum.
- Use the conservation of momentum to solve one-dimensional collision problems.

For this whale to leap out of the water, something underwater must be moving in the opposite direction, and intuition tells us it must be moving with relatively high velocity. The water that moves downward is pushed downward by the whale's tail, and that allows the whale to rise up.

Conservation of Momentum in One Dimension

When impulse and momentum were introduced, we used an example of a batted ball to discuss the impulse and momentum change that occurred with the ball. At the time, we did not consider what had happened to the bat. According to Newton's third law, however, when the bat exerted a force on the ball, the ball also exerted an equal and opposite force on the bat. Since the time of the collision between bat and ball is the same for the bat and for the ball, then we have equal forces (in opposite directions) exerted for equal times on the ball AND the bat. That means that the impulse exerted on the bat is equal and opposite (-Ft) to the impulse on the ball (Ft) and that also means that there was a change in momentum of the bat $[-\Delta(mv)_{\text{BAT}}]$ that was equal and opposite to the change in momentum of the ball $[\Delta(mv)_{\text{BALL}}]$.

The change in momentum of the ball is quite obvious because it changes direction and flies off at greater speed. However, the change in momentum of the bat is not obvious at all. This occurs primarily because the bat is more massive than the ball. Additionally, the bat is held firmly by the batter, so the batter's mass can be combined with the mass of the bat. Since the bat's mass is so much greater than that of the ball, but they have equal and opposite forces, the bat's final velocity is significantly smaller than that of the ball.

Consider another system: that of two ice skaters. If we have one of the ice skaters exert a force on the other skater, the force is called an **internal force** because both the object exerting the force and the object receiving the force are inside the system. In a closed system such as this, momentum is always conserved. The total final momentum always equals the total initial momentum in a closed system. Conversely, if we defined a system to contain just one ice skater, putting the other skater outside the system, this is not a closed system. If one skater pushes the other, the force is an external force because the receiver of the force is outside the system. Momentum is not guaranteed to be conserved unless the system is closed.

In a closed system, momentum is always conserved. Take another example: if we consider two billiard balls colliding on a billiard table and ignore friction, we are dealing with a closed system. The momentum of ball *A* before the collision plus the momentum of ball *B* before collision will equal the momentum of ball *A* after collision plus the momentum of ball *B* after collision. This is called the law of **conservation of momentum** and is given by the equation

$$p_{A\text{before}} + p_{B\text{before}} = p_{A\text{after}} + p_{B\text{after}}$$

Example Problem: Ball *A* has a mass of 2.0 kg and is moving due west with a velocity of 2.0 m/s while ball *B* has a mass of 4.0 kg and is moving west with a velocity of 1.0 m/s. Ball *A* overtakes ball *B* and collides with it from behind. After the collision, ball *A* is moving westward with a velocity of 1.0 m/s. What is the velocity of ball *B* after the collision?

Solution: Because of the law of conservation of momentum, we know that

$$p_{A\text{before}} + p_{B\text{before}} = p_{A\text{after}} + p_{B\text{after}}$$

.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$(2.0 \text{ kg})(2.0 \text{ m/s}) + (4.0 \text{ kg})(1.0 \text{ m/s}) = (2.0 \text{ kg})(1.0 \text{ m/s}) + (4.0 \text{ kg})(v_B{}' \text{ m/s})$$
$$4.0 \text{ kg} \cdot \text{m/s} + 4.0 \text{ kg} \cdot \text{m/s} = 2.0 \text{ kg} \cdot \text{m/s} + 4v_B{}' \text{ kg} \cdot \text{m/s}$$
$$4v_B{}' = 8.0 - 2.0 = 6.0$$
$$v_B{}' = 1.5 \text{ m/s}$$

After the collision, ball *B* is moving westward at 1.5 m/s.

Example Problem: A railroad car whose mass is 30,000. kg is traveling with a velocity of 2.2 m/s due east and collides with a second railroad car whose mass is also 30,000. kg and is at rest. If the two cars stick together after the collision, what is the velocity of the two cars?

Solution: Note that since the two trains stick together, the final mass is m $_A$ +m $_B$, and the final velocity for each object is the same. Thus the conservation of momentum equation, $m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$, can be rewritten $m_A v_A + m_B v_B = (m_A + m_B)\, v'$

$$(30,000. \text{ kg})(2.2 \text{ m/s}) + (30,000. \text{ kg})(0 \text{ m/s}) = (60,000. \text{ kg})(v' \text{ m/s})$$
$$66000 + 0 = 60000v'$$
$$v' = \frac{66000}{60000} = 1.1 \text{ m/s}$$

After the collision, the two cars move off together toward the east with a velocity of 1.1 m/s.

Summary

- A closed system is one in which both the object exerting a force and the object receiving the force are inside the system.
- In a closed system, momentum is always conserved.

Practice

The following video shows the Mythbusters building and using various sizes of a toy called "Newton's Cradle." Use this resource to answer the question that follows.

https://www.youtube.com/watch?v=BiLq5Gnpo8Q

MEDIA

Click image to the left for more content.

1. What is Newton's Cradle?
2. How does Newton's Cradle work?
3. How does a Newton's Cradle show conservation of momentum?

If you are interested in learning more about Newton's Cradles, visit this site: http://science.howstuffworks.com/newtons-cradle.htm

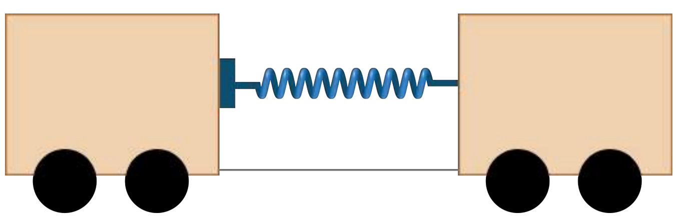
Review

1. A 0.111 kg hockey puck moving at 55 m/s is caught by a 80. kg goalie at rest. With what speed does the goalie slide on the (frictionless) ice?

2. A 0.050 kg bullet strikes a 5.0 kg stationary wooden block and embeds itself in the block. The block and the bullet fly off together at 9.0 m/s. What was the original velocity of the bullet?

3. A 0.50 kg ball traveling at 6.0 m/s due east collides head on with a 1.00 kg ball traveling in the opposite direction at -12.0 m/s. After the collision, the 0.50 kg ball moves away at -14 m/s. Find the velocity of the second ball after the collision.

4. Two carts are stationary with a compressed spring between them and held together by a thread. When the thread is cut, the two carts move apart. After the spring is released, one cart $m = 3.00$ kg has a velocity of 0.82 m/s east. What is the magnitude of the velocity of the second cart $(m = 1.70 \text{ kg})$ after the spring is released?



5. Compared to falling on a tile floor, a glass may not break if it falls onto a carpeted floor. This is because

a. less impulse in stopping.
b. longer time to stop.
c. both of these
d. neither of these.

6. A butterfly is hit by a garbage truck on the highway. The force of the impact is greater on the

a. garbage truck.
b. butterfly.
c. it is the same for both.

7. A rifle recoils from firing a bullet. The speed of the rifle's recoil is small compared to the speed of the bullet because

a. the force on the rifle is small.
b. the rifle has a great deal more mass than the bullet.
c. the momentum of the rifle is unchanged.
d. the impulse on the rifle is less than the impulse on the bullet.
e. none of these.

- **Law of Conservation of Momentum:** The total linear momentum of an isolated system remains constant regardless of changes within the system.

References

1. Courtesy of NOAA. http://commons.wikimedia.org/wiki/File:Humpback_whale_noaa.jpg . Public Domain
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

Concept 5 Conservation of Momentum in Two Dimensions

- Use the conservation of momentum and vector analysis to solve two-dimensional collision problems.
- Review vector components.

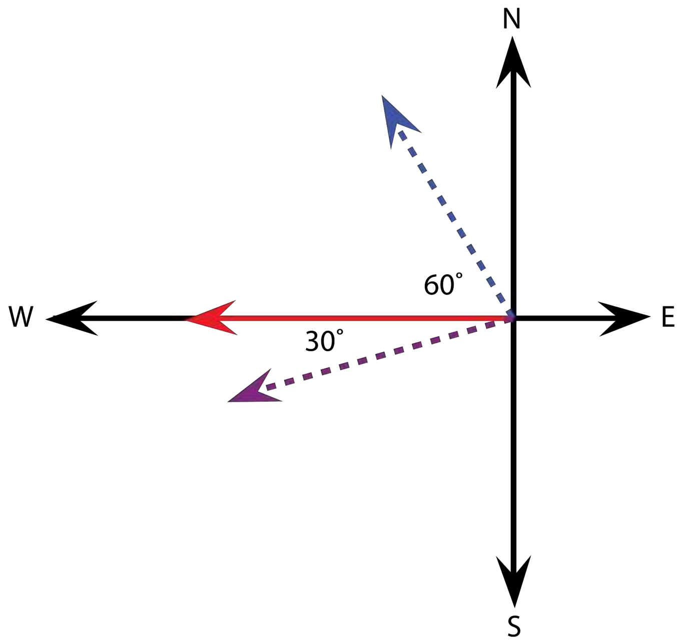
In a game of billiards, it is important to be able to visualize collisions in two dimensions – the best players not only know where the target ball is going but also where the cue ball will end up.

Conservation of Momentum in Two Dimensions

Conservation of momentum in all closed systems is valid, regardless of the directions of the objects before and after they collide. Most objects are not confined to a single line, like trains on a rail. Rather, many objects, like billiard balls or cars, can move in two dimensions. Conservation of momentum for these objects can also be calculated; momentum is a vector and collisions of objects in two dimensions can be represented by axial vector components. To review axial components, revisit Vectors: Resolving Vectors into Axial Components and Vectors: Vector Addition.

Example Problem: A 2.0 kg ball, A , is moving with a velocity of 5.00 m/s due west. It collides with a stationary ball, B , also with a mass of 2.0 kg. After the collision, ball A moves off at 30° south of west while ball B moves off at 60° north of west. Find the velocities of both balls after the collision.

Solution: Since ball B is stationary before the collision, then the total momentum before the collision is equal to momentum of ball A . The momentum of ball A before collision is shown in red below, and can be calculated to be $p = mv = (2.00 \text{ kg})(5.00 \text{ m/s}) = 10.0 \text{ kg m/s west}$



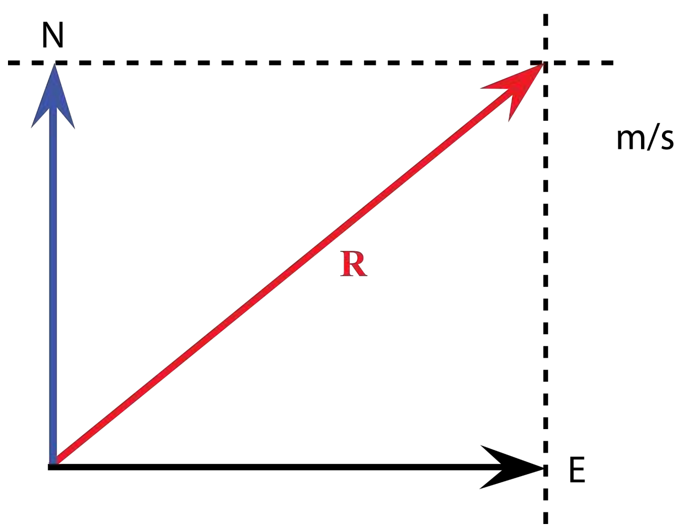
Since momentum is conserved in this collision, the sum of the momenta of balls A and B after collsion must be 10.0 kg m/s west.

$$p_{A\text{after}} = (10.0 \text{ kg m/s})(\cos 30^\circ) = (10.0 \text{ kg m/s})(0.866) = 8.66 \text{ kg m/s}$$

$$p_{B\text{after}} = (10.0 \text{ kg m/s})(\cos 60^\circ) = (10.0 \text{ kg m/s})(0.500) = 5.00 \text{ kg m/s}$$

To find the final velocities of the two balls, we divide the momentum of each by its mass. Therefore, $v_A = 4.3$ m/s and $v_B = 2.5$ m/s .

Example Problem: A 1325 kg car moving north at 27.0 m/s collides with a 2165 kg car moving east at 17.0 m/s. The two cars stick together after the collision. What is the speed and direction of the two cars after the collision?



Solution:

$$\text{Northward momentum} = (1325 \text{ kg})(27.0 \text{ m/s}) = 35800 \text{ kg m/s}$$

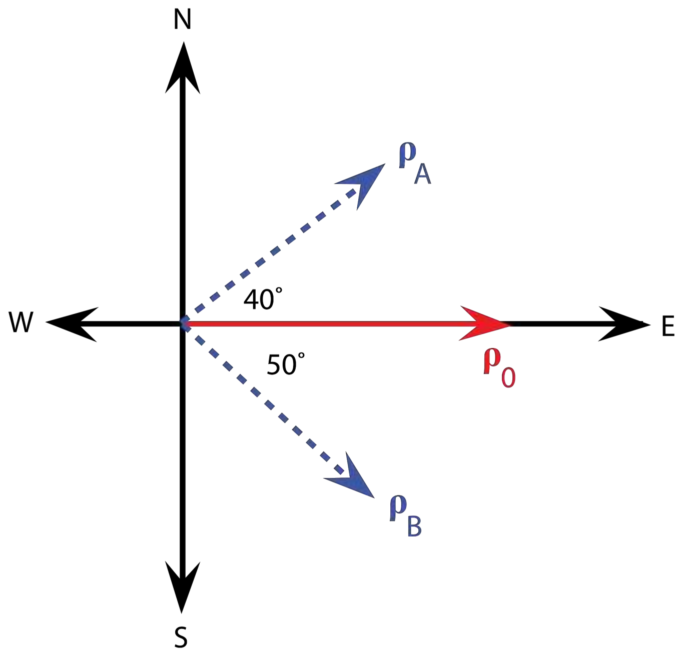
$$\text{Eastward momentum} = (2165 \text{ kg})(17.0 \text{ m/s}) = 36800 \text{ kg m/s}$$

$$R = \sqrt{(35800)^2 + (36800)^2} = 51400 \text{ kg} \cdot \text{m/s}$$

$$\theta = \sin^{-1}\frac{35800}{51400} = 44^\circ \text{ north of east}$$

$$velocity = \frac{p}{m} = \frac{51400 \text{ kg}\cdot\text{m/s}}{3490 \text{ kg}} = 14.7 \text{ m/s @ } 44^\circ \text{ N of E}$$

Example Problem: A 6.00 kg ball, A , moving at velocity 3.00 m/s due east collides with a 6.00 kg ball, B , at rest. After the collision, A moves off at 40.0° N of E and ball B moves off at 50.0° S of E.



a. What is the momentum of A after the collision?
b. What is the momentum of B after the collision?
c. What are the velocities of the two balls after the collision?

Solution: $p_{\text{initial}} = mv = (6.00 \text{ kg})(3.00 \text{ m/s}) = 18.0 \text{ kg m/s}$

This is a right triangle in which the initial momentum is the length of the hypotenuse and the two momenta after the collision are the legs of the triangle.

a. $p_A = (18.0 \text{ kg m/s})(\cos 40.0^\circ) = (18.0 \text{ kg m/s})(0.766) = 13.8 \text{ kg m/s}$
b. $p_B = (18.0 \text{ kg m/s})(\cos 50.0^\circ) = (18.0 \text{ kg m/s})(0.643) = 11.6 \text{ kg m/s}$
c. $v_A = 2.30 \text{ m/s} \qquad v_B = 1.93 \text{ m/s}$

Summary

- The conservation of momentum law holds for all closed systems regardless of the directions of the objects before and after they collide.
- Momentum is a vector; collisions in two dimensions can be represented by axial vector components.

Practice

This video shows circus performers using conservation of momentum. Use this resource to answer the questions that follow.

http://www.pbs.org/opb/circus/classroom/circus-physics/angular-momentum/

MEDIA

Click image to the left for more content.

1. Why do the fliers scrunch up in the air while spinning and twisting?
2. What happens to the rate at which they spin when they change shape in the air?

Review

1. Billiard ball *A* , mass 0.17 kg, moving due east with a velocity of 4.0 m/s, strikes stationary billiard ball *B* , also mass of 0.17 kg. After the collision, ball *A* moves off at an angle of 30° north of east with a velocity of 3.5 m/s, and ball *B* moves off at an angle of 60 ° south of east . What is the speed of ball *B* ?
2. A bomb, originally sitting at rest, explodes and during the explosion breaks into four pieces of exactly 0.25 kg each. One piece flies due south at 10 m/s while another pieces flies due north at 10 m/s.
   (a) What do we know about the directions of the other two pieces and how do we know it?
   (b) What do we know about the speeds of the other two pieces and how do we know it?
3. In a head-on collision between protons in a particle accelerator, three resultant particles were observed. All three of the resultant particles were moving to the right from the point of collision. The physicists conducting the experiment concluded there was at least one unseen particle moving to the left after the collision. Why did they conclude this?

References

1. Image copyright VitCOM Photo, 2013. http://www.shutterstock.com . Used under license from Shutterstock.com
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
4. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

CONCEPT 6 Inelastic Collisions

Students will learn how to solve problems involving inelastic collisions.

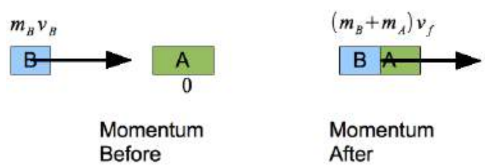
Key Equations

$$\sum p_{\text{initial}} = \sum p$$

final The total momentum does not change in closed systems

Example 1

Question : Two blocks collide on a frictionless surface. Afterwards, they have a combined mass of 10kg and a speed of 2.5m/s . Before the collision, block A, which has a mass of 8.0kg, was at rest. What was the mass and initial speed of block B?



Solution : To find mass of block B we have a simple subtraction problem. We know that the combined mass is 10kg and the mass of block A is 8.0kg.

$$10\text{kg} - 8.0\text{kg} = 2.0\text{kg}$$

Now that we know the mass of both blocks we can find the speed of block B. We will use conservation of momentum. This was a completely inelastic collision. We know this because the blocks stuck together after the collision. This problem is one dimensional, because all motion happens along the same line. Thus we will use the equation

$$(m_A + m_B)v_f = m_A \times v_A + m_B \times v_B$$

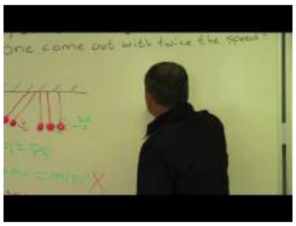
and solve for the velocity of block B.

$$(m_A + m_B)v_f = m_A \times v_A + m_B v_B \Rightarrow \frac{(m_A + m_B)(v_f) - (m_A)(v_A)}{m_B} = v_B$$

Watch this Explanation

MEDIA

Click image to the left for more content.



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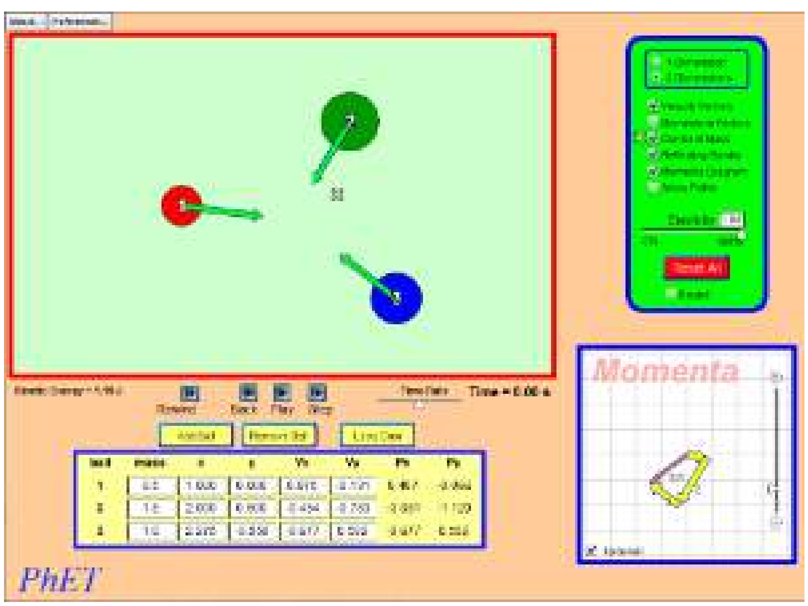
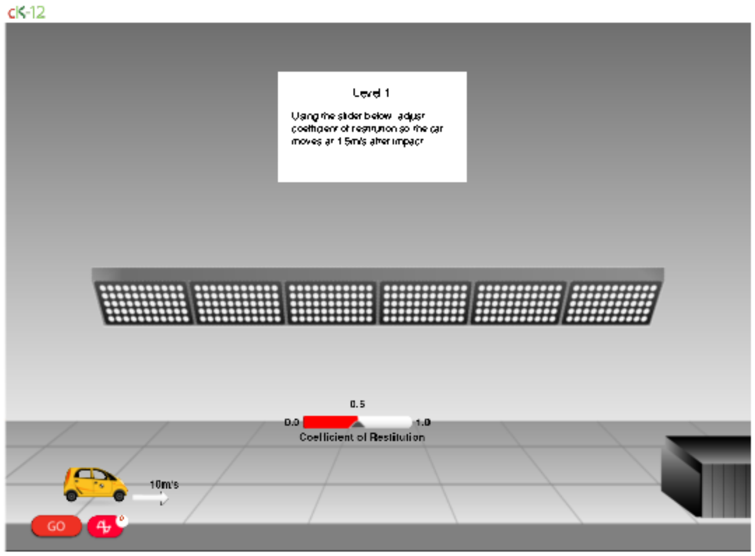
Simulation

Note: move the elasticity meter to 0% for perfectly inelastic collisions.

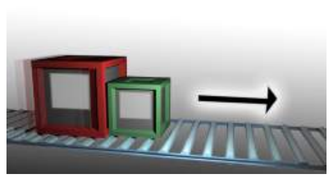
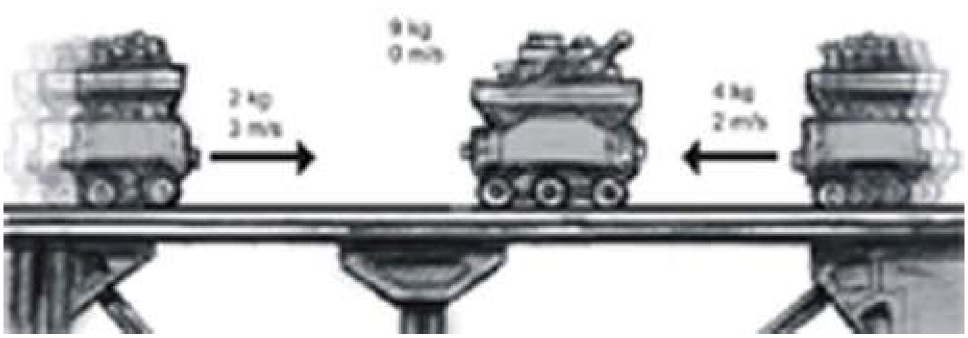


Collision Lab (PhET Simulation)



Car Collision (CK-12 Simulation)

Time for Practice

1. 
2. Two blocks collide on a frictionless surface, as shown. Afterwards, they have a combined mass of 10 kg and a speed of 2.5 m/s . Before the collision, one of the blocks was at rest. This block had a mass of 8.0 kg . What was the mass and initial speed of the second block?
3. 

4. In the above picture, the carts are moving on a level, frictionless track. After the collision all three carts stick together. Determine the direction and speed of the combined carts after the collision.
5. The train engine and its four boxcars are coasting at 40 m/s . The engine train has mass of 5,500 kg and the boxcars have masses, from left to right, of 1,000 kg , 1,500 kg , 2,000 kg, and 3,000 kg . (For this problem, you may neglect the small external forces of friction and air resistance.)

a. What happens to the speed of the train when it releases the last boxcar? (*Hint: Think before you blindly calculate.*)
b. If the train can shoot boxcars backwards at 30 m/s *relative to the train's speed,* how many boxcars does the train need to shoot out in order to obtain a speed of 58.75 m/s ?

6. In Sacramento a 4000 kg SUV is traveling 30 m/s south on Truxel crashes into an empty school bus, 7000 kg traveling east on San Juan. The collision is perfectly inelastic.
   a. Find the velocity of the wreck just after collision
   b. Find the direction in which the wreck initially moves
7. Manrico (80.0 kg) and Leonora (60.0 kg) are figure skaters. They are moving toward each other. Manrico's speed is 2.00 m/s ; Leonora's speed is 4.00 m/s . When they meet, Leonora flies into Manrico's arms.
   a. With what speed does the entwined couple move?
   b. In which direction are they moving?
   c. How much kinetic energy is lost in the collision?

Answers to Selected Problems

1. $2.0\text{ kg}, 12.5\text{ m/s}$
2. 0.13 m/s to the left
3. a. no change b. the last two cars
4. a. 15 m/s b. 49° S of E
5. a. 0.57 m/s b. the direction Leonora was originally travelling c. 297.26 J

Concept 7 Elastic Collisions

Students will learn how to solve problems involving elastic collisions

Key Equations

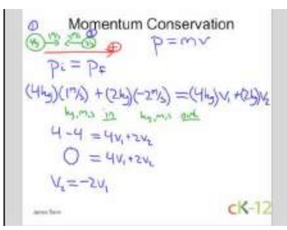
$$\sum p_{\text{initial}} = \sum p$$

final The total momentum does not change in closed systems

$$\sum KE_{\text{initial}} = \sum KE$$

final The total kinetic energy does not change in elastic collisions

Example 1



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Click image to the left for more content.

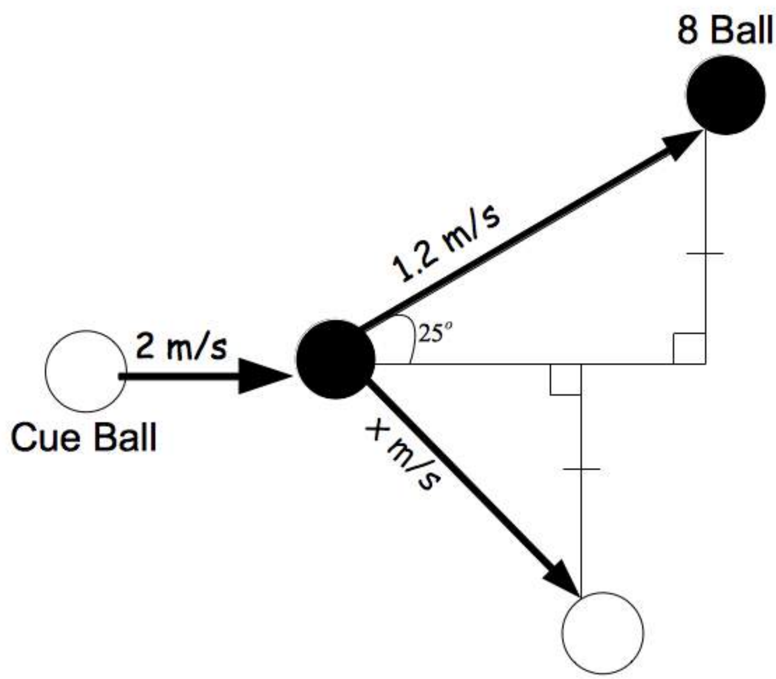
Example 2

Question : Chris and Ashley are playing pool. Ashley hits the cue ball into the 8 ball with a velocity of 1.2m/s . The cue ball (c) and the 8 ball (e) react as shown in the diagram. The 8 ball and the cue ball both have a mass of .17kg . What is the velocity of the cue ball? What is the direction (the angle) of the cue ball?

Answer : We know the equation for conservation of momentum, along with the masses of the objects in question as well two of the three velocities. Therefore all we need to do is manipulate the conservation of momentum equation so that it is solved for the velocity of the cue ball after the collision and then plug in the known values to get the velocity of the cue ball.

$$\begin{aligned}
m_c v_{ic} + m_e v_{ie} &= m_c v_{fc} + m_e v_{fe} \\
v_{fc} &= \frac{m_c v_{ic} + m_e v_{ie} - m_e v_{fe}}{m_c} \\
v_{fc} &= \frac{.17\text{kg} \times 2.0\text{m/s} + .17\text{kg} \times 0\text{m/s} - .17\text{kg} \times 1.2\text{m/s}}{.17\text{kg}} \\
v_{fc} &= .80\text{m/s}
\end{aligned}$$

Now we want to find the direction of the cue ball. To do this we will use the diagram below.



We know that the momentum in the y direction of the two balls is equal. Therefore we can say that the velocity in the y direction is also equal because the masses of the two balls are equal.

$$m_c v_c y = m_e v_e y \rightarrow v_c y = v_e y$$

Given this and the diagram, we can find the direction of the cue ball. After 1 second, the 8 ball will have traveled 1.2m . Therefore we can find the distance it has traveled in the y direction.

$$sin25^o = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{1.2\text{m}} \rightarrow x = sin25 \times 1.2\text{m} = .51\text{m}$$

Therefore, in one second the cue ball will have traveled .51m in the y direction as well. We also know how far in total the cue ball travels in one second $(.80\text{m})$. Thus we can find the direction of the cue ball.

$$sin^{-1} \frac{\text{opposite}}{\text{hypotenuse}} = sin^{-1} \frac{.51\text{m}}{.80\text{m}} = 40^o$$

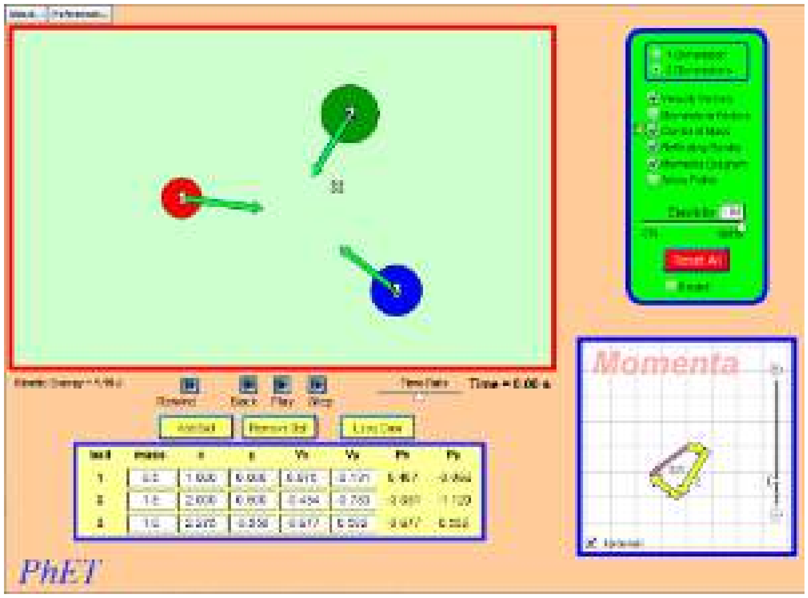
Watch this Explanation

MEDIA

Click image to the left for more content.

Simulation

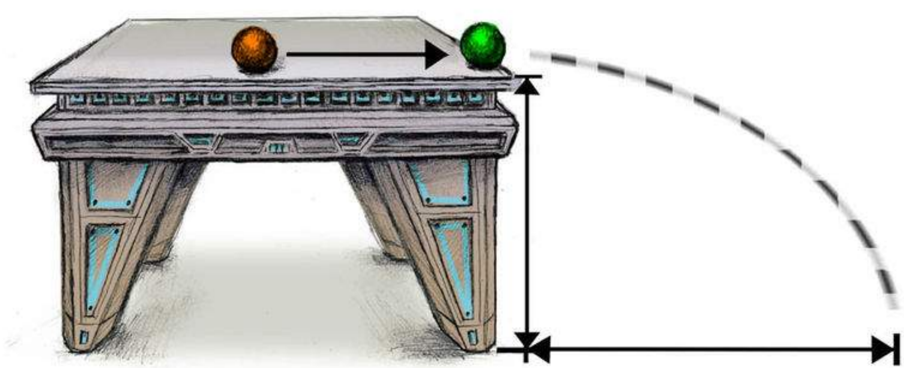
Note: move the elasticity meter to 100% for perfectly elastic collisions.



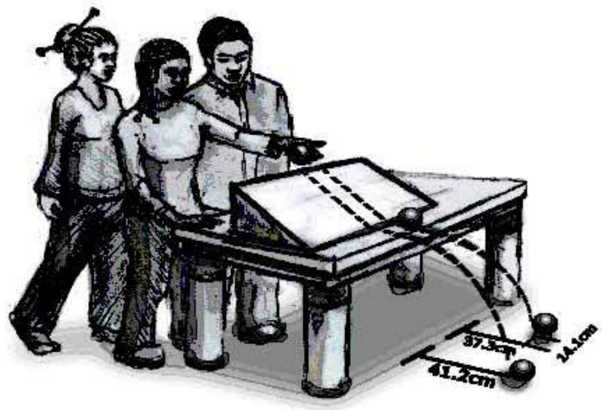
Collision Lab (PhET Simulation)

Time for Practice

1. You are playing pool and you hit the cue ball with a speed of 2 m/s at the 8 -ball (which is stationary). Assume an elastic collision and that both balls are the same mass. Find the speed and direction of both balls after the collision, assuming neither flies off at any angle.
2. A 0.045 kg golf ball with a speed of 42.0 m/s collides elastically head-on with a 0.17 kg pool ball at rest. Find the speed and direction of both balls after the collision.
3. Ball A is traveling along a flat table with a speed of 5.0 m/s , as shown below. Ball B , which has the same mass, is initially at rest, but is knocked off the table in an elastic collision with Ball A . Find the horizontal distance that Ball B travels before hitting the floor.

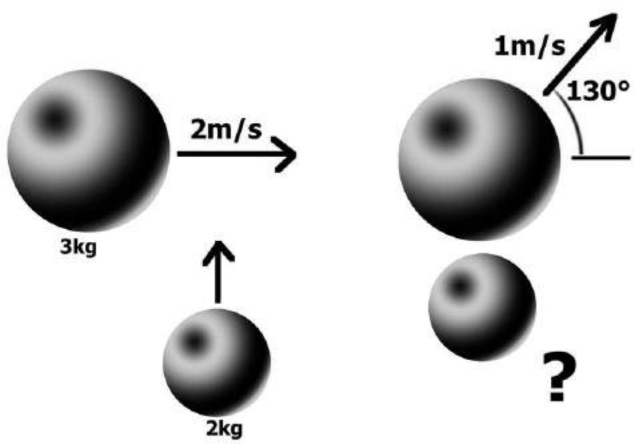


4. Students are doing an experiment on the lab table. A steel ball is rolled down a small ramp and allowed to hit the floor. Its impact point is carefully marked. Next a second ball of the same mass is put upon a set screw and a collision takes place such that both balls go off at an angle and hit the floor. All measurements are taken with a meter stick on the floor with a co-ordinate system such that just below the impact point is the origin. The following data is collected:
   (a) no collision: 41.2 cm
   (b) target ball: 37.3 cm in the direction of motion and 14.1 cm perpendicular to the direction of motion



i. From this data predict the impact position of the other ball.
ii. One of the lab groups declares that the data on the floor alone demonstrate to a 2 % accuracy that the collision was elastic. Show their reasoning.
iii. Another lab group says they can't make that determination without knowing the velocity the balls have on impact. They ask for a timer. The instructor says you don't need one; use your meter stick. Explain.
iv. Design an experiment to prove momentum conservation with balls of different masses, giving apparatus, procedure and design. Give some sample numbers.

5. A 3 kg ball is moving 2 m/s in the positive $x-$ direction when it is struck dead center by a 2 kg ball moving in the positive $y-$ direction at 1 m/s. After collision the 3 kg ball moves at 3 m/s 30 degrees from the positive $x-$ axis.



Find the velocity and direction of the 2 kg ball.

Answers to Selected Problems

1. 8 m/s same direction as the cue ball and 0 m/s
2. $v_{golf} = -24.5$ m/s; vpool $= 17.6$ m/s
3. 2.8 m
4. .
5. 1.5 m/s 54°

CONCEPT 8 Momentum and Impulse

- Define momentum.
- Define impulse.
- Given mass and velocity of an object, calculate momentum.
- Calculate the change in momentum of an object.
- State the relationship that exists between the change in momentum and impulse.
- Using the momentum-impulse theorem and given three of the four variables, calculate the fourth.

Rachel Flatt performs a layback spin at the 2 011 Rostelecom Cup in Moscow, Russia.

When an ice skater spins, angular momentum must be conserved. When her arms or feet are far away from her body, her spin slows; when she brings her arms and feet close in to her body, she spins faster.

Momentum and Impulse

If a bowling ball and a ping-pong ball are each moving with a velocity of 5 mph, you intuitively understand that it will require more effort to stop the bowling ball than the ping pong ball because of the greater mass of the bowling ball. Similarly, if you have two bowling balls, one moving at 5 mph and the other moving at 10 mph, you know it will take more effort to stop the ball with the greater speed. It is clear that both the mass and the velocity of a moving object contribute to what is necessary to change the motion of the moving object. The product of the mass and velocity of an object is called its **momentum**. Momentum is a vector quantity that has the same direction as the velocity of the object and is represented by a lowercase letter p.

$$p = mv$$

The momentum of a 0.500 kg ball moving with a velocity of 15.0 m/s will be

$$p = mv = (0.500\text{ kg})(15.0\text{ m/s}) = 7.50\text{ kg}\cdot\text{m/s}$$

You should note that the units for momentum are kg·m/s.

According to Newton's first law, the velocity of an object cannot change unless a force is applied. If we wish to change the momentum of a body, we must apply a force. The longer the force is applied, the greater the change in momentum. The **impulse** is the quantity defined as the force multiplied by the time it is applied. It is a vector quantity that has the same direction as the force. The units for impulse are N·s but we know that Newtons are also kg·m/s^2 and so N·s = (kg·m/s^2)(s) = kg·m/s. Impulse and momentum have the same units; when an impulse is applied to an object, the momentum of the object changes and the change of momentum is equal to the impulse.

$$Ft = \Delta mv$$

Example Problem: Calculating Momentum

A 0.15 kg ball is moving with a velocity of 35 m/s. Find the momentum of the ball.

Solution: $p = mv = (0.15\text{ kg})(35\text{ m/s}) = 5.25\text{ kg}\cdot\text{m/s}$

Example Problem: If a ball with mass 5.00 kg has a momentum of 5.25 kg · m/s , what is its velocity?

Solution: $v = \frac{p}{m} = \frac{5.25\ \mathbf{kg{\cdot}m/s}}{5.00\ \mathbf{kg}} = 1.05\ \mathbf{m/s}$

It should be clear from the equation relating impulse to change in momentum, $Ft = \Delta mv$, that any amount of force would (eventually) bring a moving object to rest. If the force is very small, it must be applied for a long time, but a greater force can bring the object to rest in a shorter period of time.

If you jump off a porch and land on your feet with your knees locked in the straight position, your motion would be brought to rest in a very short period of time and thus the force would need to be very large – large enough, perhaps, to damage your joints or bones.

Suppose that when you hit the ground, your velocity was 7.0 m/s and that velocity was brought to rest in 0.05 seconds. If your mass is 100. kg, what force was required to bring you to rest?

$$F = \frac{\Delta mv}{t} = \frac{(100.\text{ kg})(7.0\text{ m/s})}{0.050\text{ s}} = 14{,}000\text{ N}$$

If, on the other hand, when your feet first touched the ground, you allowed your knees to flex so that the period of time over which your body was brought to rest is increased, then the force on your body would be smaller and it would be less likely that you would damage your legs.

Suppose that when you first touch the ground, you allow your knees to bend and extend the stopping time to 0.50 seconds. What force would be required to bring you to rest this time?

$$F = \frac{\Delta mv}{t} = \frac{(100.\text{ kg})(7.0\text{ m/s})}{0.50\text{ s}} = 1400\text{ N}$$

With the longer period of time for the force to act, the necessary force is reduced to one-tenth of what was needed before.

Extending the period of time over which a force acts in order to lessen the force is a common practice in design. Padding in shoes and seats allows the time to increase. The front of automobiles are designed to crumple in an accident; this increases the time the car takes to stop. Similarly, barrels of water or sand in front of abutments on the highway and airbags serve to slow down the stoppage time. These changes all serve to decrease the amount of force it takes to stop the momentum in a car crash, which consequently saves lives.

Example Problem: An 0.15 kg baseball is thrown horizontally at 40. m/s and after it is struck by a bat, it is traveling at -40. m/s.

(a) What impulse did the bat deliver to the ball?

(b) If the contact time of the bat and bat was 0.00080 seconds, what was the average force the bat exerted on the ball?

(c) Calculate the average acceleration of the ball during the time it was in contact with the bat.

Solution: We can calculate the change in momentum and give the answer as impulse because we know that the impulse is equal to the change in momentum.

(a)

$$\begin{aligned} p = m\Delta v &= (0.15 \text{ kg})(-40. \text{ m/s} - 40. \text{ m/s}) \\ &= (0.15 \text{ kg})(-80. \text{ m/s}) = -12 \text{ kg} \cdot \text{m/s} \end{aligned}$$

The minus sign indicates that the impulse was in the opposite direction of the original throw.

(b) $F = \frac{\Delta mv}{t} = \frac{-12 \text{ kg}\cdot\text{m/s}}{0.00080 \text{ s}} = -15000 \text{ N}$

Again, the negative sign indicates the force was in the opposite direction of the original throw.

(c) $a = \frac{F}{m} = \frac{-15000 \text{ N}}{0.15 \text{ kg}} = -100,000 \text{ m/s}^2$

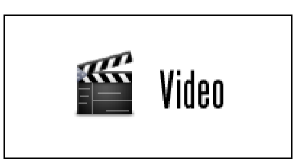
Summary

- The product of the mass and velocity of an object is called momentum, given by the equation $\rho = mv$.
- Momentum is a vector quantity that has the same direction as the velocity of the object.
- The quantity of force multiplied by the time it is applied is called impulse.
- Impulse is a vector quantity that has the same direction as the force.
- Momentum and impulse have the same units: kg·m/s.
- The change of momentum of an object is equal to the impulse. $Ft = \Delta mv$

Practice

Use this resource to answer the question that follows.

https://www.youtube.com/watch?v=3g4v8x7xggU



MEDIA
Click image to the left for more content.

1. Why don't the glasses of water spill when the tablecloth is pulled out from under them?
2. How does the video get from momentum to impulse?

Review

1. A small car with a mass of 800. kg is moving with a velocity of 27.8 m/s.
   (a) What is the momentum of the car?
   (b) What velocity is needed for a 2400. kg car in order to have the same momentum?
2. A scooter has a mass of 250. kg. A constant force is exerted on it for 60.0 s. During the time the force is exerted, the scooter increases its speed from 6.00 m/s to 28.0 m/s.
   (a) What is the change in momentum?
   (b) What is the magnitude of the force exerted on the scooter?
3. The brakes on a 15,680 N car exert a stopping force of 640. N. The car's velocity changes from 20.0 m/s to 0 m/s.
   (a) What is the car's mass?
   (b) What was its initial momentum?
   (c) What was the change in momentum for the car?
   (d) How long does it take the braking force to bring the car to rest?

- **momentum:** A measure of the motion of a body equal to the product of its mass and velocity. Also called linear momentum **.**
- **impulse:** The product obtained by multiplying the average value of a force by the time during which it acts. The impulse equals the change in momentum produced by the force in this time interval.

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1. User:deerstop/Wikimedia Commons. http://commons.wikimedia.org/wiki/File:Flatt-3.jpg . Public Domain

CK-12 FlexBook

Physics Unit 9: Circular Motion

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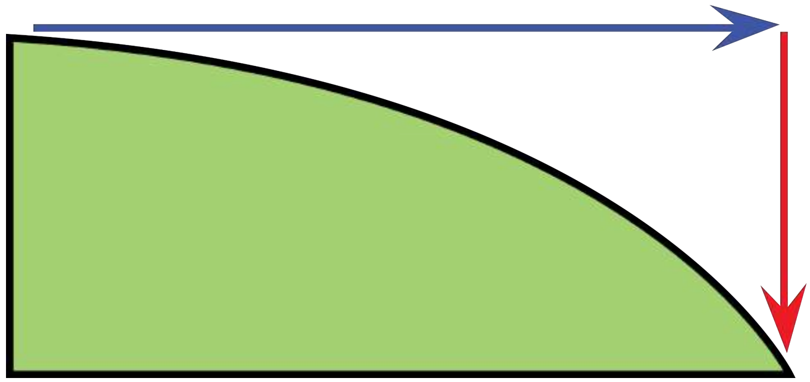
CONCEPT 1 Circular Motion

- Define centripetal acceleration.
- Understand the theory of the centripetal acceleration equation.
- Use the centripetal acceleration equation.
- Understand the angular relationship between velocity and centripetal acceleration.
- Use the equations for motion in two directions and Newton's Laws to analyze circular motion.

Weather satellites, like the one shown above, are found miles above the earth's surface. Satellites can be polar orbiting, meaning they cover the entire Earth asynchronously, or geostationary, in which they hover over the same spot on the equator.

Circular Motion

The earth is a sphere. If you draw a horizontal straight line from a point on the surface of the earth, the surface of the earth drops away from the line. The distance that the earth drops away from the horizontal line is very small – so small, in fact, that we cannot represent it well in a drawing. In the sketch below, if the blue line is 1600 m, the amount of drop (the red line) would be 0.20 m. If the sketch were drawn to scale, the red line would be too short to see.



When an object is launched exactly horizontally in projectile motion, it travels some distance horizontally before it strikes the ground. In the present discussion, we wish to imagine a projectile fired horizontally on the surface of the earth such that while traveling 1600 m horizontally, the object would fall exactly 0.20 m. If this could occur, then the object would fall exactly the amount necessary during its horizontal motion to remain at the surface of the earth, but not touching it. In such a case, the object would travel all the way around the earth continuously and circle the earth, assuming there were no obstacles, such as mountains.

What initial horizontal velocity would be necessary for this to occur? We first calculate the time to fall the 0.20 m:

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{(2)(0.20\text{ m})}{9.80\text{ m/s}^2}} = 0.20\text{ s}$$

The horizontal velocity necessary to travel 1600 m in 0.20 s is 8000 m/s. Thus, the necessary initial horizontal velocity is 8000 m/s.

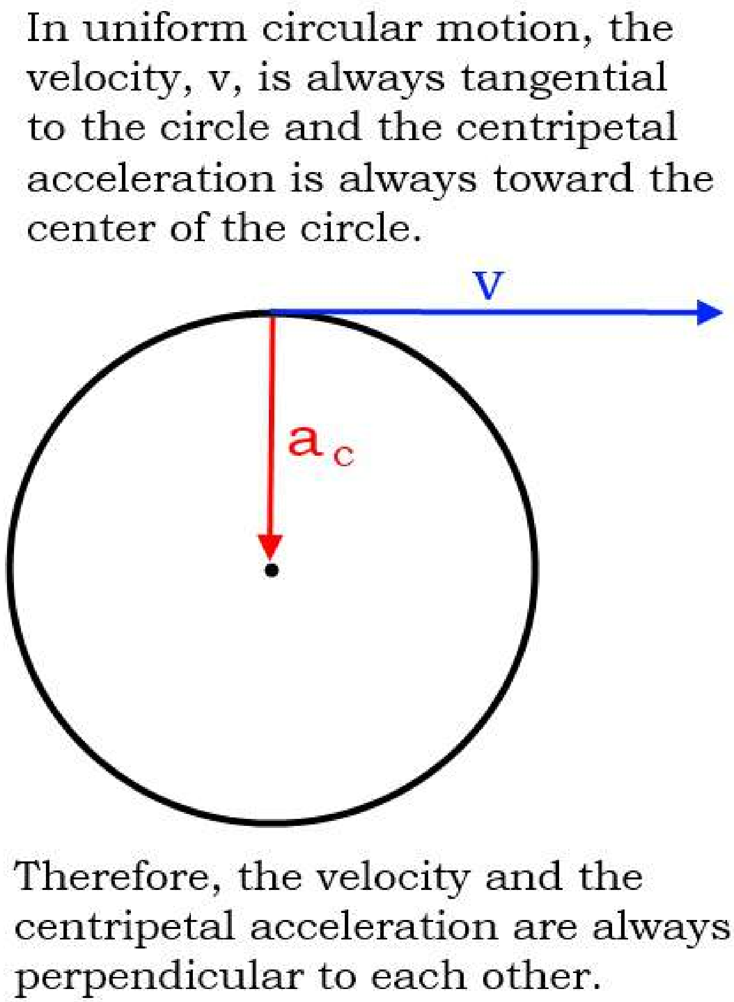
In order to keep an object traveling in a circular path, there must be an acceleration toward the center of the circle. This acceleration is called **centripetal acceleration.** In the case of satellites orbiting the earth, the centripetal acceleration is caused by gravity. If you were swinging an object around your head on a string, the centripetal acceleration would be caused by your hand pulling on the string toward the center of the circle.

It is important to note that the object traveling in a circle has a constant speed but does not have a constant velocity. This is because direction is part of velocity; when an object changes its direction, it is changing its velocity. Hence the object's acceleration. The acceleration in the case of uniform **circular motion** is the change in the direction of the velocity, but not its magnitude.

For an object traveling in a circular path, the centripetal acceleration is directly related to the square of the velocity of the object and inversely related to the radius of the circle.

$$a_c = \frac{v^2}{r}$$

Taking a moment to consider the validity of this equation can help to clarify what it means. Imagine a yo-yo. Instead of using it normally, let it fall to the end of the string, and then spin it around above your head. If we were to increase the speed at which we rotate our hand, we increase the velocity of the yo-yo - it is spinning faster. As it spins faster, it also changes direction faster. The acceleration increases. Now let's think about the bottom of the equation: the radius. If we halve the length of the yo-yo string (bring the yo-yo closer to us), we make the yo-yo's velocity greater. Again, it moves faster, which increases the acceleration. If we make the string longer again, this decreases the acceleration. We now understand why the relationship between the radius and the acceleration is an inverse relationship - as we decrease the radius, the acceleration increases, and visa versa.



Example Problem: A ball at the end of a string is swinging in a horizontal circle of radius 1.15 m. The ball makes exactly 2.00 revolutions per second. What is its centripetal acceleration?

Solution: We first determine the velocity of the ball using the facts that the circumference of the circle is $2\pi r$ and the ball goes around exactly twice per second.

$v = \frac{(2)(2\pi r)}{t} = \frac{(2)(2)(3.14)(1.15 \text{ m})}{1.00 \text{ s}} = 14.4 \text{ m/s}$

We then use the velocity and radius in the centripetal acceleration equation.

$a_c = \frac{v^2}{r} = \frac{(14.4 \text{ m/s})^2}{1.15 \text{ m}} = 180. \text{ m/s}^2$

Example Problem: The moon's nearly circular orbit around the earth has a radius of about 385,000 km and a period of 27.3 days. Calculate the acceleration of the moon toward the earth.

Solution:

$v = \frac{2\pi r}{T} = \frac{(2)(3.14)(3.85\times10^8 \textbf{ m})}{(27.3 \textbf{ d})(24.0 \textbf{ h/d})(3600 \textbf{ s/h})} = 1020 \textbf{ m/s}$

$a_c = \frac{v^2}{r} = \frac{(1020 \textbf{ m/s})^2}{3.85\times10^8 \textbf{ m}} = 0.00273 \textbf{ m/s}^2$

As shown in the previous example, the velocity of an object traveling in a circle can be calculated by

$$v = \frac{2\pi r}{T}$$

Where r is the radius of the circle and T is the period (time required for one revolution).

This equation can be incorporated into the equation for centripetal acceleration as shown below.

$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$

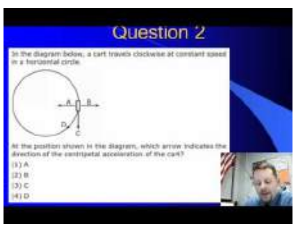
Summary

- In order to keep an object traveling in a circular path, there must be an acceleration toward the center of the circle. This acceleration is called centripetal acceleration **.**
- The acceleration in the case of uniform circular motion changes the direction of the velocity but not its magnitude.
- Formulas for centripetal acceleration are $a_c = \frac{v^2}{r}$ and $a_c = \frac{4\pi^2 r}{T^2}$.

Practice

This video is a demonstration of centripetal force using balloons and trays of water. Use this resource to answer the questions that follow.

https://www.youtube.com/watch?v=EX5DZ2MHlV4



MEDIA

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1. What does centripetal mean?
2. What is uniform circular motion?
3. Why is centripetal acceleration always towards the center?

Review

1. An automobile rounds a curve of radius 50.0 m on a flat road at a speed of 14 m/s. What centripetal acceleration is necessary to keep the car on the curve?
2. An object is swung in a horizontal circle on a length of string that is 0.93 m long. If the object goes around once in 1.18 s, what is the centripetal acceleration?

- **circular motion:** A movement of an object along the circumference of a circle or rotation along a circular path.
- **centripetal acceleration:** The acceleration toward the center that keeps an object following a circular path.

References

1. Greg Goebel . http://www.public-domain-image.com/space-public-domain-images-pictures/weather-satellite.jpg.html . Public Domain
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Richard Parsons. . CC-BY-NC-SA 3.0

CONCEPT 2 Circular Motion

Students will learn that in circular motion there is always an acceleration (and hence a force) that points to the center of the circle defined by the objects motion. This force changes the direction of the velocity vector of the object but not the speed. Students will also learn how to calculate that speed using the period of motion and the distance of its path (circumference of the circle it traces out).

Vocabulary

- **centripetal acceleration:** The inward acceleration that keeps an object in circular motion.
- **centripetal force:** The inward force that keeps an object in circular motion.

Introduction

A satellite orbits around the Earth in **Figure** below . A car travels around a curve in **Figure** below . All of these objects are engaged in circular motion. Let us consider the satellite first. The satellite is held in place by the Earth's gravity. The gravity holds the satellite in its orbit. In what direction does this force act? If the earth were "magically" gone, the satellite would fly off tangent to its motion at the instant gravity no longer held it. The force preventing this from happening must keep pulling the satellite toward the center of the circle to maintain circular motion.

FIGURE 2.1

What is the force in **Figure** above that prevents the car from skidding off the road? If you guessed "the friction between the tires and the road" you'd be correct. But is it static or kinetic friction? Unless the tires skid, there can be no kinetic friction. It is static friction that prevents the tires from skidding, just as it is static friction that permits you to walk without slipping. In **Figure** below , you can see the foot of a person who walks toward the right by pushing their foot backward with a horizontal component of force F . They move forward because the ground exerts a horizontal component force f_s in the opposite direction. (Note that vertical forces are ignored.) The force the ground exerts on the person's foot is a static friction force. Because the foot does not slide, we know that F and f_s are equal opposed forces. We can easily see which direction the static friction force must act when we walk, but what about a car performing circular motion? In what direction does the static friction act on the car in **Figure** above ?

FIGURE 2.2

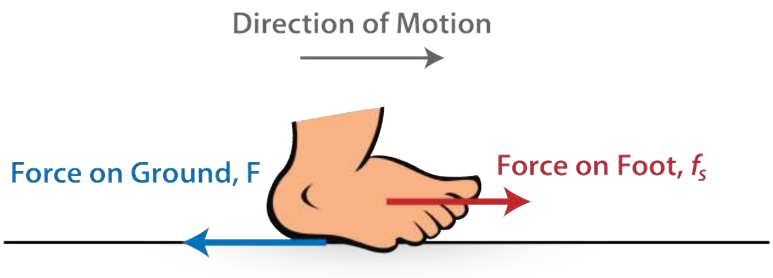

FIGURE 2.3

Figure below shows the top view of a car moving around a circular track with a constant speed. Since acceleration is defined as $a = \frac{\Delta v}{\Delta t}$, you may be tempted to say that since the speed remains constant, $\Delta v = 0$, the acceleration must also be zero. But that conclusion would be incorrect because Δv represents a change in velocity, not a change in speed. The velocity of the car is not constant since it is continuously changing its direction. How then do we find the acceleration of the car?

Figure below shows the instantaneous velocity vectors for the car in two different positions a very small time apart.

Notice that the vector ΔV points toward the center of the circle. (Recall that ΔV can be thought of as the sum of the vectors $V_2 + (-V_1)$.) The direction of the acceleration points in the direction of ΔV since acceleration is defined as $a = \frac{\Delta \vec{v}}{\Delta t}$. This is reasonable, since if there were no force directed toward the center of the circle, the car would move off tangent to the circle.

We call the inward force that keeps an object in circular motion a "center seeking", or **centripetal** force and the acceleration, centripetal acceleration. The centripetal acceleration is often denoted as a_c In order to find the correct expression for the magnitude of the centripetal acceleration we'll need to use a little geometric reasoning. **Figure** below and **Figure** below show two "almost" similar triangles.

The magnitudes of $-V_1$ and V_2 are equal, and the change in location of the car occurs over a very small increment in time, Δt . The velocities are tangent to the circle and therefore perpendicular to the radius of the circle. As such, the "radius" triangle and the "velocity" triangle are approximately similar (see the figures above). We construct an approximate ratio between the two triangles by assuming that during the time Δt , the car has traveled a distance Δs

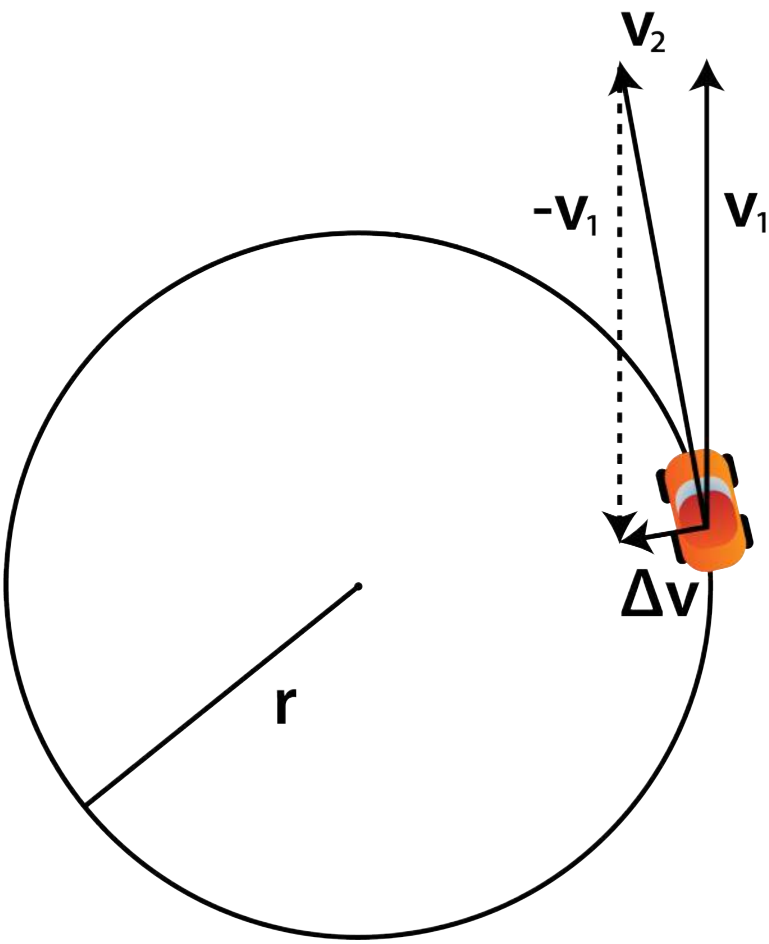

FIGURE 2.4

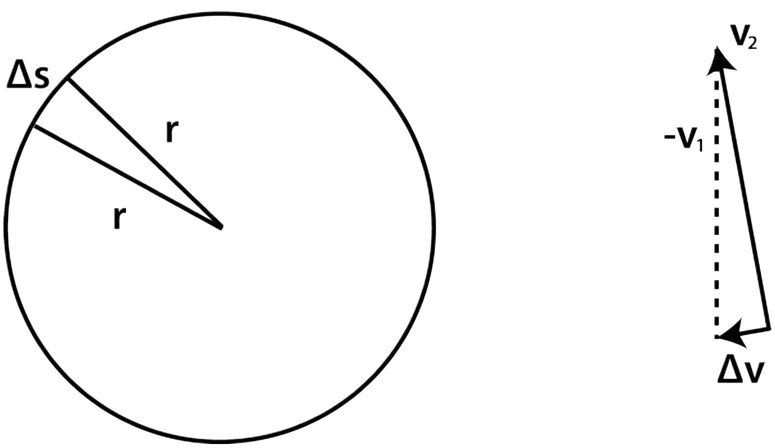

FIGURE 2.5

along the circle. The ratio below is constructed in order to determine the acceleration.

$\frac{\Delta s}{r} \doteq \frac{\Delta v}{v}$, which leads to, $v\Delta s \doteq r\Delta v$.

Dividing both sides of the equation by Δt , we have: $v\frac{\Delta s}{\Delta t} \doteq r\frac{\Delta v}{\Delta t}$.

But $\frac{\Delta s}{\Delta t}$ is the speed v of the car and $\frac{\Delta v}{\Delta t}$ is the acceleration of the car.

If we allow the time to become infinitesimally small, then the approximation becomes exact and we have: $v^2 = ra, a = \frac{v^2}{r}$. Thus, the magnitude of the centripetal acceleration for an object moving with constant speed in circular motion is $a_c = \frac{v^2}{r}$, and its direction is toward the center of the circle.

Illustrative Examples using Centripetal Acceleration and Force

Example 1A: A 1000 kg car moves with a constant speed 13.0 m/s around a flat circular track of radius 40.0 m. What is the magnitude and direction of the centripetal acceleration?

Answer: The magnitude of the car's acceleration is $a_c = \frac{v^2}{r} = \frac{132}{40} = 4.225 = 4.23\ m/s^2$ and the direction of its acceleration is toward the center of the track.

Example 1b: Determine the force of static friction that acts upon the car in **Figure** below

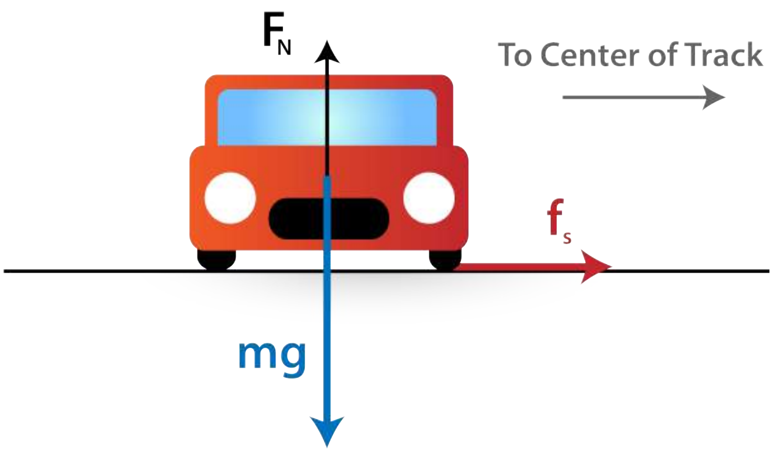

FIGURE 2.6

Answer:

Using Newton's Second Law: $\sum F = f_s = ma = 1000(4.225) = 4225 = 4230\ N$

Example 1c: Determine the minimum necessary coefficient of static friction between the tires and the road.

Answer:

$\sum_y F = F_N - Mg = 0, F_N = Mg$ but $f_s = \mu_s F_N = ma$

Thus, $\mu_s Mg = Ma, \mu_s g = a, \mu_s = \frac{a}{g} = \frac{4.225}{9.8} = 0.431 = 0.43$

Check Your Understanding

True or False?

1. Kinetic friction is responsible for the traction (friction) between the tires and the road.

Answer : False. As long as the car does not skid, there is no relative motion between the instantaneous contact area of the tire and the road.

2. True or False? The force of static friction upon an object can vary.

Answer : True. In attempting to move an object, a range of forces of different magnitudes can be applied until the maximum static friction between the object and the surface it rests upon is overcome and the object is set into motion. Recall that the magnitude of static friction is represented by the inequality: $f_s \leq \mu_s F_N$

3. The greater the mass of the car, the greater the coefficient of friction.

Answer : False. The coefficient of friction is independent of the mass of an object. Recall that it is the ratio of the friction force to the normal force. As such, it is a pure number dependent only upon the nature of the materials in contact with each other- in this case rubber and asphalt.

References

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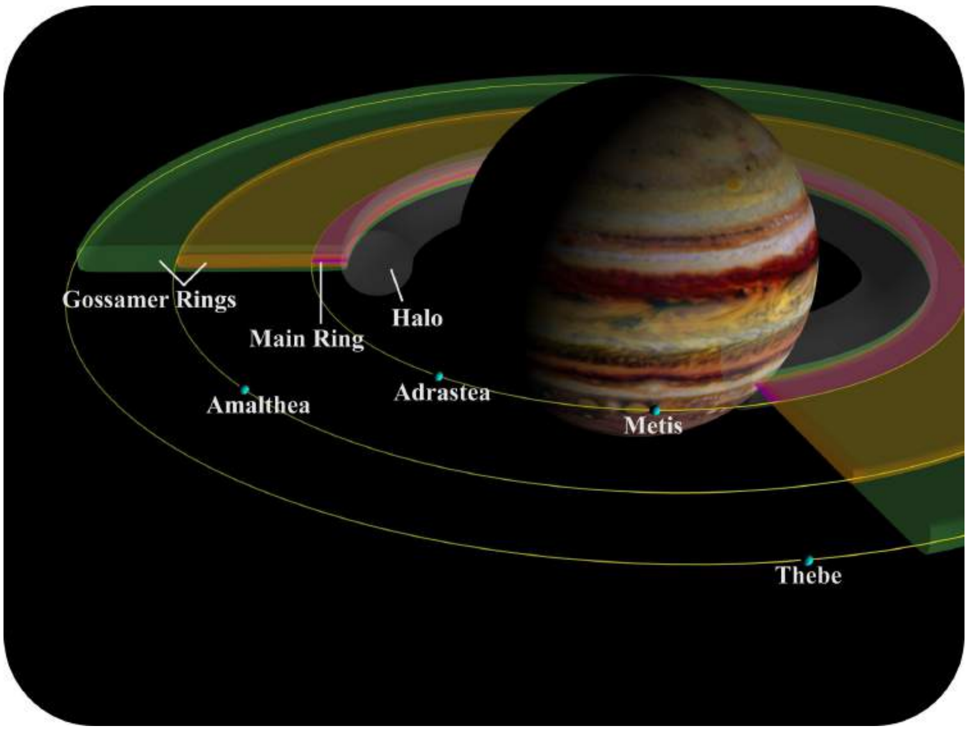
CONCEPT 3 Centripetal Force

- Define centripetal force.
- Solve problems involving centripetal force.
- Explain the difference between centripetal and centrifugal forces.



Jupiter's moons and ring materials follow all the laws of physics, including centripetal force and centripetal acceleration.

Centripetal Force

Centripetal force is, simply, the force that causes centripetal acceleration. Objects that move in uniform circular motion all have an acceleration toward the center of the circle and therefore, they must also suffer a force toward the center of the circle. That force is the centripetal force. For orbiting satellites, such as the moon orbiting the earth or the earth orbiting the sun, the centripetal force is produced by gravity. When an Olympic hammer thrower whirls a massive ball on a chain, the centripetal force is created by the athlete and transmitted by the chain.

Newton's second law shows the relationship between force and acceleration, $F = ma$. Since we have formulas expressing the relationships for centripetal acceleration, they can easily be altered to show the relationships for centripetal force.

$a_c = \frac{v^2}{r} \text{ and } F = ma \text{ so } F_c = \frac{mv^2}{r}$

and

$a_c = \frac{4\pi^2 r}{T^2}$ so $F_c = \frac{4\pi^2 rm}{T^2}$

Common Misconceptions

Many people incorrectly use the term *centrifugal force* instead of *centripetal force* . Often, you will hear the term centrifugal force used to describe the outward force pushing an object away from the center of a circle. In reality, however, centrifugal forces are inertial, or fictional, forces. They only exist in the frame of reference of the object that is moving and, even then, are theoretical. Physicists dealing in a moving frame of reference use centrifugal forces to ease calculations.

For a great explanation of the difference between centrifugal and centripetal force, see this video:

https://www.youtube.com/watch?v=DLgy6rVV-08

MEDIA

Click image to the left for more content.

Summary

- Centripetal force is the force that causes centripetal acceleration.
- Equations for centripetal force are $F_c = \frac{mv^2}{r}$ and $F_c = \frac{4\pi^2 rm}{T^2}$.

Practice

A video of physics students riding a roller coaster. Use this resource to answer the questions that follow.

http://www.teachersdomain.org/asset/phy03_vid_roller/

1. Does the roller coaster in the video have a complete circle as part of its path?
2. What is it that keeps the glass of water on the tray as it swings over the student's head?

Review

1. A runner moving at a speed of 8.8 m/s rounds a bend with a radius of 25 m. What is the centripetal force needed to keep this runner on the curve and what supplies this force?
2. A 1000. kg car rounds a curve of 50.0 m radius on a flat road with a speed of 14.0 m/s.
   (a) Will the car make the turn successfully if the pavement is dry and the coefficient of friction is 0.60?
   (b) Will the car make the turn successfully if the pavement is wet and the coefficient of friction is 0.20?
3. An 0.500 kg object tied to a string is swung around a person's head in a horizontal circle. The length of the string is 1.00 m and the maximum force the string can withstand without breaking is 25.0 N. What is the maximum speed the object may be swung without breaking the string?

- **centripetal force:** The component of force acting on a body in curvilinear motion that is directed toward the center of curvature or axis of rotation.

References

1. Courtesy of NASA/JPL/Cornell University. http://www.nasa.gov/centers/goddard/multimedia/largest/EduImageGallery.html . Public Domain

CK-12 FlexBook

Physics Unit 10: Waves

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CONCEPT 1 Measuring Waves

Lesson Objectives

- Define wave amplitude and wavelength.
- Relate wave speed to wave frequency and wavelength.

Lesson Vocabulary

- hertz (Hz)
- wave amplitude
- wave frequency
- wavelength
- wave speed

Introduction

Tsunamis, or the waves caused by earthquakes, are unusually large ocean waves. You can see an example of a tsunami in **Figure** 1.1 . Because tsunamis are so big, they can cause incredible destruction and loss of life. The tsunami in the figure crashed into Thailand, sending people close to shore running for their lives. The height of a tsunami or other wave is just one way of measuring its size. You'll learn about this and other ways of measuring waves in this lesson.

FIGURE 1.1
This tsunami occurred in Thailand on December 26, 2004.

Wave Amplitude and Wavelength

The height of a wave is its amplitude. Another measure of wave size is wavelength. Both wave amplitude and wavelength are described in detail below. **Figure** 1.2 shows these wave measures for both transverse and longitudinal waves. You can also simulate waves with different amplitudes and wavelengths by doing the interactive animation at this URL: http://sci-culture.com/advancedpoll/GCSE/sine%20wave%20simulator.html .

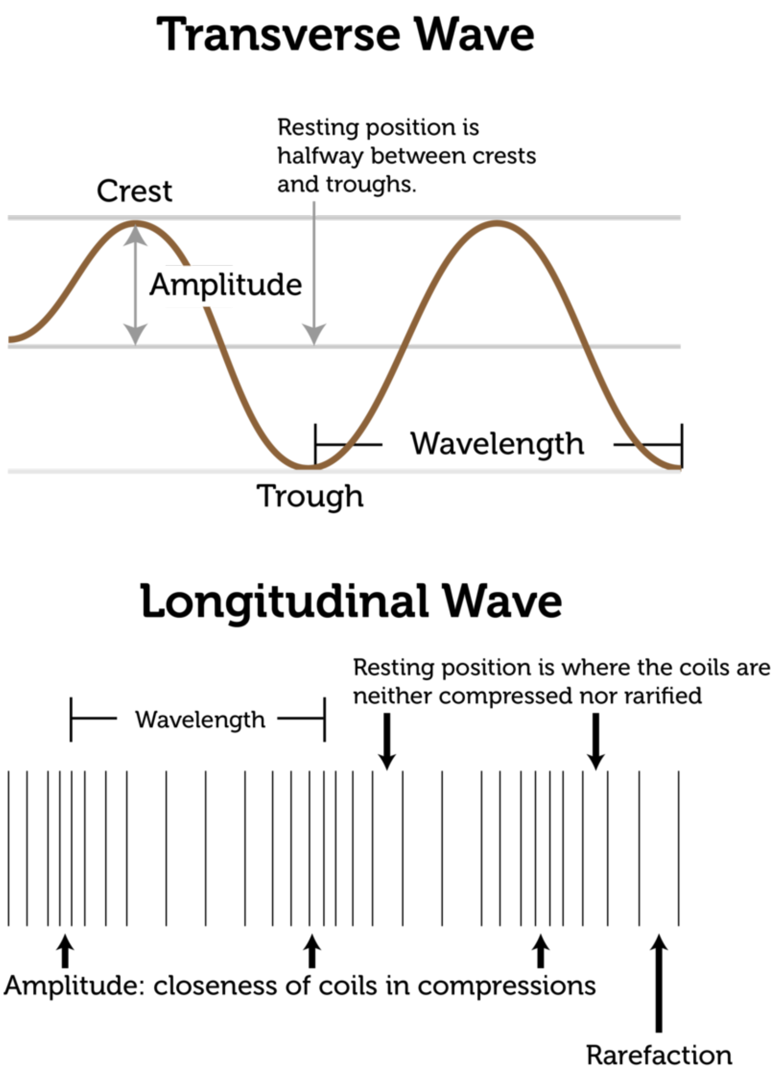


FIGURE 1.2
Wave amplitude and wavelength are two important measures of wave size.

Wave Amplitude

Wave amplitude is the maximum distance the particles of a medium move from their resting position when a wave passes through. The resting position is where the particles would be in the absence of a wave.

- In a transverse wave, wave amplitude is the height of each crest above the resting position. The higher the crests are, the greater the amplitude.
- In a longitudinal wave, amplitude is a measure of how compressed particles of the medium become when the wave passes through. The closer together the particles are, the greater the amplitude.

What determines a wave's amplitude? It depends on the energy of the disturbance that causes the wave. A wave caused by a disturbance with more energy has greater amplitude. Imagine dropping a small pebble into a pond of still water. Tiny ripples will move out from the disturbance in concentric circles, like those in **Figure** above . The ripples are low-amplitude waves. Now imagine throwing a big boulder into the pond. Very large waves will be generated by the disturbance. These waves are high-amplitude waves.

Wavelength

Another important measure of wave size is wavelength. **Wavelength** is the distance between two corresponding points on adjacent waves (see **Figure** 1.2). Wavelength can be measured as the distance between two adjacent crests of a transverse wave or two adjacent compressions of a longitudinal wave. It is usually measured in meters. Wavelength is related to the energy of a wave. Short-wavelength waves have more energy than long-wavelength waves of the same amplitude. You can see examples of waves with shorter and longer wavelengths in **Figure** 1.3 .

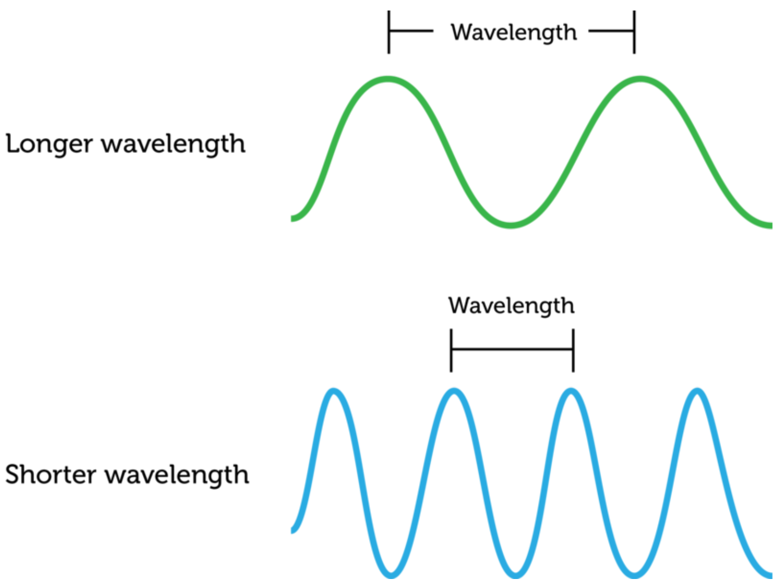


FIGURE 1.3
Both of these waves have the same amplitude, but they differ in wavelength. Which wave has more energy?

Wave Frequency and Speed

Imagine making transverse waves in a rope, like the waves in **Figure** above . You tie one end of the rope to a doorknob or other fixed point and move the other end up and down with your hand. You can move the rope up and down slowly or quickly. How quickly you move the rope determines the frequency of the waves.

Wave Frequency

The number of waves that pass a fixed point in a given amount of time is **wave frequency** . Wave frequency can be measured by counting the number of crests or compressions that pass the point in 1 second or other time period. The higher the number is, the greater is the frequency of the wave. The SI unit for wave frequency is the **hertz (Hz)** , where 1 hertz equals 1 wave passing a fixed point in 1 second. **Figure** 1.4 shows high-frequency and low-frequency transverse waves. You can simulate transverse waves with different frequencies at this URL: http://zonalandeducation.com/mstm/physics/waves/partsOfAWave/waveParts.htm .

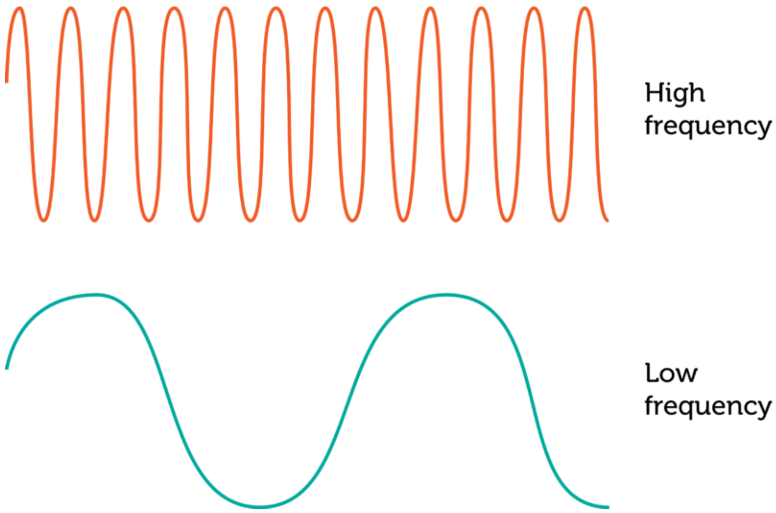


FIGURE 1.4
A transverse wave with a higher frequency has crests that are closer together.

The frequency of a wave is the same as the frequency of the vibrations that caused the wave. For example, to generate a higher-frequency wave in a rope, you must move the rope up and down more quickly. This takes more energy, so a higher-frequency wave has more energy than a lower-frequency wave with the same amplitude.

Wave Speed

Assume that you move one end of a rope up and down just once. How long will take the wave to travel down the rope to the other end? This depends on the speed of the wave. **Wave speed** is how far the wave travels in a given amount of time, such as how many meters it travels per second. Wave speed is not the same thing as wave frequency, but it is related to frequency and also to wavelength. This equation shows how the three factors are related:

$$\text{Speed} = \text{Wavelength} \times \text{Frequency}$$

In this equation, wavelength is measured in meters and frequency is measured in hertz, or number of waves per second. Therefore, wave speed is given in meters per second.

The equation for wave speed can be used to calculate the speed of a wave when both wavelength and wave frequency are known. Consider an ocean wave with a wavelength of 3 meters and a frequency of 1 hertz. The speed of the wave is:

$$\text{Speed} = 3 \text{ m} \times 1 \text{ wave/s} = 3 \text{ m/s}$$

You Try It!

Problem: Jera made a wave in a spring by pushing and pulling on one end. The wavelength is 0.1 m, and the wave frequency is 0.2 m/s. What is the speed of the wave?

If you want more practice calculating wave speed from wavelength and frequency, try the problems at this URL: http://www.physicsclassroom.com/class/waves/u10l2e.cfm .

The equation for wave speed (above) can be rewritten as:

$$\text{Frequency} = \frac{\text{Speed}}{\text{Wavelength}} \text{ or Wavelength} = \frac{\text{Speed}}{\text{Frequency}}$$

Therefore, if you know the speed of a wave and either the wavelength or wave frequency, you can calculate the missing value. For example, suppose that a wave is traveling at a speed of 2 meters per second and has a wavelength of 1 meter. Then the frequency of the wave is:

$$\text{Frequency} = \frac{2 \text{ m/s}}{1 \text{ m}} = 2 \text{ waves/s, or } 2 \text{ Hz}$$

You Try It!

Problem: A wave is traveling at a speed of 2 m/s and has a frequency of 2 Hz. What is its wavelength?

The Medium Matters

The speed of most waves depends on the medium through which they are traveling. Generally, waves travel fastest through solids and slowest through gases. That's because particles are closest together in solids and farthest apart in gases. When particles are farther apart, it takes longer for the energy of the disturbance to pass from particle to particle.

Lesson Summary

- Wave amplitude is the maximum distance the particles of a medium move from their resting positions as a wave passes through. Wavelength is the distance between two corresponding points of adjacent waves. Waves with greater amplitudes or shorter wavelengths have more energy.
- Wave frequency is the number of waves that pass a fixed point in a given amount of time. Higher frequency waves have more energy. Wave speed is calculated as wavelength multiplied by wave frequency. Wave speed is affected by the medium through which a wave travels.

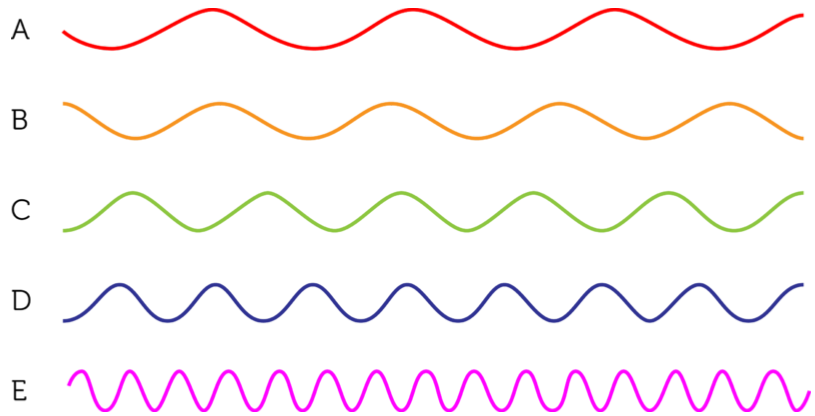
Lesson Review Questions

Recall

1. How is wave amplitude measured in a transverse wave?
2. Describe the wavelength of a longitudinal wave.
3. Define wave frequency.

Apply Concepts

4. All of the waves in the sketch below have the same amplitude and speed. Which wave has the longest wavelength? Which has the highest frequency? Which has the greatest energy?



5. A wave has a wavelength of 0.5 m/s and a frequency of 2 Hz. What is its speed?

Think Critically

6. Relate wave amplitude, wavelength, and wave frequency to wave energy.
7. Waves A and B have the same speed, but wave A has a shorter wavelength. Which wave has the higher frequency? Explain how you know.

Points to Consider

You read in this lesson that waves travel at different speeds in different media.

- When a wave enters a new medium, it may speed up or slow down. What other properties of the wave do you think might change when it enters a new medium?
- What if a wave reaches a type of matter it cannot pass through? Does it just stop moving? If not, where does it go?

References

1. David Rydevik. http://commons.wikimedia.org/wiki/File:2004-tsunami.jpg . Public Domain
2. Christopher Auyeung. CK-12 Foundation .
3. Christopher Auyeung. CK-12 Foundation .
4. Christopher Auyeung. CK-12 Foundation .

CONCEPT 2 Mechanical Wave

- Describe mechanical waves.
- Define the medium of a mechanical wave.
- Identify three types of mechanical waves.

No doubt you've seen this happen. Droplets of water fall into a body of water, and concentric circles spread out through the water around the droplets. The concentric circles are waves moving through the water.

Waves in Matter

The waves in the picture above are examples of mechanical waves. A **mechanical wave** is a disturbance in matter that transfers energy through the matter. A mechanical wave starts when matter is disturbed. A source of energy is needed to disturb matter and start a mechanical wave.

Q: Where does the energy come from in the water wave pictured above?

A: The energy comes from the falling droplets of water, which have kinetic energy because of their motion.

The Medium

The energy of a mechanical wave can travel only through matter. The matter through which the wave travels is called the **medium** (*plural* , media). The medium in the water wave pictured above is water, a liquid. But the medium of a mechanical wave can be any state of matter, even a solid.

Q: How do the particles of the medium move when a wave passes through them?

A: The particles of the medium just vibrate in place. As they vibrate, they pass the energy of the disturbance to the particles next to them, which pass the energy to the particles next to them, and so on. Particles of the medium don't actually travel along with the wave. Only the energy of the wave travels through the medium.

Types of Mechanical Waves

There are three types of mechanical waves: transverse, longitudinal, and surface waves. They differ in how particles of the medium move. You can see this in the **Figure** 2.1 and in the animation at the following URL. http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

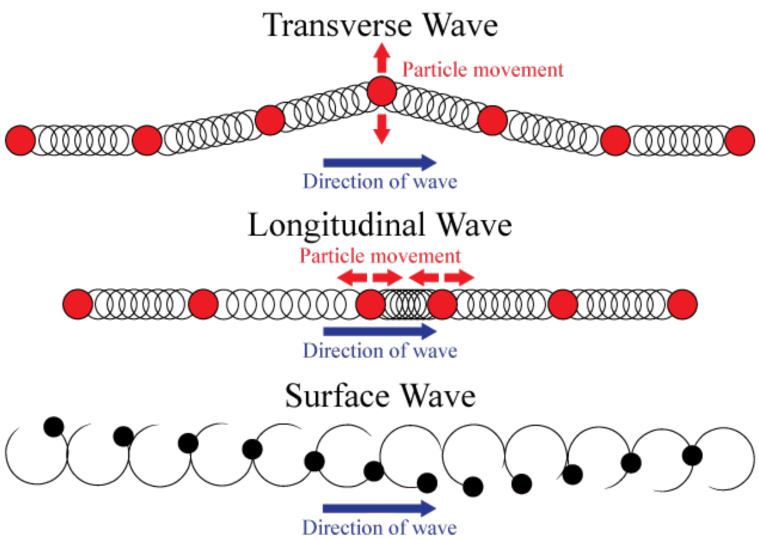


FIGURE 2.1

- In a transverse wave, particles of the medium vibrate up and down perpendicular to the direction of the wave.
- In a longitudinal wave, particles of the medium vibrate back and forth parallel to the direction of the wave.
- In a surface wave, particles of the medium vibrate both up and down and back and forth, so they end up moving in a circle.

Q: How do you think surface waves are related to transverse and longitudinal waves?

A: A surface wave is combination of a transverse wave and a longitudinal wave.

Summary

- A mechanical wave is a disturbance in matter that transfers energy through the matter.
- The matter through which a mechanical wave travels is called the medium (*plural* , media).
- There are three types of mechanical waves: transverse, longitudinal, and surface waves. They differ in how particles of the medium move when the energy of the wave passes through.

Vocabulary

- **mechanical wave** : Disturbance in matter that transfers energy from one place to another.
- **medium** (plural, **media**): Matter through which a mechanical wave moves.

Practice

At the following URL, read the short introduction to waves and watch the animations. Then answer the questions below. http://www.acs.psu.edu/drussell/Demos/waves-intro/waves-intro.html

1. The article gives a dictionary definition of wave. What is the most important part of this definition?
2. What happens to particles of the medium when a wave passes?
3. How is "doing the wave" in a football stadium like a mechanical wave?

Review

1. Define mechanical wave.
2. What is the medium of a mechanical wave?
3. List three types of mechanical waves.
4. If you shake one end of a rope up and down, a wave passes through the rope. Which type of wave is it?

References

1. Zachary Wilson. . CC BY-NC 3.0

CONCEPT 3 Transverse Waves

- Describe transverse waves.
- Explain how waves transfer energy without transferring matter.
- Define wavelength, frequency, and period of a transverse wave.
- State the relationship between speed, wavelength, and frequency.
- Solve problems using the relationships between speed, wavelength, frequency, and period.

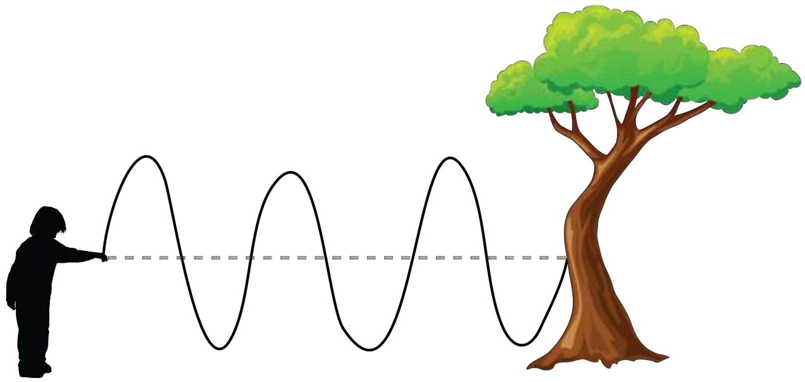
Professional surfer Marcio Freire rides a giant wave at the legendary big wave surf break known as as "Jaws" during one the largest swells of the winter March 13, 2011 in Maui, HI. Massive waves, such as this one, transfer huge amounts of energy.

Transverse Waves

Types of Waves

Water waves, sound waves, and the waves that travel along a rope are **mechanical waves** . Mechanical waves require a material medium such as water, air, or rope. Light waves, however, are **electromagnetic waves** and travel without a material medium. They are not mechanical waves.

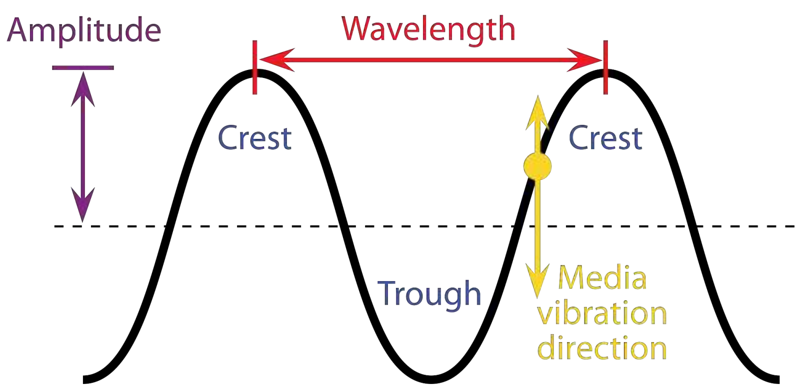
In all types of mechanical waves, energy moves from one place to another while the media carrying the wave only vibrates back and forth in position. One type of mechanical wave is the **transverse wave** . In the case of transverse waves, the movement of the medium is perpendicular to the direction of the energy movement.



In the sketch above, consider the transverse wave produced when the boy jerks one end of a rope up and down while the other end is tied to a tree. The energy spent by the boy transfers permanently down the rope to the tree. The rope, however, only moves up and down. If we stuck a piece of tape somewhere on the rope, we would see that the particles of medium do not travel with the energy. After the wave has passed by, the piece of tape would still be in the same place it was before the wave approached. In all transverse waves, the movement media vibrates perpendicularly to the direction of wave motion, and the medium is not permanently moved from one place to another.

Frequency, Wavelength, and Velocity

Waves are identified by several characteristics. There is a center line where the medium would be if there were no wave, which is sometimes describes as the undisturbed position. The displacement of the medium above this undisturbed position is called a **crest** and the displacement below the undisturbed position is called a **trough** . The maximums of the crest and trough are equal and are called the **amplitude** . The distance between equivalent positions on succeeding waves is called the **wavelength** . The wavelength could be measured from a crest to the next crest or from a trough to the next trough, and is commonly represented with the Greek letter lambda, λ .



The time interval required for one complete wave to pass a point is called the **period** . During the period of the wave, an entire wavelength from one crest to the next crest passes a position. The number of waves that pass a single position in one second is called the **frequency** . The period of a wave and its frequency are reciprocals of each other.

$$f = \frac{1}{T}$$

The units for the period are seconds and the units for frequency are s^{-1} or $\frac{1}{s}$. This unit has also been given the name Hertz (Hz).

Another important characteristic of a wave is its velocity. The wave velocity is different from the velocity of the medium; the wave velocity is the velocity of the linearly transferred energy. Since the energy travels one wavelength, λ , in one period, T , the velocity can be expressed as distance over time:

$$v = \frac{\lambda}{T}.$$

Since period and frequency are reciprocals, the speed of the wave could also be expressed as $v = \lambda f$.

Example Problem: A sound wave has a frequency of 262 Hz. What is the time lapse between successive wave crests?

Solution: The time lapse between successive crests would be the period and the period is the reciprocal of the frequency.

$$T = \frac{1}{f} = \frac{1}{262 \text{ s}^{-1}} = 0.00382 \text{ s}$$

Example Problem: A sound wave has a frequency of 262 Hz has a wavelength of 1.29 m. What is the velocity of the wave?

Solution: $v = \lambda f = (1.29 \text{ m})(262 \text{ s}^{-1}) = 338 \text{ m/s}$

Summary

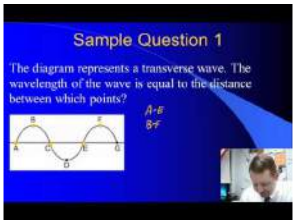
- Mechanical waves require a material medium such as water, air, or rope.
- In all types of mechanical waves, energy moves from one place to another while the media carrying the wave only vibrates back and forth in position.
- One type of mechanical wave is the transverse wave, in which the movement of the medium is perpendicular to the direction of the energy propagation.
- The maximum displacement of the medium is the distance from the undisturbed position to the top of a crest, or the amplitude.
- The distance along the line of motion of the wave from equivalent positions on succeeding waves is the wavelength.
- The time interval required for one entire wave to pass a point is the period.
- The number of periods per second is the wave's frequency.
- The period of a wave and its frequency are reciprocals of each other.
- The velocity of the wave's energy transfer is given by $v = \lambda f$ or

$$v = \frac{\lambda}{T}.$$

Practice

The following video explains wave characteristics. Pause the video before each practice question and try to solve it yourself before moving on.

http://www.youtube.com/watch?v=5ENLxaPiJJI



MEDIA
Click image to the left for more content.

1. What is the distance between the base line and crest called?
2. What symbol is used for wavelength?
3. What is the relationship between period and frequency?

Review

1. A sound wave produced by a chime 515 m away is heard 1.50 s later.
   (a) What is the speed of sound in air?
   (b) The sound wave has a frequency of 436 Hz. What is its period?
   (c) What is the wavelength of the sound?
2. A hiker shouts toward a vertical cliff 685 m away. The echo is heard 4.00 s later.
   (a) What is the speed of sound in air?
   (b) Why is this speed of sound slightly different from the previous answer?
   (c) The wavelength of the sound is 0.750 m. What is the frequency?
   (d) What is the period of the wave?
3. The speed of light in air is 3.00×10^{8} m/s. If a light wave has a wavelength of 5.80×10^{-7} m, what is its frequency?

- **transverse wave:** A transverse wave is a moving wave that consists of oscillations occurring perpendicular (or right angled) to the direction of energy transfer
- **undisturbed position:** The equilibrium or rest position of the medium in a wave.
- **crest:** A crest is the point on a wave with the maximum value or upward displacement within a cycle
- **trough:** A trough is the opposite of a crest, so the minimum or lowest point in a cycle
- **amplitude:** The maximum displacement from a zero value during one period of an oscillation.
- **wavelength:** The distance between corresponding points of two consecutive waves.
- **frequency:** The number of waves that pass a fixed point in unit time.
- **period:** A period is the time required for one complete cycle of vibration to pass a given point.

References

1. Image copyright EpicStockMedia, 2013. http://www.shutterstock.com . Used under license from Shutterstock.com
2. Kid: Image copyright Richcat, 2013; Tree: Image copyright AlexeyZet, 2013; Composite created by CK-12 Foundation - Samantha Bacic. http://www.shutterstock.com/ . Used under licenses from Shutterstock.com
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CONCEPT 4 Longitudinal Waves

- Describe longitudinal waves.

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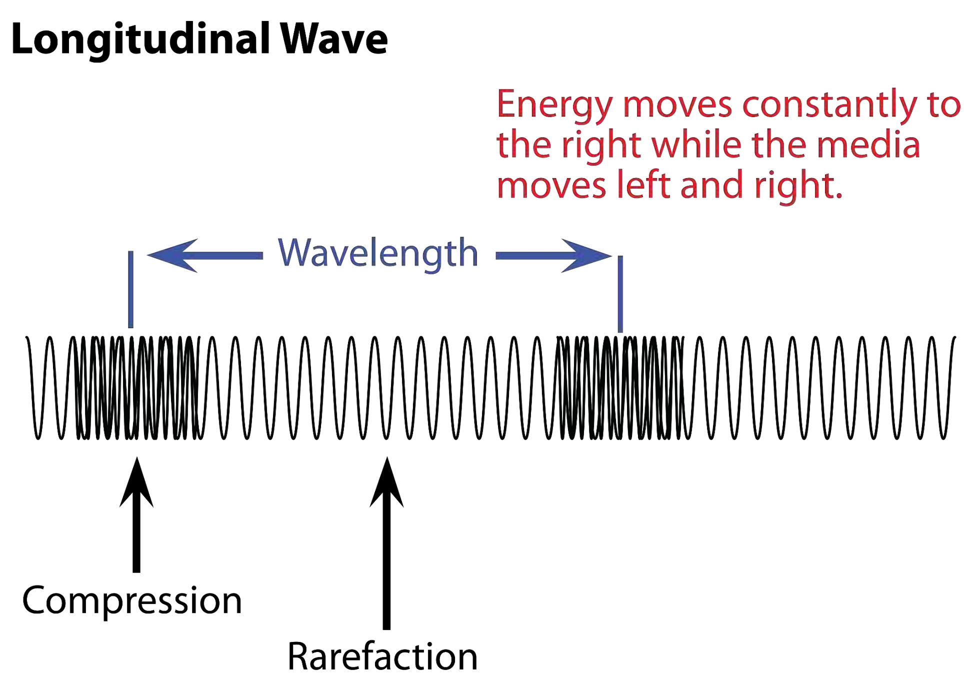
Playing with a Slinky is a childhood tradition, but few children realize they are actually playing with physics.

Longitudinal Waves

Like transverse waves, **longitudinal waves** are mechanical waves, which means they transfer energy through a medium. Unlike transverse waves, longitudinal waves cause the particles of medium to move parallel to the direction of the wave. They are most common in springs, where they are caused by the pushing an pulling of the spring. Although the surface waves on water are transverse waves, fluids (liquids, gases, and plasmas) usually transmit longitudinal waves.

As shown in the image below, longitudinal waves are a series of compressions and **rarefactions** , or expansions. The wavelength of longitudinal waves is measured by the distance separating the densest compressions. The amplitude of longitudinal waves is the difference in media density between the undisturbed density to the highest density in a compression.



Example Problem: A sonar signal (sonar is sound waves traveling through water) of 1.00×10^6 Hz frequency has a wavelength of 1.50 mm in water. What is the speed of sound in water?

Solution: $v = \lambda f = (0.00150 \text{ m})(1.00 \times 10^6 \text{ s}^{-1}) = 1500 \text{ m/s}$

Example Problem: A sound wave of wavelength 0.70 m and velocity 330 m/s is produced for 0.50 s.

a. What is the frequency of the wave?
b. How many complete waves are emitted in this time interval?
c. After 0.50 s, how far is the wave front from the source of the sound?

Solution:

a. $f = \frac{v}{\lambda} = \frac{330 \text{ m/s}}{0.70 \text{ m}} = 470 \text{ s}^{-1}$
b. complete waves $= (470 \text{ cycles/s})(0.50 \text{ s}) = 235$ cycles
c. distance $= (330 \text{ m/s})(0.50 \text{ s}) = 115$ m

Summary

- Longitudinal waves cause the particles of medium to move parallel to the direction of the wave.

Practice

The following video explains how a tuning fork creates sound . Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=bomzzHC-59k

MEDIA
Click image to the left for more content.

1. In your own words, how are compressions and rarefactions produced by the tuning fork?
2. Make a guess why sound can easily travel around corners (Hint: think of its medium).

Review

1. Some giant ocean waves have a wavelength of 25 m long, and travel at speeds of 6.5 m/s. Determine the frequency and period of such a wave.
2. Bats use sound echoes to navigate and hunt. They emit pulses of high frequency sound waves which reflect off obstacles in the surroundings. By detecting the time delay between the emission and return of a pulse, a bat can determine the location of the object. What is the time delay between the sending and return of a pulse from an object located 12.5 m away? The approximate speed of sound is 340 m/s.
3. Sachi is listening to her favorite radio station which broadcasts radio signals with a frequency of 1.023×10^8 Hz . If the speed of the signals in air is 2.997×10^8 m/s , what is the wavelength of these radio signals?
4. A longitudinal wave is observed to be moving along a slinky. Adjacent crests are 2.4 m apart. Exactly 6 crests are observed to move past a given point in 9.1 s. Determine the wavelength, frequency, and speed of this wave.
5. A sonar signal leaves a submarine, travels through the water to another submarine and reflects back to the original submarine in 4.00 s. If the frequency of the signal was 512 cycles per second and the wavelength of the signal was 2.93 m, how far away is the second submarine?

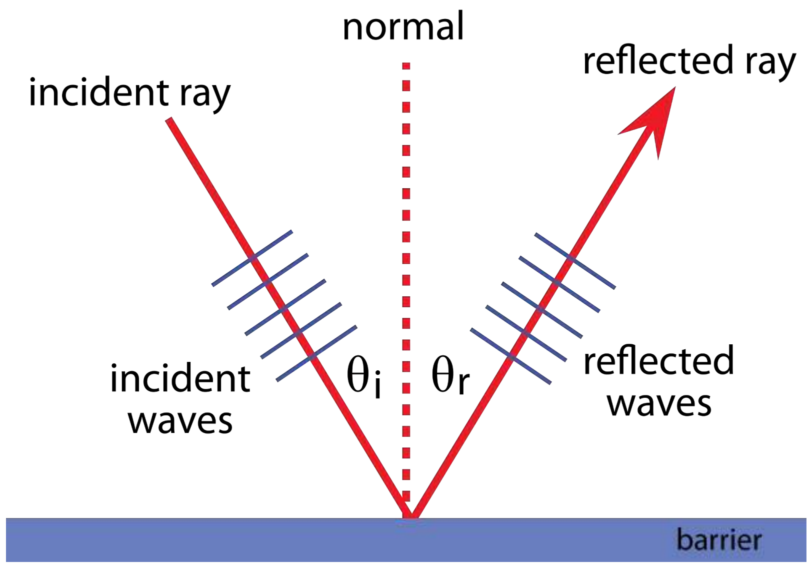
- **longitudinal wave:** A wave in which the direction of media displacement is the same as the direction of wave propagation.

References

1. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

CONCEPT 5 Reflection of Mechanical Waves

- State the law of reflection.
- Solve problems using the law of reflection.
- Given data about the media on either side of a barrier, determine whether the reflected wave will be upright or inverted.



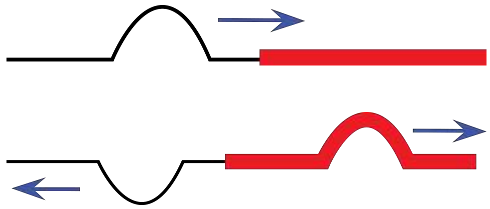
When mechanical waves strike a barrier, at least part of the energy of the waves will be reflected back into the media from which they came. You experience this every single day, when you look in the mirror and see your own reflection.

Reflection of Mechanical Waves

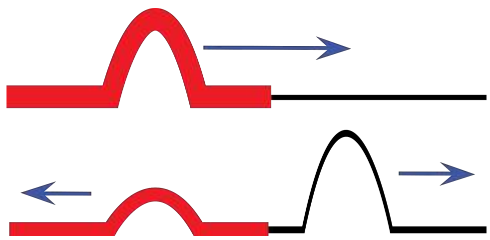
When a wave strikes an obstacle or comes to the end of the medium it is traveling in, some portion of the wave is reflected back into the original medium. It reflects back at an equal angle that it came in. These angles are called the **angle of incidence** and the **angle of reflection** . The normal line, the incident and reflected rays, and the angles of incidence and reflection are all shown in the diagram sketched above. The **law of reflection** states that the angle of incidence equals the angle of reflection. These rules of **reflection** apply in the cases of water waves bouncing off the side of a pool, sound waves echoing off a distant cliff, or wave pulses traveling down a rope or a slinky.

Consider the change that would occur with a light rope joined to a heavier rope. When a wave pulse travels down the rope and encounters the media change, a reflection will occur. Look at the image below. In the top sketch, we see a lightweight (black) rope attached to a heavier rope (red). There is a wave pulse traveling down the rope from left to right. When the wave pulse encounters the barrier (the change in rope weight), part of the wave moves into the new medium and part of the wave is reflected back into the old medium.

As you can see in the bottom half of the diagram, the transmitted portion of the wave continues into the new medium right side up. The transmitted wave is somewhat diminished because some of the energy of the wave was reflected and also because the rope to be lifted is heavier. The reflected wave is also diminished because some of the energy was transmitted through the barrier. The reflected wave is also inverted (upside down). This is a general rule for mechanical waves passing from a less dense medium into a more dense medium, that is, the reflected wave will be inverted.



The situation changes when the wave is passing from a more dense medium into a less dense medium. As you can see in the sketch below, when a wave pulse moving in denser medium encounters a media interface to a medium of less density, the reflected wave is upright rather than inverted.



It is also possible for a mechanical wave to encounter an impenetrable barrier, that is, a barrier which does not allow any transmission at all. In such a case, the complete wave pulse will be reflected and the reflected wave will be inverted.

Summary

- When a wave strikes an obstacle or comes to the end of the medium it is traveling in, some part of the wave is reflected back into the original medium.
- The law of reflection states that the angle of incidence equals the angle of reflection.
- The general rule, for mechanical waves passing from a less dense medium into a more dense medium, the reflected wave will be inverted.
- When a wave pulse moving in denser medium encounters a media interface to a medium of lesser density, the reflected wave is upright rather than inverted.
- When a mechanical wave encounters an impenetrable barrier, the complete wave pulse will be reflected and the reflected wave will be inverted.

Practice

The following video shows a wave machine in action. Use this video to answer the questions that follow.

http://www.youtube.com/watch?v=YQHbRw_hyz4

MEDIA
Click image to the left for more content.

1. What happens to the wave when it is reflected from an open end?
2. What happens to the wave when it is reflected from a fixed end?

Review

1. Draw a diagram showing a surface with a normal line. On the diagram, show a wave ray striking the surface with an angle of incidence of 60° . Draw the reflection ray on the diagram and label the angle of reflection.
2. Light strikes a mirror's surface at 30° to the normal. What will the angle of reflection be?
3. If the angle between the incident ray and the reflected ray is 90° , what is the angle of incidence?
4. When a water wave is reflected from a concrete wall, will the reflected wave be inverted or upright?
5. If you tie a heavy spring to a light spring and send a wave pulse down the heavy spring, some of the wave will be reflected when the wave passes into the lighter spring. Will the reflected pulse be upright or inverted?

- **reflection:** The change in direction of a wavefront at an interface between two different media so that the wavefront returns into the medium from which it originated.
- **law of reflection:** Says the angle at which the wave is incident on the surface equals the angle at which it is reflected.
- **angle of incidence:** The angle formed by a ray incident on a surface and a perpendicular to the surface at the point of incidence.
- **angle of reflection:** The angle formed by a reflected ray and a perpendicular to the surface at the point of reflection.

References

1. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
4. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

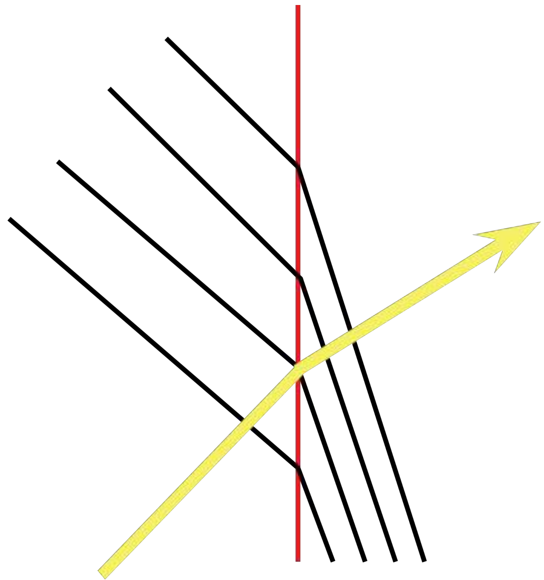
Concept 6 Refraction of Mechanical Waves

- Define and describe refraction of mechanical waves.
- State and use the law of refraction.

A straw in a glass of water seen from the side often appears broken, even though it is not. The apparent break is due to the bending of light rays leaving the straw; as the light passes from the water to the glass and from the glass to the air, the light rays are bent. Nonetheless, your eye traces the light ray backward as if the light has followed a straight path from its origin at the straw. Since the light appears to have come from a different place, your eye sees the straw as being broken.

Refraction of Mechanical Waves

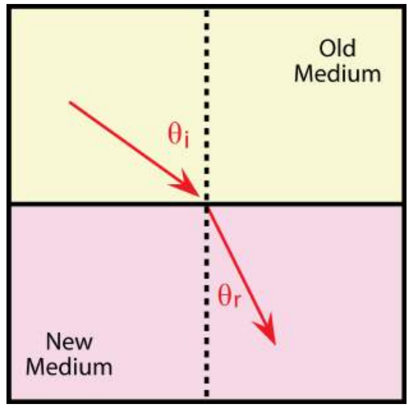
When any wave strikes a boundary between media, some of the energy is reflected and some is transmitted. When the wave strikes the media interface at an angle, the transmitted wave will move in a slightly different direction than the incident wave. This phenomenon is known as **refraction** .



Consider the image sketched above. Suppose that the waves represented here are water waves. The wave crests are represented by the black lines in the image. As such, the distance between two consecutive black lines is the wavelength. Let the red line represent a transition from deep to shallow water. This transition is called the **media interface** . As the waves hit the boundary, the waves slow down. The right side of the wave reaches the boundary before the left side of the wave, causing the left side to catch up and the angle of propagation to change slightly. This change in direction can be seen in the yellow line, which is slightly angled at the boundary.

The refraction of waves across boundaries operates similarly to the method by which tanks are steered. Tanks do not have a steering wheel. Instead, they have an accelerator to produce forward motion and separate brakes on each tread. The operator uses brakes on both treads at the same time in order to stop, but brakes on only one tread to turn the tank. By braking one side, the operator causes that side to slow down or stop while the other side continues at the previous speed, causing the tank to turn towards the slower tread.

This sketch shows a wave ray striking an interface between old medium and new medium. A normal line has been drawn as a dotted line perpendicular to the interface. The angle between the incident ray and the normal line is called the **angle of incidence** , shown as θ_i , and the angle between the refracted ray and the normal line is called the **angle of refraction** , θ_r .



We already understand that the change in the wave direction at the border depends on the difference between the two velocities. This relationship is conveniently expressed in a mathematical relationship:

$$\frac{\sin\theta_r}{\sin\theta_i} = \frac{v_r}{v_i} = \frac{\lambda_r}{\lambda_i}$$

The ratio of the sine of the angle of refraction to the sine of the angle of incidence is the same as the ratio of the velocity of the wave in the new medium to the velocity of the wave in the old medium and equal to the ratio of wavelength (λ) in the old medium to the wavelength in the new medium.

Example Problem: A water wave with a wavelength of 3.00 m is traveling in deep water at 16.0 m/s. The wave strikes a sharp interface with shallow water with an angle of incidence of 53.0° . The wave refracts into the shallow water with an angle of refraction of 30.0° . What is the velocity of the wave in shallow water and what is its wavelength in the new medium?

Solution: $\frac{\sin\theta_r}{\sin\theta_i} = \frac{v_r}{v_i}$ so $\frac{\sin 30^\circ}{\sin 53^\circ} = \frac{v_r}{16.0\ m/s}$ and $v_r = 10.0\ m/s$.

$\frac{v_r}{v_i} = \frac{\lambda_r}{\lambda_i}$ so $\frac{10.0\ m/s}{16.0\ m/s} = \frac{\lambda_r}{3.00\ m}$ and $\lambda_r = 1.88\ m$.

Example Problem: The ratio of the $\sin\theta_r$ to $\sin\theta_i$ is 0.769 . If the wavelength of a wave in a new medium is $5.00 \times 10^{-9}\ m$, what is its wavelength in the original medium?

Solution:

$$0.769 = \frac{\lambda_r}{\lambda_i}\ so\ \lambda_i = \frac{5.00 \times 10^{-9}\ m}{0.769} = 6.50 \times 10^{-9}\ m$$

Summary

- When any wave strikes a boundary between media, some of the energy is reflected and some is transmitted.
- When a wave strikes the media interface at an angle, the transmitted wave will move in a different direction than the incident wave. This phenomenon is known as refraction.
- At any media interface, $\frac{\sin\theta_r}{\sin\theta_i} = \frac{v_r}{v_i} = \frac{\lambda_r}{\lambda_i}$

Practice

http://www.youtube.com/watch?v=mH9VwivqjmE

Follow up questions.

1. What causes refraction?
2. What doesn't change during refraction?

Review

1. A laser beam passes through water and enters a glass block at an angle. The ratio of the speed of the wave in glass to the speed in water is 0.866 . If the angle of incidence to the interface is 60° , what is the angle of refraction?
2. A ray of light is traveling from air into glass at an angle of 30.0° to the normal line. The speed of the light in air is $3.00 \times 10^8\ m/s$ and in glass the speed drops to $2.00 \times 10^8\ m/s$. What is the angle of refraction?
3. Which of the following change when a water wave moves across a boundary at an angle between deep water and shallow water?
   a. frequency
   b. wavelength
   c. speed
   d. wave direction
   e. period
4. Which of the following change when a water wave moves across a boundary exactly along the media interface between deep water and shallow water?
   a. frequency
   b. wavelength
   c. speed
   d. wave direction
   e. period
5. The speed of sound is 340 m/s. A particular sound wave has a frequency of 320. Hz.
   a. What is the wavelength of this sound in air?
   b. If this sound refracts into water where the speed of sound is 4 times faster, what will be the new wavelength?
   c. What will be the new frequency?
6. When a light ray passes from air into diamond, the angle of incidence is 45.0° and the angle of refraction is 16.7° . If the speed of light in air is $3.00 \times 10^8\ m/s$, what is the speed of light in diamond?

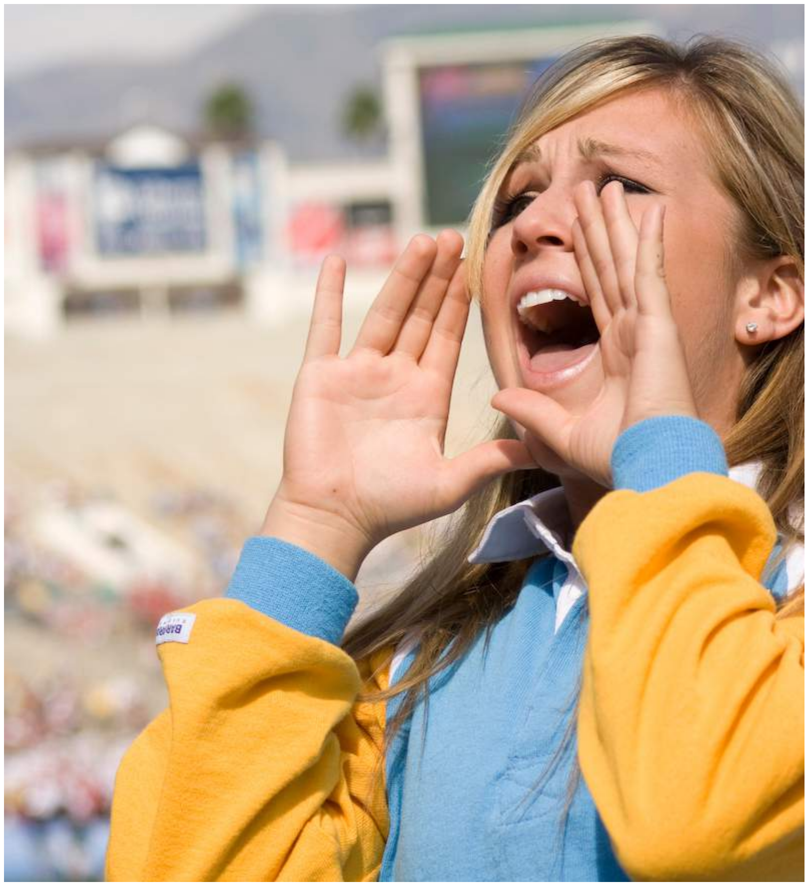
- **refraction:** The turning or bending of a wave direction when it passes from one medium to another of different density.

References

1. Image copyright cheyennezj, 2013. http://www.shutterstock.com . Used under license from Shutterstock.com
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. Greg Goebel. http://www.public-domain-image.com/full-image/transportation-vehicles-public-domain-images-pictures/tanks-public-domain-images-pictures/m7-priest-self-propelled-105-millimeter-howitzer-tank.jpg-copyright-friendly-image.html . Public Domain
4. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

CONCEPT 7 Wave Interactions

- Identify ways that waves can interact with matter.
- Define and give examples of wave reflection, refraction, and diffraction.



Did you ever hear an echo of your own voice? An echo occurs when sound waves bounce back from a surface that they can't pass through. The woman pictured here is trying to create an echo by shouting toward a large building. When the sound waves strike the wall of the building, most of them bounce back toward the woman, and she hears an echo of her voice. An echo is just one example of how waves interact with matter.

How Waves Interact with Matter

Waves interact with matter in several ways. The interactions occur when waves pass from one medium to another. The types of interactions are reflection, refraction, and diffraction. Each type of interaction is described in detail below. You can see animations of the three types at this URL: http://www.acoustics.salford.ac.uk/schools/teacher/lesson3/flash/whiteboardcomplete.swf

Reflection

An echo is an example of wave reflection. **Reflection** occurs when waves bounce back from a surface they cannot pass through. Reflection can happen with any type of waves, not just sound waves. For example, light waves can also be reflected. In fact, that's how we see most objects. Light from a light source, such as the sun or a light bulb, shines on the object and some of the light is reflected. When the reflected light enters our eyes, we can see the object.

Reflected waves have the same speed and frequency as the original waves before they were reflected. However, the direction of the reflected waves is different. When waves strike an obstacle head on, the reflected waves bounce straight back in the direction they came from. When waves strike an obstacle at any other angle, they bounce back at the same angle but in a different direction. This is illustrated in diagram below. In this diagram, waves strike a wall at an angle, called the angle of incidence. The waves are reflected at the same angle, called the angle of reflection, but in a different direction. Notice that both angles are measured relative to a line that is perpendicular to the wall.

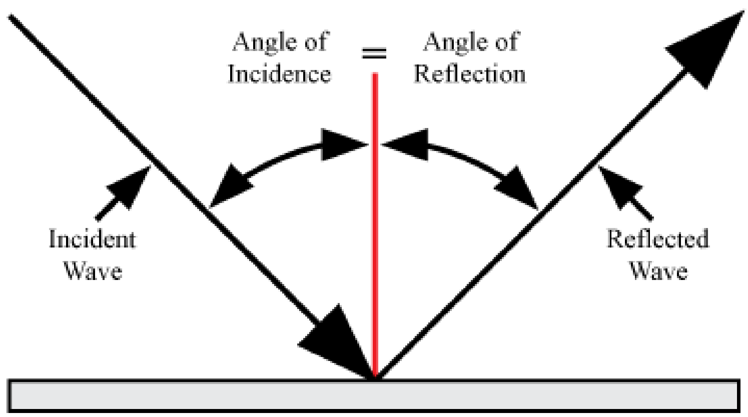


FIGURE 7.1

Refraction

Refraction is another way that waves interact with matter. **Refraction** occurs when waves bend as they enter a new medium at an angle. You can see an example of refraction in the picture below. Light bends when it passes from air to water or from water to air. The bending of the light traveling from the fish to the man's eyes causes the fish to appear to be in a different place from where it actually is.

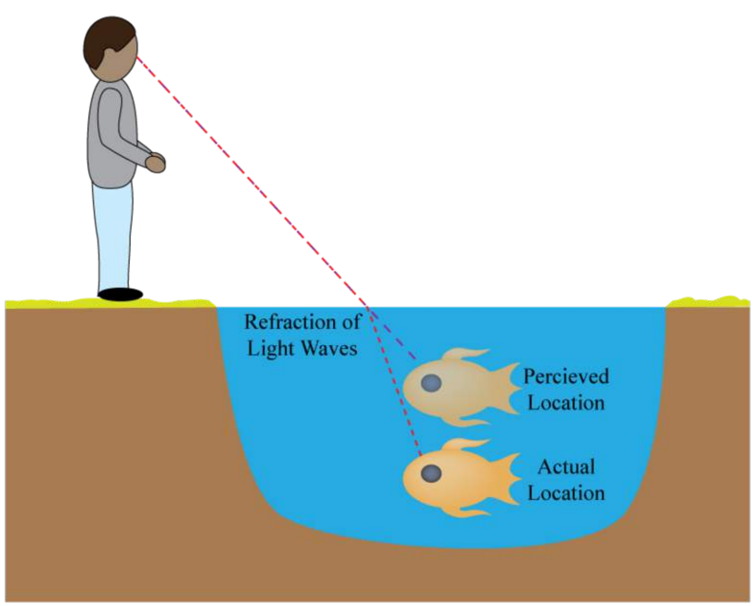


FIGURE 7.2

Waves bend as they enter a new medium because they start traveling at a different speed in the new medium. For example, light travels more slowly in water than in air. This causes it to refract when it passes from air to water or from water to air.

Q: Where would the fish appear to be if the man looked down at it from straight above its actual location?

A: The fish would appear to be where it actually is because refraction occurs only when waves (in this case light waves from the fish) enter a new medium at an angle other than 90°.

Diffraction

Did you ever notice that you can hear sounds around the corners of buildings even though you can't see around them? The **Figure** 7.3 shows why this happens. As you can see from the figure, sound waves spread out and travel around obstacles. This is called **diffraction** . It also occurs when waves pass through an opening in an obstacle. All waves may be diffracted, but it is more pronounced in some types of waves than others. For example, sound waves bend around corners much more than light does. That's why you can hear but not see around corners.

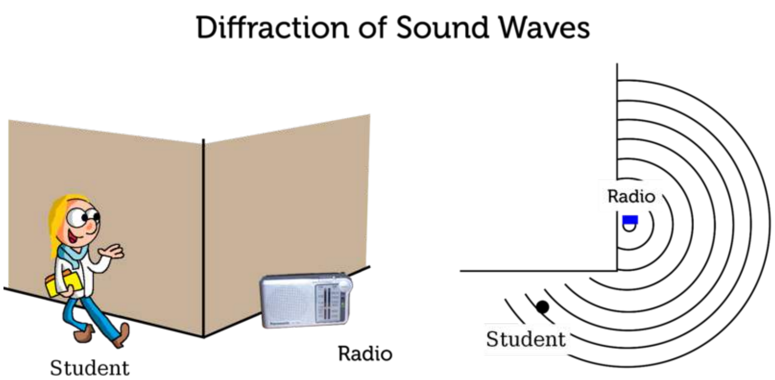


FIGURE 7.3

For a given type of waves, such as sound waves, how much the waves diffract depends on the size of the obstacle (or opening in the obstacle) and the wavelength of the waves. The **Figure** 7.4 shows how the amount of diffraction is affected by the size of the opening in a barrier. Note that the wavelength of the wave is the distance between the vertical lines.

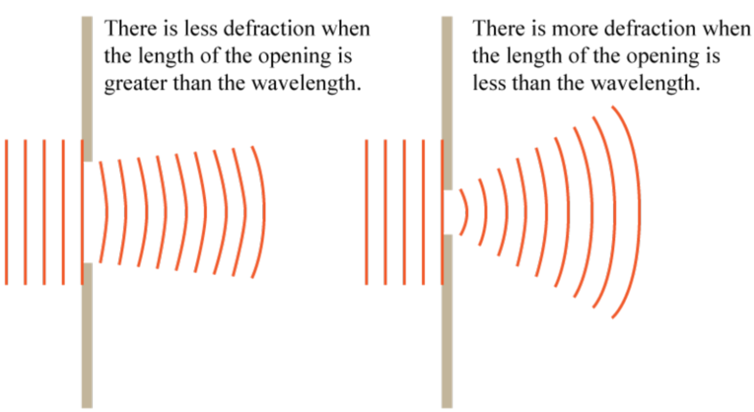


FIGURE 7.4

Summary

- Three ways that waves may interact with matter are reflection, refraction, and diffraction.
- Reflection occurs when waves bounce back from a surface that they cannot pass through.
- Refraction occurs when waves bend as they enter a new medium at an angle and start traveling at a different speed.
- Diffraction occurs when waves spread out as they travel around obstacles or through openings in obstacles.

Vocabulary

- **diffraction** : Bending of a wave around an obstacle or through an opening in an obstacle.
- **reflection** : Bouncing back of waves from a barrier they cannot pass through.
- **refraction** : Bending of waves as they enter a new medium at an angle and change speed.

Practice

Make a crossword puzzle of terms relating to wave interactions. Include at least seven different terms. You can use the puzzle maker at the following URL. Then exchange and solve puzzles with a classmate. http://puzzlemaker.discoveryeducation.com/CrissCrossSetupForm.asp

Review

1. What is reflection? What happens if waves strike a reflective surface at an angle other than 90°?
2. Define refraction. Why does refraction occur?
3. When does diffraction occur? How is wavelength related to diffraction?

References

1. Zachary Wilson. . CC BY-NC 3.0
2. Zachary Wilson. . CC BY-NC 3.0
3. Student: Flickr:MaxTorrt; Radio: Flickr:Kansir. . CC BY 2.0
4. Zachary Wilson. . CC BY-NC 3.0

CONCEPT 8 Wave Interference

- Define wave interference.
- Compare and contrast constructive and destructive interference.
- Explain how standing waves occur.

When raindrops fall into still water, they create tiny waves that spread out in all directions away from the drops. What happens when the waves from two different raindrops meet? They interfere with each other.

When Waves Meet

When two or more waves meet, they interact with each other. The interaction of waves with other waves is called **wave interference** . Wave interference may occur when two waves that are traveling in opposite directions meet. The two waves pass through each other, and this affects their amplitude. Amplitude is the maximum distance the particles of the medium move from their resting positions when a wave passes through. How amplitude is affected by wave interference depends on the type of interference. Interference can be constructive or destructive.

Constructive Interference

Constructive interference occurs when the crests, or highest points, of one wave overlap the crests of the other wave. You can see this in the **Figure** 8.1 . As the waves pass through each other, the crests combine to produce a wave with greater amplitude. You can see an animation of constructive interference at this URL: http://phys23p.sl.psu.edu/phys_anim/waves/embederQ1.20100.html

Destructive Interference

Destructive interference occurs when the crests of one wave overlap the troughs, or lowest points, of another wave. The **Figure** 8.2 shows what happens. As the waves pass through each other, the crests and troughs cancel each other out to produce a wave with zero amplitude. You can see an animation of destructive interference at this URL: http://phys23p.sl.psu.edu/phys_anim/waves/embederQ1.20200.html

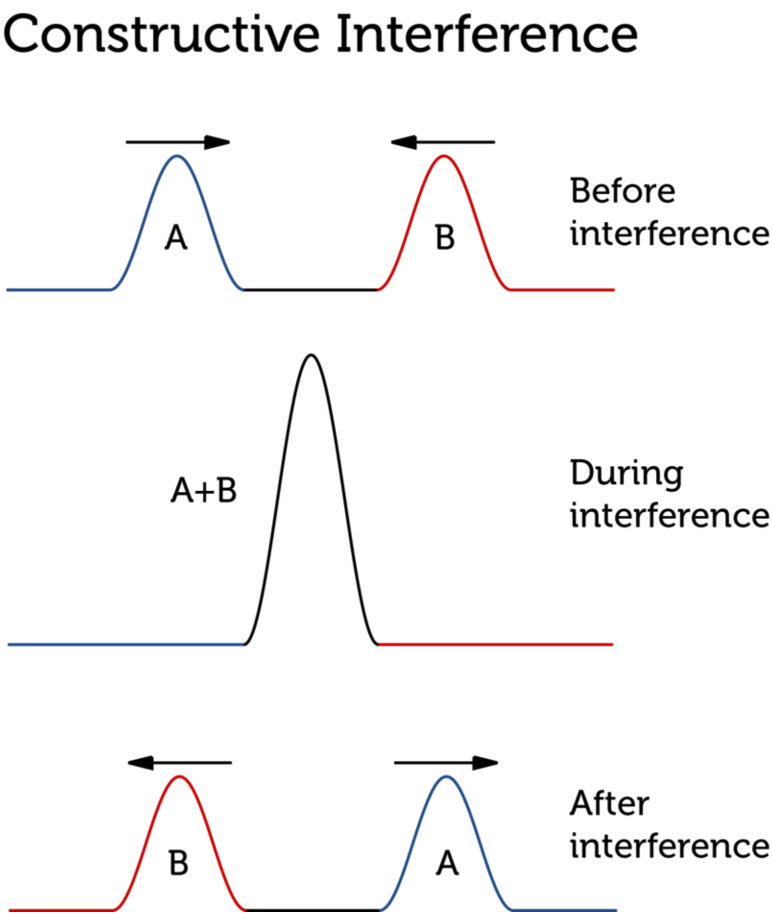


FIGURE 8.1

Standing Waves

Waves may reflect off an obstacle that they are unable to pass through. When waves are reflected straight back from an obstacle, the reflected waves interfere with the original waves and create **standing waves** . These are waves that appear to be standing still. Standing waves occur because of a combination of constructive and destructive interference. You can see animations of standing waves at the URLs below.

http://skullsinthestars.com/2008/05/04/classic-science-paper-otto-wieners-experiment-1890/ http://www.physicsclassroom.com/mmedia/waves/swf.cfm

Q : How could you use a rope to produce standing waves?

A : You could tie one end of the rope to a fixed object, such as doorknob, and move the other end up and down to generate waves in the rope. When the waves reach the fixed object, they are reflected back. The original waves and the reflected waves interfere to produce a standing wave. Try it yourself and see if the waves appear to stand still.

Summary

- Wave interference is the interaction of waves with other waves.
- Constructive interference occurs when the crests of one wave overlap the crests of the other wave, causing an increase in wave amplitude.
- Destructive interference occurs when the crests of one wave overlap the troughs of the other wave, causing a decrease in wave amplitude.

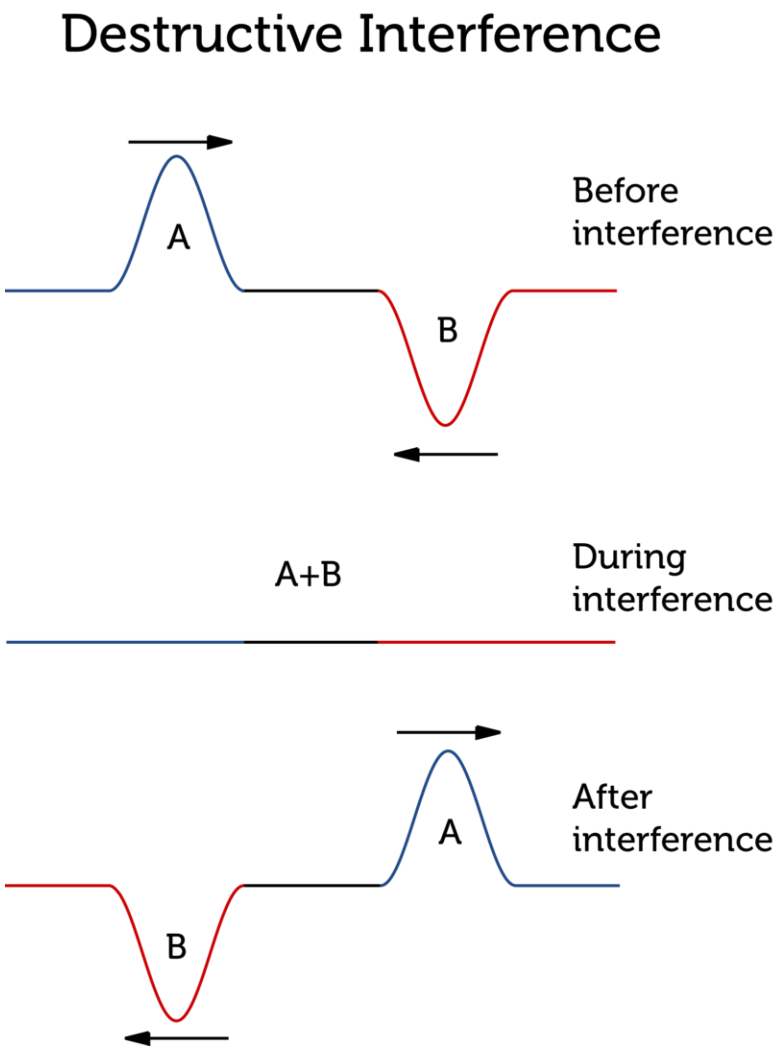


FIGURE 8.2

- When waves are reflected straight back from an obstacle, the reflected waves interfere with the original waves and create standing waves.

Vocabulary

- **standing wave** : Wave appearing to stand still that forms when a wave and its reflected wave interfere.
- **wave interference** : Interaction of waves with other waves.

Practice

Review wave interference at the following URL. Then do the Check Your Understanding problem at the bottom of the Web page. Be sure to check your answers. http://www.physicsclassroom.com/class/waves/u10l3c.cfm

Review

1. What is wave interference?
2. Create a table comparing and contrasting constructive and destructive interference.

3. What are standing waves? How do they form?

References

1. Christopher Auyeung. . CC BY-NC 3.0
2. Christopher Auyeung. . CC BY-NC 3.0

CONCEPT 9 Wave Speed

Objective

The student will:

- Solve problems involving wavelength, wave speed, and frequency.

Vocabulary

- **wave equation:** Relates wavelength and wave speed. Distance is speed times time, $x = vt$, so wavelength is wave speed times period $\lambda = vT$.
- **wave speed:** How quickly the peak of each wave is moving forward.

The Wave Equation

Simple harmonic motion, moving back and forth in place, has amplitude along with period and frequency. Wave motion means that the back-and-forth change is also moving through space. This means that there are two further qualities of a wave.

- The wavelength is the distance between two compressions in the direction of motion of the wave, and is represented by the Greek letter lambda, λ . For an ocean wave, it would be the distance in meters from the top of one wave to the next.
- The **wave speed** is how quickly the peak of each wave is moving forward, and is represented by v (for velocity, although for our current purpose the direction is not important).

These two are related by the period T of the wave. The period is the time it takes for a wave to complete one cycle, which is the time it takes for a peak to move forward one wavelength. The **wave equation** expresses this. Distance is speed times time, $x = vt$, so wavelength is wave speed times period $\lambda = vT$.

This can alternately be expressed in terms of frequency. Suppose there are three waves every second. This is frequency $f = 3.0 \text{ Hz}$, equivalent to period $T = \frac{1}{3} \text{ s}$. This means that during one second, three waves come out from the source. Since each peak is one wavelength, λ , ahead of the other, this means that during that one second, the lead wave has gone ahead three wavelengths. The distance the wave goes in one second is the wave speed. So, the wave speed is equal to the frequency times the wavelength, $v = \lambda f$.

Because $f = \frac{1}{T}$, these are mathematically the same:

$\lambda = vT \rightarrow v = \frac{\lambda}{T} = \lambda \frac{1}{T} = \lambda f$

The wave equation says distance wavelength is equal to wave speed multiplied by period T .

http://demonstrations.wolfram.com/SpeedOfSound/

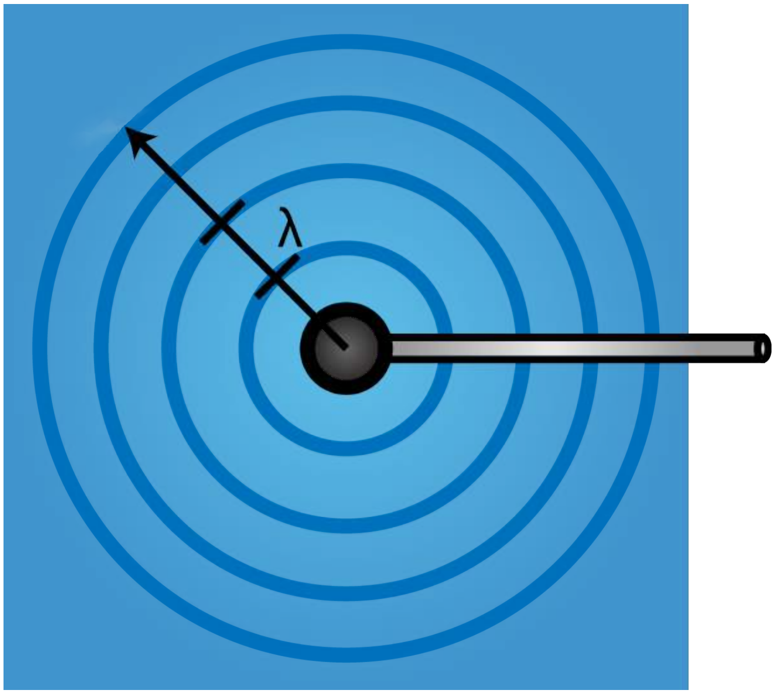


FIGURE 9.1

Illustrative Example 1

a. The ripple tank arm in **Figure** 9.1 has a period of 0.25 s and the length of the arrow in the figure is 3.76 cm. What is the wavelength of the water waves if the picture is to scale?

Answer: There are four wavelengths from the start to the end of the arrow. Wavelength is: $\lambda = \frac{3.76\text{cm}}{4} = 0.94\text{cm}$

b. What is the velocity of the wave?

Answer:

Since the period is $T = 0.25$ s , the frequency is , $f = \frac{1}{T} = \frac{1}{0.25\text{ s}} = 4.0$ Hz or four cycles per second. *So* the velocity is $v = \lambda f = (0.94\text{ cm})(4.0\text{ Hz}) = 3.76\ \frac{\text{cm}}{\text{s}}$.

Check Your Understanding

1. A sound wave travels at the speed of 343.0 m/s through the air at room temperature, $20.0°C(68.0°F)$. If the frequency of the sound is 261.6 Hz (a middle-C note), what is the wavelength of the note?

Answer: $v = \lambda f \rightarrow \lambda = \frac{v}{f} = \frac{343.0\text{ m/s}}{261.6\text{ Hz}} = 1.311$ m

2. X-rays are electromagnetic waves. A particular type of x-ray has a frequency of 3.0×10^{17} Hz and a wavelength of 1.0×10^{-9} m . What is the velocity of this type of x-ray?

Answer: $v = \lambda f = \left(1.0 \times 10^{-9}\text{ m}\right)\left(3.0 \times 10^{17}\ \frac{1}{\text{s}}\right) = 3.0 \times 10^{8}$ m/s

This result is true for any electromagnetic radiation traveling in a vacuum, including visible light.

3. Compared to the speed of sound in air at a temperature of $20°C$, how many times faster is the speed of light through the air at the same temperature?

Answer: We'll assume that the velocity of light $v_L = 3.0 \times 10^{8}\frac{m}{s}$ is approximately the same through the air as it is through a vacuum. Number 1 above gave the velocity of sound for a temperature of $20°C$ as $v_s = 343.0\frac{m}{s}$.

Answer: $\frac{V_L}{V_s} = 874,635.57 \rightarrow 8.7 \times 10^5$. The velocity of light is indeed a good deal greater than the velocity of sound in air. In fact, the velocity of light in vacuum is the greatest velocity that exists. No material object can travel at this velocity! We will discuss these ideas in Chapter 23 (The Special Theory of Relativity).

4. At the moment a lightning flash is seen, a person begins to count off seconds. If he hears the thunder after seven seconds, approximately how far away from the person did the lightning flash originate?

Answer : We'll assume that the sound travels at a velocity of 343 m/s. In seven seconds the sound has traveled $x = vt \rightarrow x = \left(343\frac{m}{s}\right)(7s) = 2401 \rightarrow 2400\ m$.

5. Sound travels about 1,500 m/s in water. A destroyer ship locates an enemy submarine using a sonar signal which takes a 4.3 s to travel to, reflect, and return to the destroyer. How far is the sub from the destroyer?

Answer : The time for the signal to reach the sub is half of the total time of travel, $\rightarrow \frac{4.3\ s}{2} = 2.15 \rightarrow 2.2\ s$. Using $x = vt \rightarrow x = \left(1,500\frac{m}{s}\right)(2.15\ s) = 3225 \rightarrow 3200\ m$ or 3.2 km.

References

1. CK-12 Foundation - Raymond Chou. . CC-BY-NC-SA 3.0

CONCEPT 10 Sound Waves

- Define sound.
- Describe sound waves and how they are generated.
- Identify media through which sound waves can travel.

Crack! Crash! Thud! That's what you'd hear if you were in the forest when this old tree cracked and came crashing down to the ground. But what if there was nobody there to hear the tree fall? Would it still make these sounds? This is an old riddle. To answer the riddle correctly, you need to know the scientific definition of sound.

Defining Sound

In science, **sound** is defined as the transfer of energy from a vibrating object in waves that travel through matter. Most people commonly use the term sound to mean what they hear when sound waves enter their ears. The tree above generated sound waves when it fell to the ground, so it made sound according to the scientific definition. But the sound wasn't detected by a person's ears if there was nobody in the forest. So the answer to the riddle is both yes and no!

How Sound Waves Begin

All sound waves begin with vibrating matter. Look at the first guitar string on the left in the **Figure** 10.1 . Plucking the string makes it vibrate. The diagram below the figure shows the wave generated by the vibrating string. The moving string repeatedly pushes against the air particles next to it, which causes the air particles to vibrate. The vibrations spread through the air in all directions away from the guitar string as longitudinal waves. In longitudinal waves, particles of the medium vibrate back and forth parallel to the direction that the waves travel. You can see an animation of sound waves traveling through air at this URL: http://www.mediacollege.com/audio/01/sound-waves.html

Q: If there were no air particles to carry the vibrations away from the guitar string, how would sound reach the ear?

A: It wouldn't unless the vibrations were carried by another medium. Sound waves are mechanical waves, so they can travel only though matter and not through empty space.

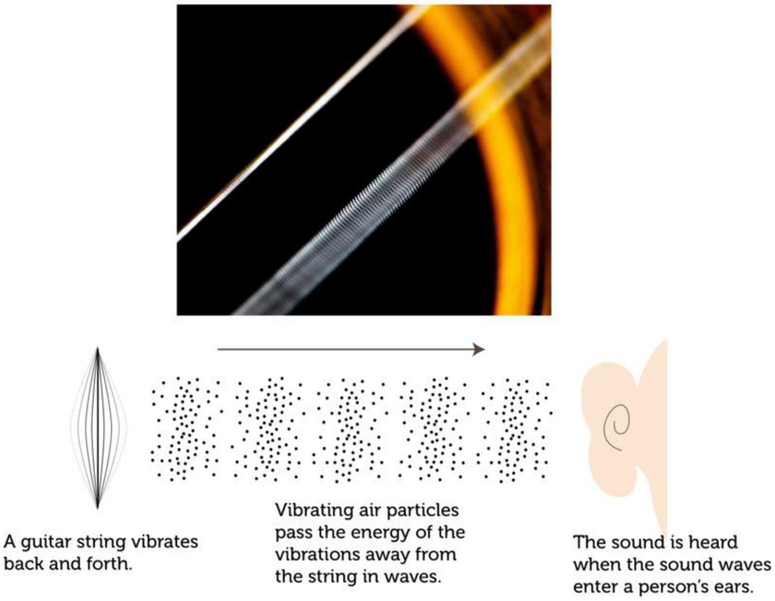


FIGURE 10.1

A Ticking Clock

The fact that sound cannot travel through empty space was first demonstrated in the 1600s by a scientist named Robert Boyle. Boyle placed a ticking clock in a sealed glass jar. The clock could be heard ticking through the air and glass of the jar. Then Boyle pumped the air out of the jar. The clock was still ticking, but the ticking sound could no longer be heard. That's because the sound couldn't travel away from the clock without air particles to pass the sound energy along. You can see an online demonstration of the same experiment—with a modern twist—at this URL: http://www.youtube.com/watch?v=b0JQt4u6-XI

MEDIA

Click image to the left for more content.

Sound Waves and Matter

Most of the sounds we hear reach our ears through the air, but sounds can also travel through liquids and solids. If you swim underwater—or even submerge your ears in bathwater—any sounds you hear have traveled to your ears through the water. Some solids, including glass and metals, are very good at transmitting sounds. Foam rubber and heavy fabrics, on the other hand, tend to muffle sounds. They absorb rather than pass on the sound energy.

Q: How can you tell that sounds travel through solids?

A: One way is that you can hear loud outdoor sounds such as sirens through closed windows and doors. You can also hear sounds through the inside walls of a house. For example, if you put your ear against a wall, you may be able to eavesdrop on a conversation in the next room—not that you would, of course.

Summary

- In science, sound is defined as the transfer of energy from a vibrating object in waves that travel through matter.
- All sound waves begin with vibrating matter. The vibrations generate longitudinal waves that travel through matter in all directions.
- Most sounds we hear travel through air, but sounds can also travel through liquids and solids.

Vocabulary

- **sound** : Transfer of energy from a vibrating object in longitudinal waves that travel through matter.

Practice

Watch the video "How Sound Waves Travel" at the following URL. Then explain how sound waves begin and how they travel, using the human voice as an example. http://www.youtube.com/watch?v=_vYYqRVi8vY

MEDIA
Click image to the left for more content.

Review

1. How is sound defined in science? How does this definition differ from the common meaning of the word?
2. Hitting a drum, as shown in the **Figure** 10.2 , generates sound waves. Create a diagram to show how the sound waves begin and how they reach a person's ears.

FIGURE 10.2

3. How do you think earplugs work?

References

1. Guitar string photo by Flickr:jar(); illustration by Christopher Auyeung (CK-12 Foundation). . CC BY 2.0
2. S.L. Ratigan. . CC BY 2.0

CONCEPT 11 Frequency and Pitch of Sound

- Define the pitch of sound.
- Relate the pitch of sound to the frequency of sound waves.
- Identify infrasound and ultrasound.

A marching band passes you as it parades down the street. You heard it coming from several blocks away. Now that the different instruments have finally reached you, their distinctive sounds can be heard. The tiny piccolos trill their bird-like high notes, and the big tubas rumble out their booming bass notes. Clearly, some sounds are higher or lower than others.

High or Low

How high or low a sound seems to a listener is its **pitch** . Pitch, in turn, depends on the frequency of sound waves. **Wave frequency** is the number of waves that pass a fixed point in a given amount of time. High-pitched sounds, like the sounds of the piccolo in the **Figure** 11.1 , have high-frequency waves. Low-pitched sounds, like the sounds of the tuba **Figure** 11.1 , have low-frequency waves. For a video demonstration of frequency and pitch, go to this URL: http://www.youtube.com/watch?v=irqfGYD2UKw

Can You Hear It?

The frequency of sound waves is measured in hertz (Hz), or the number of waves that pass a fixed point in a second. Human beings can normally hear sounds with a frequency between about 20 Hz and 20,000 Hz. Sounds with frequencies below 20 hertz are called **infrasound** . Infrasound is too low-pitched for humans to hear. Sounds with frequencies above 20,000 hertz are called **ultrasound** . Ultrasound is too high-pitched for humans to hear.

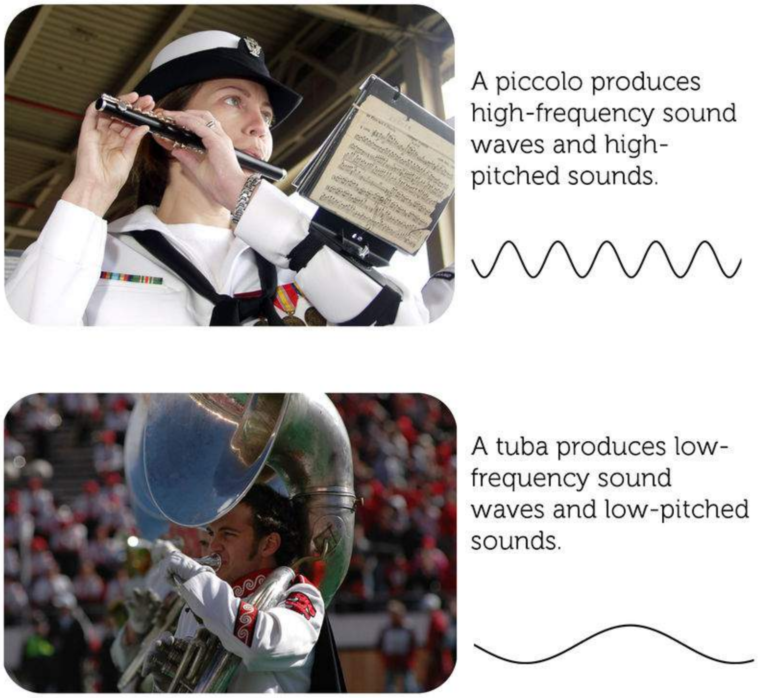


FIGURE 11.1

Some other animals can hear sounds in the ultrasound range. For example, dogs can hear sounds with frequencies as high as 50,000 Hz. You may have seen special whistles that dogs—but not people—can hear. The whistles produce sounds with frequencies too high for the human ear to detect. Other animals can hear even higher-frequency sounds. Bats, like the one pictured in the **Figure** 11.2 , can hear sounds with frequencies higher than 100,000 Hz!

FIGURE 11.2

Q: Bats use ultrasound to navigate in the dark. Can you explain how?

A: Bats send out ultrasound waves, which reflect back from objects ahead of them. They sense the reflected sound waves and use the information to detect objects they can't see in the dark. This is how they avoid flying into walls and trees and also how they find flying insects to eat.

Summary

- How high or low a sound seems to a listener is its pitch. Pitch, in turn, depends on the frequency of sound waves.
- High-frequency sound waves produce high-pitched sounds, and low-frequency sound waves produce low-pitched sounds.

- Infrasound has wave frequencies too low for humans to hear. Ultrasound has wave frequencies too high for humans to hear.

Vocabulary

- **infrasound** : Sound with a frequency below the range of human hearing (less than 20 hertz).
- **pitch** : How high or low a sound seems to a listener.
- **ultrasound** : Sound with a frequency above the range of human hearing (greater than 20,000 hertz).
- **wave frequency** : Number of waves that pass a fixed point in a given amount of time.

Practice

At the following URL, complete the interactive module to review and test your knowledge of the frequency and pitch of sound. http://www.engineeringinteract.org/resources/oceanodyssey/flash/concepts/pitch.htm

Review

1. What is the pitch of sound?
2. How is the pitch of sound related to the frequency of sound waves?
3. Define infrasound and ultrasound.

References

1. Piccolo: U.S. Navy photo by Chief Mass Communication Specialist David Rush; Tuba: Bob Fishbeck. . Public Domain
2. . . Public Domain

CONCEPT 12 Speed of Sound

- Give the speed of sound in dry air at 20 °C.
- Describe variation in the speed of sound in different media.
- Explain the effect of temperature on the speed of sound.

Has this ever happened to you? You see a flash of lightning on the horizon, but several seconds pass before you hear the rumble of thunder. The reason? The speed of light is much faster than the speed of sound.

What Is the Speed of Sound?

The **speed of sound** is the distance that sound waves travel in a given amount of time. You'll often see the speed of sound given as 343 meters per second. But that's just the speed of sound under a certain set of conditions, specifically, through dry air at 20 °C. The speed of sound may be very different through other matter or at other temperatures.

Speed of Sound in Different Media

Sound waves are mechanical waves, and mechanical waves can only travel through matter. The matter through which the waves travel is called the medium (plural, media). The **Table** 12.1 gives the speed of sound in several different media. Generally, sound waves travel most quickly through solids, followed by liquids, and then by gases. Particles of matter are closest together in solids and farthest apart in gases. When particles are closer together, they can more quickly pass the energy of vibrations to nearby particles. You can explore the speed of sound in different media at this URL:

http://www.ltscotland.org.uk/resources/s/sound/speedofsound.asp?strReferringChannel=resources&strReferringPageID=tcm:4-248291-64

TABLE 12.1: speed of sound

Medium (20 °C)	Speed of Sound Waves (m/s)
Dry Air	343
Water	1437
Wood	3850
Glass	4540
Aluminum	6320

Q: The table gives the speed of sound in dry air. Do you think that sound travels more or less quickly through air that contains water vapor? (Hint: Compare the speed of sound in water and air in the table.)

A: Sound travels at a higher speed through water than air, so it travels more quickly through air that contains water vapor than it does through dry air.

Temperature and Speed of Sound

The speed of sound also depends on the temperature of the medium. For a given medium, sound has a slower speed at lower temperatures. You can compare the speed of sound in dry air at different temperatures in the following **Table** 12.2 . At a lower temperature, particles of the medium are moving more slowly, so it takes them longer to transfer the energy of the sound waves.

TABLE 12.2: speed of sound

Temperature of Air	Speed of Sound Waves (m/s)
0 °C	331
20 °C	343
100 °C	386

Q: What do you think the speed of sound might be in dry air at a temperature of -20 °C?

A: For each 1 degree Celsius that temperature decreases, the speed of sound decreases by 0.6 m/s. So sound travels through dry, -20 °C air at a speed of 319 m/s.

Summary

- The speed of sound is the distance that sound waves travel in a given amount of time. The speed of sound in dry air at 20 °C is 343 meters per second.
- Generally, sound waves travel most quickly through solids, followed by liquids, and then by gases.
- For a given medium, sound waves travel more slowly at lower temperatures.

Vocabulary

- **speed of sound** : Speed at which sound waves travel, which is 343 m/s in dry air at 20 °C.

Practice

At the following URL, read about the speed of sound in different materials. Be sure to play the animation. Then answer the questions below. http://www.ndt-ed.org/EducationResources/HighSchool/Sound/speedinmaterials.htm

1. Describe what you hear when you play the animation. Explain your observations.
2. Name two properties of materials that affect the speed of sound waves. How do they affect the speed of sound?
3. Explain why sound waves moves more quickly through warmer air than cooler air.

Review

1. What is the speed of sound in dry air at 20 °C?
2. Describe variation in the speed of sound through various media.
3. Explain how temperature affects the speed of sound.

CONCEPT 13 Resonance with Sound Waves

Objectives

The student will:

- Understand the conditions for resonance.
- Solve problems with strings and pipes using the condition for resonance.

Vocabulary

- **beat frequency**
- **natural frequency:** The frequency at which a system vibrates normally when given energy without outside interference.
- **resonance:** Timing force to be the same as natural frequency.
- **sympathetic vibrations**

Introduction

Many systems have a tendency to vibrate. When the forced vibration frequency is the same as the natural frequency, the amplitude of vibration can increase tremendously. A well-known example of this situation is pushing a person on a swing, **Figure** below . We know from study of simple pendulums that without being pushed, the person in the swing rocks back and forth with a frequency that depends on gravity and the length of the chain.

$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

This is one example of a **natural frequency** – the frequency at which a system vibrates normally when given energy without outside interference.

Pushing on the person in the swing will affect the amplitude of the swinging. This is called forced vibration – when a periodic force from one object (the person pushing) affects the vibration of another object (the person swinging). To get the most effect, the person pushing will start just at the very back of the swing. In other words, the frequency of how often they push is exactly the same as the frequency of the swing. Suppose they do not push at the right time, but instead push at some other frequency. That would mean that sometimes they are pushing forward when the swing is still going backward. In that case, the swing would slow down – i.e. the amplitude of the swing will be reduced.

Timing the pushes to be the same as the natural frequency is called **resonance** . For this reason, the natural frequency is also known as the resonant frequency. If the pushes are timed just right, then even if each individual push is small, the vibration will get larger with each push.

FIGURE 13.1

A classic example of an unfortunate consequence of a forced vibration at resonant frequency is what happened to the Tacoma Narrows Bridge, in 1940. See the link below.

http://www.youtube.com/watch?v=3mclp9QmCGs

In **Figure** below , the bridge is beginning to resonate, in part, due to the frequency of vibration of the wind gusts. In **Figure** below , we see that the bridge is no longer able to respond elastically to the tremendous amplitude of vibration from the forced vibration of wind energy (at its resonant frequency), and it is torn apart.

FIGURE 13.2

Modern bridges are built to avoid this effect, but through history there are a number of documented situations where a forced vibration at resonance had dire results. The Broughton Suspension Bridge (1831) and the Angers Bridge (1850) are two examples of bridges believed to have collapsed due to the effect of soldiers marching at a regular pace that caused resonance. The Albert Bridge in West London, England has been nicknamed The Trembling Lady because it has been set into resonance so often by marching soldiers. Though soldiers no longer march across the bridge, there still remains a sign of concern as shown in **Figure** below .

FIGURE 13.3

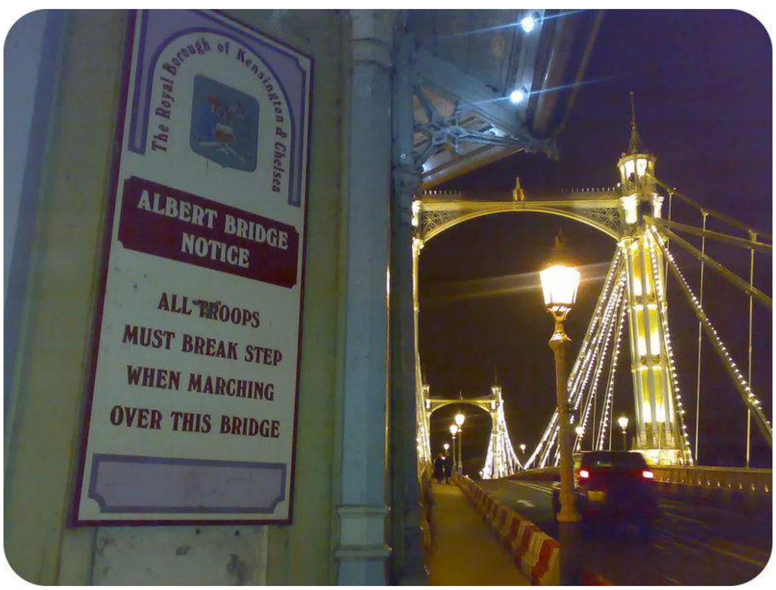


FIGURE 13.4

Sympathetic Vibrations

There is a typical classroom physics demonstration where one tuning fork is set into motion and an identical tuning fork, if placed closed enough, will also be set vibrating, though with smaller amplitude. The same effect occurs when tuning a guitar. One string is plucked and another, whose length is shortened by holding it down some distance from the neck of the guitar, will also be set into vibration. When this condition is met, both strings are vibrating with the same frequency. We call this phenomenon **sympathetic vibration** .

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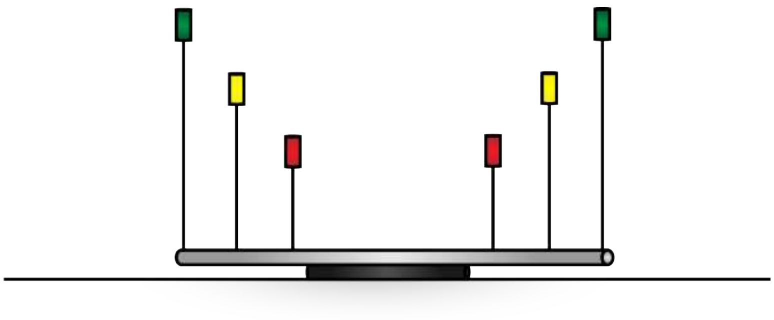

FIGURE 13.5

https://www.youtube.com/watch?v=tnS0SYF4pYE

In **Figure** above , a set of pendulums are fixed to a horizontal bar that can be easily jostled. Pendulums A and E have the same length. If one of them is set swinging, the horizontal bar will be forced into moving with a period equal to that of the pendulum, which, in turn, will cause the other pendulum of the same length to begin swinging. Any pendulum that is close in length to pendulums A and E , for example, pendulum D , will also begin to swing. Pendulum D will swing with smaller amplitude than pendulums A and E since its resonant frequency is not quite the same as pendulums A and E . Pendulums with lengths dramatically different from pendulums A and E will hardly move at all. You can try a similar demonstration out yourself with the following simulation:

http://phet.colorado.edu/en/simulation/resonance

Resonance is a very common phenomenon, especially with sound. The length of any instrument is related to what note it plays. If you blow into the top of a bottle, for example, the note will vary depending on the height of air in the bottle. This plays an important role in human voice generation. The length of the human vocal tube is between 17 cm and 18 cm. The typical frequencies of human speech are in the range of 100 Hz to 5000 Hz.

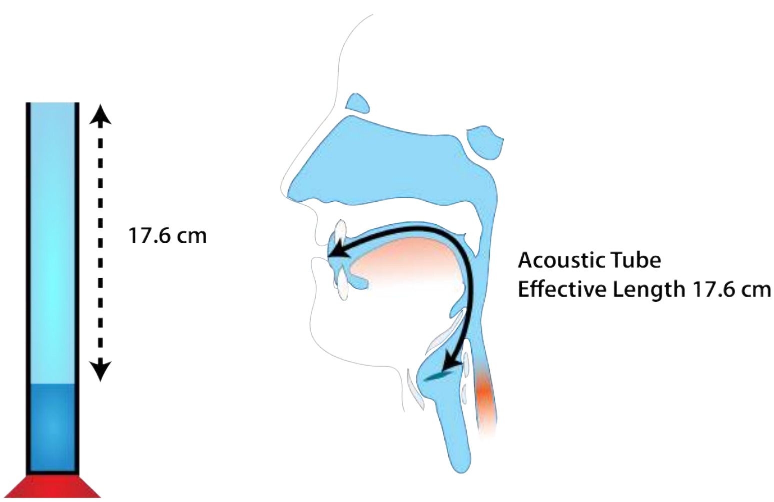

FIGURE 13.6

By using the muscles in their throat, singers change the note they sing. A dramatic example of this is breaking glass with the human voice. By singing at exactly the resonant frequency of a delicate wine glass, the glass will resonate with the note and shatter.

http://www.youtube.com/watch?v=7YmuOD5X4L8

The resonance of sound is also a mechanical analogue to how a radio set receives a signal. The **Figure** below shows one of the earliest radio designs, called a crystal radio because the element which detected the radio waves was a crystalline mineral such as galena.

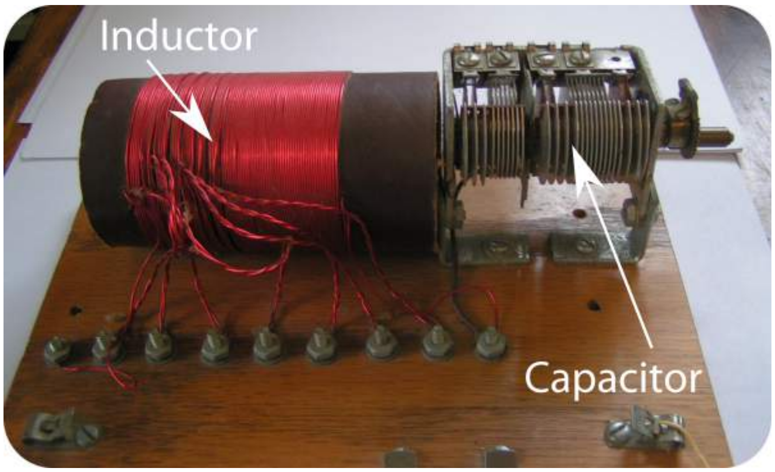

FIGURE 13.7
An old crystal radio set.

In modern times, the air is filled with all manner of radio waves. In order to listen to your favorite radio station, you must tune your radio to resonate with only the frequency of the radio station. When you hear the tuning number of a radio station, such as "101.3 FM", that is the measure in *megahertz,* $101.3 \text{ MHz} = 1.013 \times 10^8 \text{ Hz}$. The coiled wire (called an inductor) and the capacitor in **Figure** above act together to tune in a specific radio station . Effectively, the capacitor and inductor act analogously to a pendulum of a specific length that will only respond to vibrations of another pendulum of the same length. So when tuned, only a specific radio frequency will cause resonance in the radio antenna.

Strings Fixed at Both Ends

A case of natural frequency that you can observe plainly is when you pluck a string or stretched rubber band. Normally, the string will vibrate at a single widest point in the middle. This is called the fundamental or first harmonic resonance of the string. This is the same as the natural frequency of a simple pendulum or mass on a spring. Because it vibrates all along its length, though, the string also lets us see further patterns of resonance.

By vibrating the end of the string rather than just plucking it, we can force vibration at frequencies other than the first harmonic. When the string is set into vibration, energy will travel down the string and reflect back toward the end where the waves are being generated. This steady pattern of vibration is called a standing wave . The points where the reflecting waves interfere destructively with the "generated' waves are called nodes. The points where the reflecting waves interfere constructively with the generated waves are called anti-nodes.

Figure below shows a string fixed at both ends vibrating in its fundamental mode. There are two nodes shown and one antinode. The dashed segment represents the reflected wave.

If you compare the wave shape of the first harmonic to the wave shape of **Figure** above , it will be apparent that the first harmonic contains one-half of a wavelength, λ . Therefore L , the length of the unstretched string, is equal to one-half the wavelength, which is $\frac{1}{2}\lambda_1 = L \rightarrow \lambda_1 = 2L$.

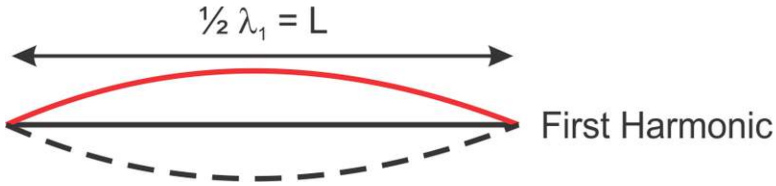

FIGURE 13.8

The second harmonic contains an entire wavelength $\frac{2}{2}\lambda_2 = L \rightarrow \lambda_2 = L$ as shown in **Figure** below .

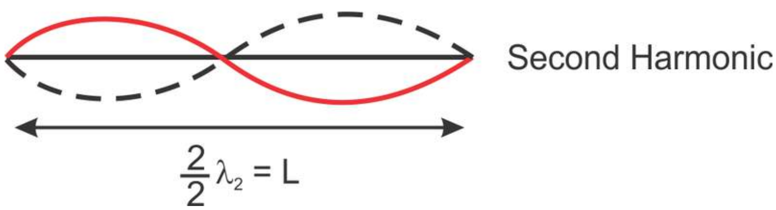

FIGURE 13.9

And the third harmonic contains one and one-half wavelengths $\frac{3}{2}\lambda_3 = L \rightarrow \lambda_3 = \frac{2}{3}L$.

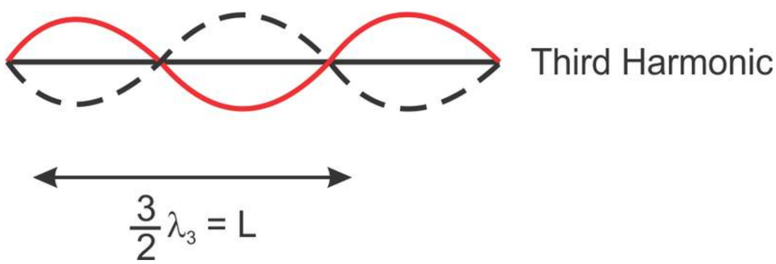

FIGURE 13.10

If the pattern continues then the fourth harmonic will have a wavelength of $\frac{4}{2}\lambda_4 = L \rightarrow \lambda_4 = \frac{1}{2}L$. Looking at the expressions for the length of the string in terms of the wavelength, a simple pattern emerges: $L = \frac{1}{2}\lambda_1, \frac{2}{2}\lambda_2, \frac{3}{2}\lambda_3, \frac{4}{2}\lambda_4 \ldots$. We can express the condition for standing waves (and of resonance, as well) as $L = \frac{n}{2}\lambda_n$ or $\lambda_n = \frac{2}{n}L$, where L is the length of string and $n = 1, 2, 3 \ldots$.

Check Your Understanding

1. How many nodes and anti-nodes are shown in **Figure** above .?

Answer: There are three nodes and two anti-nodes.

2. If the length of the unstretched string is 20 cm, what is the wavelength for the 10th harmonic?

Answer: $\lambda_n = \frac{2}{n}L \rightarrow \lambda_{10} = \frac{2}{10}L = \frac{1}{5}(20 \text{ cm}) = 4 \text{ cm}$

Strings Fixed at One End and Opened at One End

A string fixed at one only end displays a different standing wave pattern. In this case, the unbounded end of a string of length L is an antinode. The fundamental mode (the first harmonic) for the length L of string contains only one-fourth of a wavelength as shown in **Figure** below below. Therefore L, the length of the unstretched string, is equal to one-quarter the wavelength, which is $\frac{1}{4}\lambda_1 = L \rightarrow \lambda_1 = 4L$.

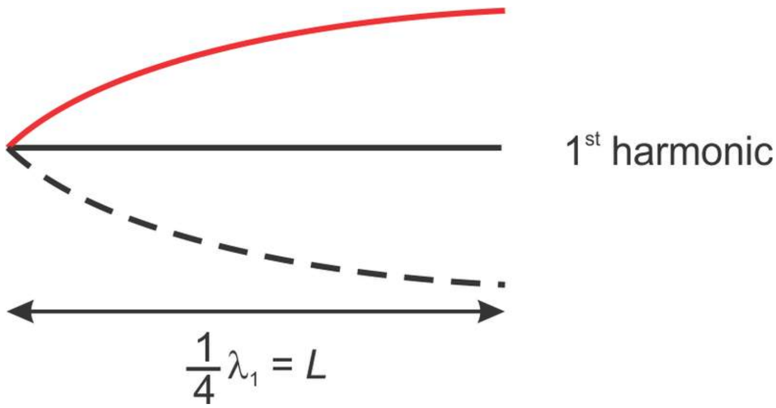

FIGURE 13.11

The second harmonic contains three-quarters of a wavelength $\frac{3}{4}\lambda_2 = L \rightarrow \lambda_2 = \frac{4}{3}L$ as shown in **Figure** above

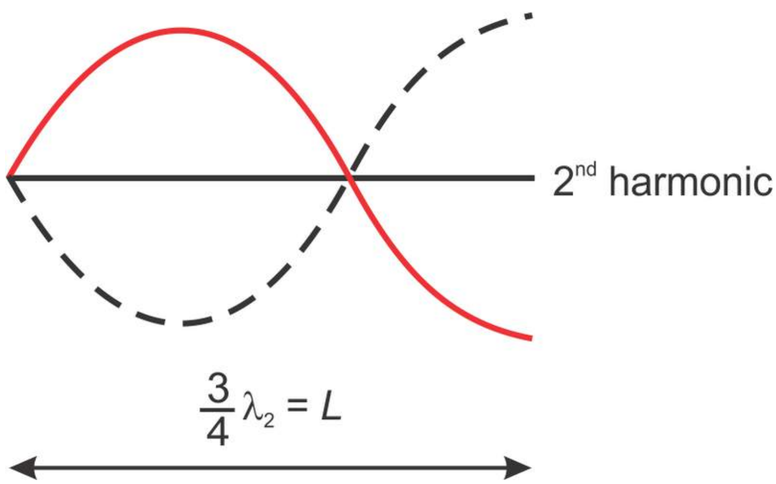

FIGURE 13.12

The third harmonic contains five-fourths of a wavelength $\frac{5}{4}\lambda_2 = L \rightarrow \lambda_2 = \frac{4}{5}L$ as shown in **Figure** above

The third harmonic contains five-fourths wavelengths as shown in **Figure** below .

If the pattern continues, then the fourth harmonic will have a wavelength of $\frac{7}{4}\lambda_4 = L \rightarrow \lambda_4 = \frac{4}{7}L$. Looking at the expressions for the length of the string in terms of the wavelength, a simple pattern emerges $\frac{1}{4}\lambda_1, \frac{3}{4}\lambda_2, \frac{5}{4}\lambda_3, \frac{7}{4}\lambda_4 \ldots$. We can express the condition for resonance as $L = \frac{n}{4}\lambda_n$ or $\lambda_n = \frac{4}{n}L$, where L is the length of string and $n = 1, 3, 5 \ldots$.

As long as the tension in the string remains fixed, the velocity of the wave along the string remains constant. Does it seem reasonable that a sagging string will not support the same wave velocity as a taut string? Since $v = \lambda f$ product λf is constant as long as the wave velocity remains constant. Therefore, for a string vibrating in many different modes, we have $v = \lambda_1 f_1 = \lambda_2 f_2 = \lambda_3 f_3 \ldots$.

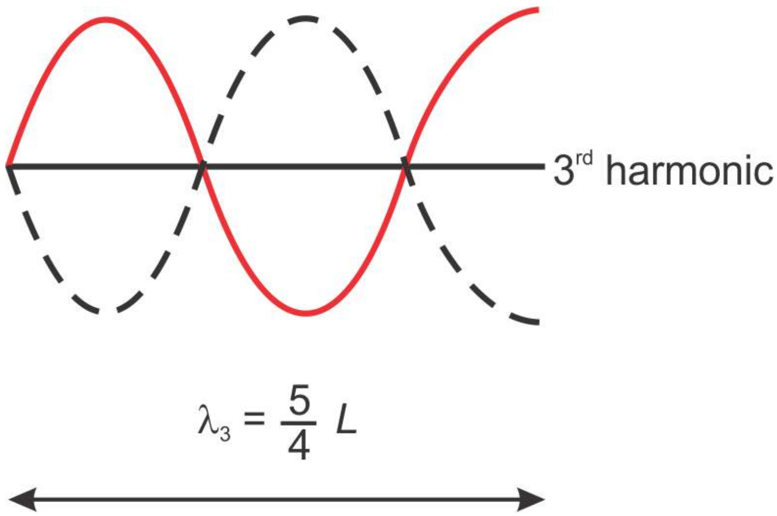

FIGURE 13.13

Illustrative Example 1

1a. If the frequency of the first harmonic for a string fixed at both ends is f_1 , determine the frequency for successive harmonics in terms of f_1 .

Answer: We know that $\lambda_n = \frac{2}{n}L$ and $v = \lambda f$. Combining, we have $v = \left(\frac{2}{n}L\right) f \rightarrow f_n = \frac{n}{2L}v$. But v can be expressed $v = \lambda_1 f_1 = 2Lf_1$ and substituted into $f_n = \frac{n}{2L}v$, giving

$f_n = \frac{n}{2L}(2Lf_1) = nf_1 \rightarrow f_n = nf_1$

1b. If the first harmonic has frequency of 261 Hz, what frequencies do the second and third harmonics have?

Answer:

Since

$$f_n = nf_1 \rightarrow 2(261 \text{ Hz}) = 522 \text{ Hz}$$
$$f_n = nf_1 \rightarrow 3(261 \text{ Hz}) = 783 \text{ Hz}$$

All whole number multiples of the first harmonic (the fundamental) are called harmonics. String instruments, as well as non-string instruments, can actually vibrate with many different frequencies simultaneously (called modes). For example, a string may vibrate with frequencies 261 Hz, 522 Hz and 783 Hz simultaneously.

One of attributes of the "quality" or "timbre" of musical instruments depends upon the combination of the various overtones produced by the instrument.

Check Your Understanding

1. A tuning fork has a frequency of 512 Hz stamped on it. When it is struck, a student claims she can hear higher frequencies from the tuning fork. Is this possible?

Answer : Yes, it is. The tuning fork may be producing harmonics, in which case the student may be hearing frequencies in multiples of 512 Hz, such as 1,024 Hz and 1,536 Hz.

2. A string with a fundamental frequency of 220 Hz vibrates in its third harmonic with a wavelength of 60 cm. What is the wave velocity on the string?

Answer : $v = \lambda f$ but $f = 3f_1 = 3(220 \text{ Hz}) = 660 \text{ Hz}$, so

$$\lambda = 0.60 \text{ m}$$
$$v = (660 \text{ Hz})(0.60 \text{ m}) = 396 \text{ m/s}$$

.

Open and Closed Pipes and Tubes

In our discussions of pipes, the length of the pipe will be assumed to be much greater than the diameter of the pipe.

An open pipe, as the name implies, has both ends open. Though open pipes have antinodes at their ends, the resonant conditions for standing waves in an open pipe are the same as for a string fixed at both ends. Thus for an open pipe we have: for $n = 1, 2, 3..., L = \frac{n}{2}\lambda_n$, or $\lambda_n = \frac{2}{n}L$.

There is a simple experiment your instructor may have you do in class that demonstrates resonance in an open tube. Roll two sheets of long paper into two separate tubes and use a small amount of tape to keep them rolled. Have the diameter of one tube just small enough to fit inside the other tube so the inside tube can freely slide back and forth. Hold a struck tuning fork (your instructor will make sure the frequency is adequate) close to the end of the outer tube while the inside tube is moved slowly. When the total length of the tubes is the proper length to establish resonance, you'll hear a noticeable increase in the volume of the sound. At this moment, there are standing waves present in the tubes.

A closed pipe is closed at only one end. Closed pipes have the same standing wave patterns as a string fixed at one end and unbound at the other end. They therefore have the same resonant conditions as a string fixed at only one end, for $n = 1, 3, 5 \ldots, L = \frac{n}{4}\lambda_n$ or $\lambda_n = \frac{4}{n}L$.

A closed pipe supporting the first harmonic (the fundamental frequency) will fit one-fourth of the wavelength, the second harmonic will fit three-fourths, and so on, as shown in **Figure** below . Compare these pictures to those in **Figure** above for a string fixed at only one end

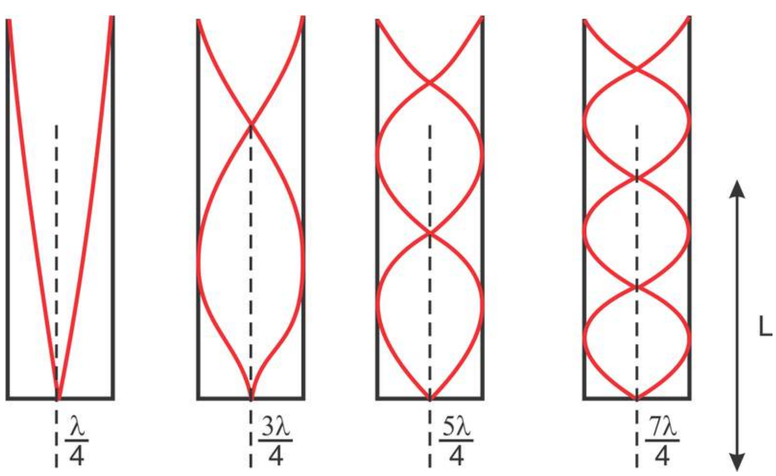

FIGURE 13.14

A standard physics laboratory experiment is to find the velocity of sound by using a tuning fork that vibrates over a closed pipe as shown in Figure **Figure** above . The water level in a pipe is slowly changed until the first harmonic is heard.

http://demonstrations.wolfram.com/ResonanceInOpenAndClosedPipes/

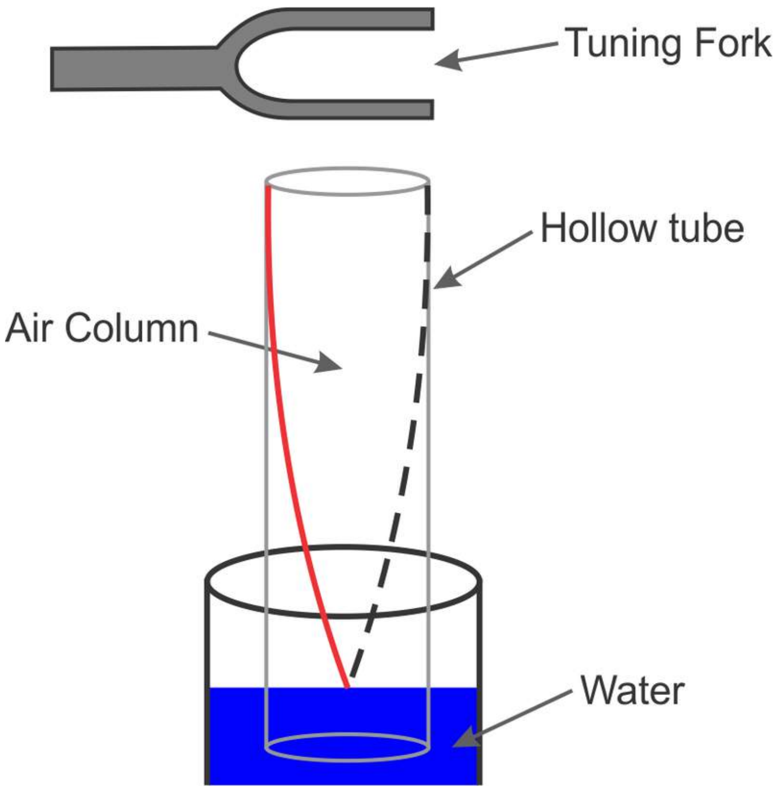

FIGURE 13.15

Illustrative Example 2

Resonance is established in a hollow tube similar to that shown in **Figure** above with a tuning fork of 512 Hz. The distance from the tube opening to the water level is 16.8 cm.

a. What is the velocity of sound according to this experiment?

Answer: The wave velocity equation is $v = \lambda f$. One-fourth of the wave occupies the length of the tube for the first harmonic. So the wavelength of the resonant wave must be four times the length of the hollow tube. That is,

$$\lambda_1 = 4L = 4 \times 16.8 \text{ cm} = 67.2 \text{ cm} = 0.672 \text{ m}$$
$$v = (0.672 \text{ m})(512 \text{ Hz}) = 344 \text{ m/s}$$

b. The velocity of sound changes with temperature as given by the formula $v = 330 + 0.6T$, where T is the temperature in degrees centigrade. Using the result of part A, determine the temperature at the location the experiment was conducted.

Answer: We simply set the result of part A equal to the given equation:

$344 = 330 + 0.6T \rightarrow T = 23.3°C$ (or about $74°F$).

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Concept 14 Sound in a Tube

Students will learn how to analyze and solve problems where standing waves (and hence sound) is produced in a tube.

Students will learn how to analyze and solve problems where standing waves (and hence sound) is produced in a tube.

Key Equations

$v = \lambda f$

for a tube closed at one end

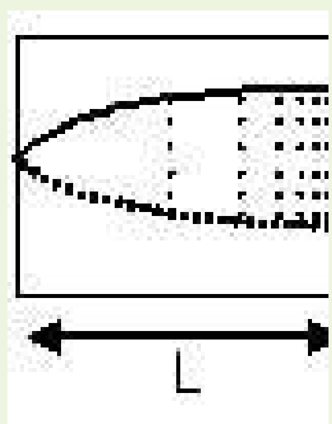
$f = nv/4L$, where n is always odd

for a tube open at both ends

$f = nv/2L$, where n is an integer

Guidance

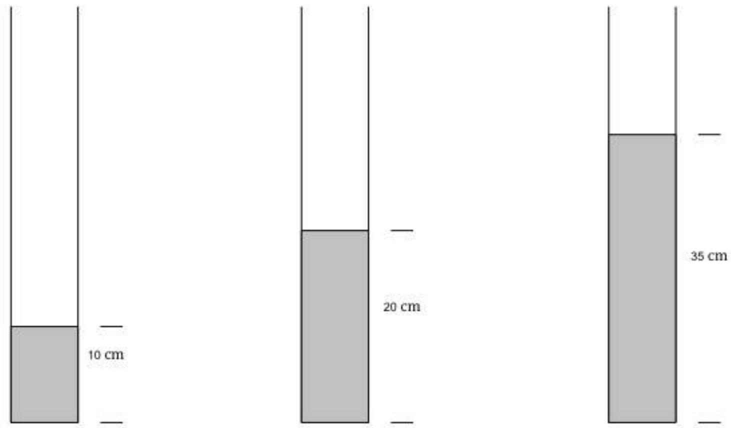
In the case of a tube that is open at one end, a node is forced at the closed end (no air molecules can vibrate up and down) and an antinode occurs at the open end (here, air molecules are free to move). A different spectrum of standing waves is produced. For instance, the fundamental standing sound wave produced in a tube closed at one end is shown below. In this case, the amplitude of the standing wave is referring to the magnitude of the air pressure variations.



This standing wave is the first harmonic and one can see that the wavelength is $\lambda = 4L$. Since $v = \lambda f$, the frequency of oscillation is $f = v/4L$. In general, the frequency of oscillation is $f = nv/4L$, where n is always odd.

Example 1

Question The objects A, B, and C below represent graduated cylinders of length 50 cm which are filled with water to the depths of 10, 20 and 30 cm, respectively as shown.



a) If you blow in each of these tubes, which (A,B,C) will produce the highest frequency sound?

b) What is the wavelength of the 1st harmonic (i.e. fundamental) of tube B?

c) The speed of sound at room temperature is about 343 m/s. What is the frequency of the 1st harmonic for tube B?

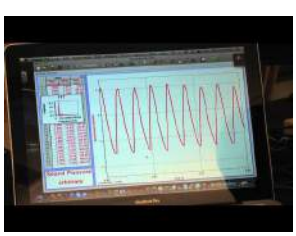
Solution

a) The water forms the bottom of the tube and thus where the node of the wave will be. Thus the air column is where the sound wave can exist. The larger the air column, the larger the wavelength. Frequency is inversely proportional to wavelength, thus the tube with the smallest air column will have the highest frequency. So the answer is tube C.

b) The air column is 50 cm - 20 cm = 30 cm. The first harmonic has a quarter wavelength in the tube. Thus $\lambda = 4 \times L$. Thus, $\lambda = 4 \times 30\text{cm} = 120\text{cm}$

c) Using the wave equation for the first harmonic (thus, n = 1) of a tube open at one end we get $f = \frac{v}{4L} = \frac{343\text{m/s}}{1.2\text{m}} =$ 286Hz

Watch this Explanation



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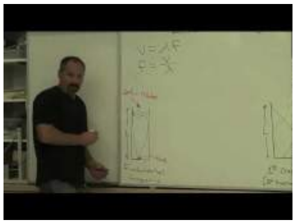

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Time for Practice

1. 
2. Aborigines, the native people of Australia, play an instrument called the Didgeridoo like the one shown above. The Didgeridoo produces a low pitch sound and is possibly the world's oldest instrument. The one shown above is about 1.3 m long and open at both ends.
   a. Knowing that when a tube is open at both ends there must be an antinode at both ends, draw the first 3 harmonics for this instrument.
   b. Calculate the frequency of the first 3 harmonics assuming room temperature and thus a velocity of sound of 340 m/s. Then take a shot at deriving a generic formula for the frequency of the n th standing wave mode for the Didgeridoo, as was done for the string tied at both ends and for the tube open at one end.
3. Students are doing an experiment to determine the speed of sound in air. They hold a tuning fork above a large empty graduated cylinder and try to create resonance. The air column in the graduated cylinder can be adjusted by putting water in it. At a certain point for each tuning fork a clear resonance point is heard. The students adjust the water finely to get the peak resonance then carefully measure the air column from water to top of air column. (The assumption is that the tuning fork itself creates an anti-node and the water creates a node.) The following data were collected:

TABLE 14.1:

Frequency of tuning fork (Hz)	**Length of air column (cm)**	**Wavelength (m)**	**Speed of sound (m/s)**
184	46		
328	26		
384	22		
512	16		
1024	24		

(a) Fill out the last two columns in the data table.

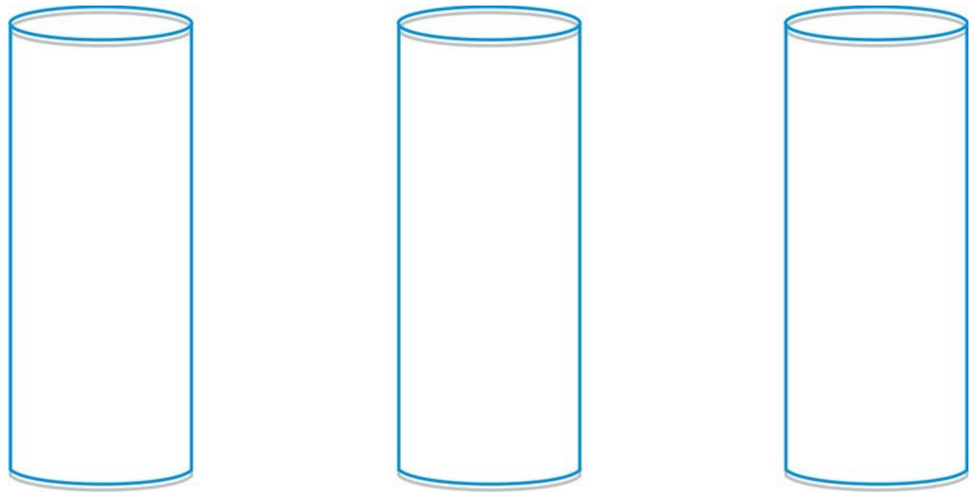
(b) Explain major inconsistencies in the data or results.

(c) The graduated cylinder is 50 cm high. Were there other resonance points that could have been heard? If so what would be the length of the wavelength?

(d) What are the inherent errors in this experiment?

3. Peter is playing tones by blowing across the top of a glass bottle partially filled with water. He notices that if he blows softly he hears a lower note, but if he blows harder he hears higher frequencies. (a) In the 120 cm long tubes below draw three diagrams showing the first three harmonics produced in the tube. Please draw the waves as transverse even though we know sound waves are longitudinal (reason for this, obviously, is that it

is much easier to draw transverse waves rather than longitudinal). Note that the tube is CLOSED at one end and OPEN at the other.



(b) Calculate the frequencies of the first three harmonics played in this tube, if the speed of sound in the tube is 340 m/s. (c) The speed of sound in carbon dioxide is lower than in air. If the bottle contained $CO2$ instead of air, would the frequencies found above be higher or lower? Knowing that the pitch of your voice gets higher when you inhale helium, what can we say about the speed of sound in He .

Answers:

1. (b) 131 Hz, 262 Hz, 393 Hz; formula is same as closed at both ends
2. Discuss in class
3. (b) 70.8 Hz, 213 Hz, 354 Hz (c) voice gets lower pitch. Speed of sound in He must be faster by same logic.

CK-12 FlexBook

Physics Unit 11: Electromagnetic Waves

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CONCEPT 1 Electromagnetic Waves

- Define electromagnetic wave and electromagnetic radiation.
- Describe the electric and magnetic fields of an electromagnetic wave.
- Explain how electromagnetic waves begin and how they travel.
- State how electromagnetic waves may interact with matter.
- Identify sources of electromagnetic waves.

Did you ever wonder how a microwave works? It directs invisible waves of radiation toward the food placed inside of it. The radiation transfers energy to the food, causing it to get warmer. The radiation is in the form of microwaves, which are a type of electromagnetic waves.

What Are Electromagnetic Waves?

Electromagnetic waves are waves that consist of vibrating electric and magnetic fields. Like other waves, electromagnetic waves transfer energy from one place to another. The transfer of energy by electromagnetic waves is called **electromagnetic radiation** . Electromagnetic waves can transfer energy through matter or across empty space. For an excellent video introduction to electromagnetic waves, go to this URL: http://www.youtube.com/watch?v=cfXzwh3KadE

MEDIA
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Q: How do microwaves transfer energy inside a microwave oven?

A: They transfer energy through the air inside the oven to the food.

May the Force Be with You

A familiar example may help you understand the vibrating electric and magnetic fields that make up electromagnetic waves. Consider a bar magnet, like the one in the **Figure** 1.1 . The magnet exerts magnetic force over an area all around it. This area is called a magnetic field. The field lines in the diagram represent the direction and location of the magnetic force. Because of the field surrounding a magnet, it can exert force on objects without touching them. They just have to be within its magnetic field.

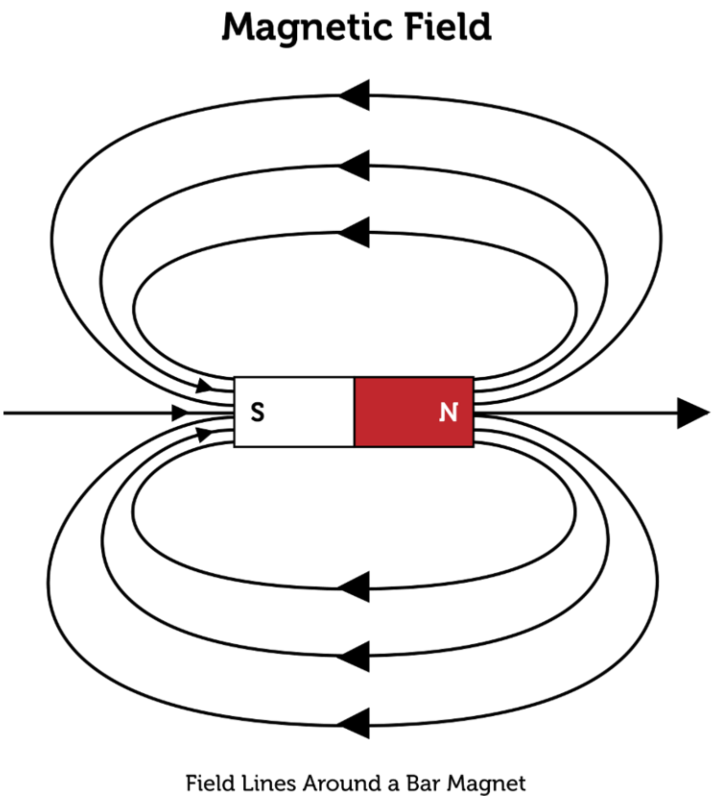


FIGURE 1.1

Q: How could you demonstrate that a magnet can exert force on objects without touching them?

A: You could put small objects containing iron, such as paper clips, near a magnet and show that they move toward the magnet.

An electric field is similar to a magnetic field. It is an area of electrical force surrounding a positively or negatively charged particle. You can see electric fields in the following **Figure** 1.2 . Like a magnetic field, an electric field can exert force on objects over a distance without actually touching them.

How an Electromagnetic Wave Begins

An electromagnetic wave begins when an electrically charged particle vibrates. The **Figure** 1.3 shows how this happens. A vibrating charged particle causes the electric field surrounding it to vibrate as well. A vibrating electric field, in turn, creates a vibrating magnetic field. The two types of vibrating fields combine to create an electromagnetic wave. You can see animations of electromagnetic waves at these URLs: http://www.youtube.com/watch?v=Qju7QnbrOhM&feature=related http://www.phys.hawaii.edu/~teb/java/ntnujava/emWave/emWave.html

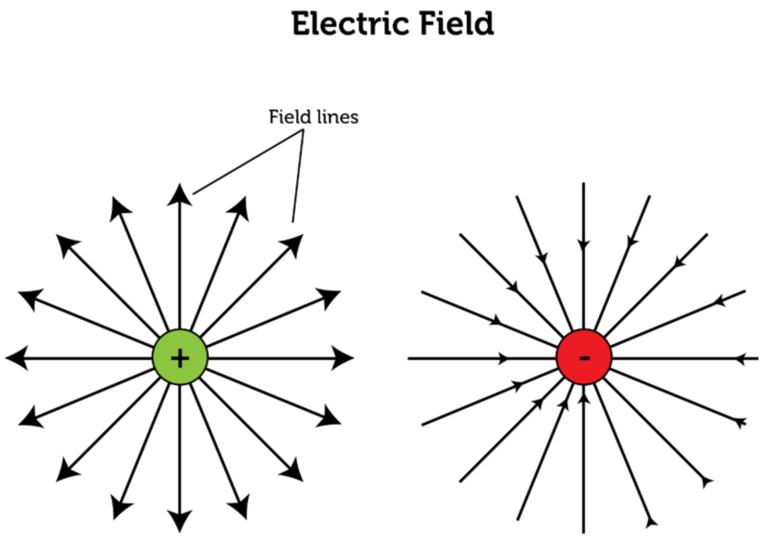


FIGURE 1.2

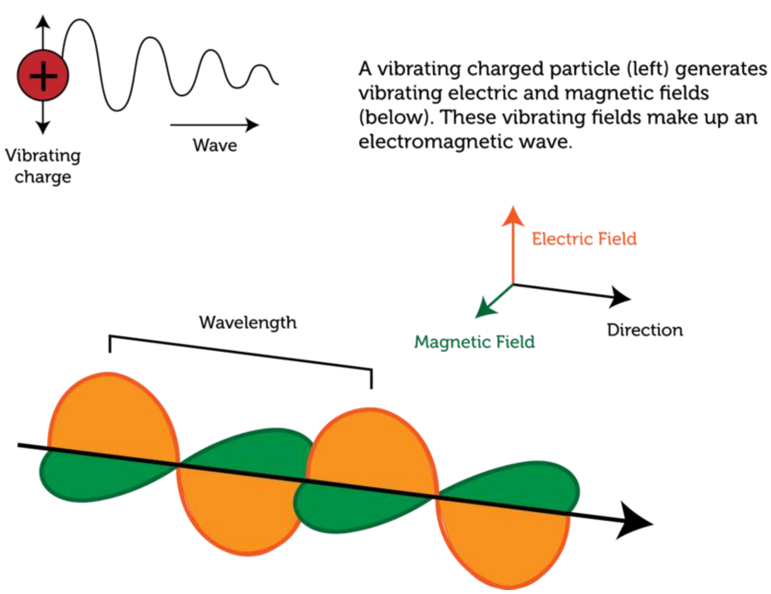


FIGURE 1.3

How an Electromagnetic Wave Travels

As you can see in the diagram above, the electric and magnetic fields that make up an electromagnetic wave are perpendicular (at right angles) to each other. Both fields are also perpendicular to the direction that the wave travels. Therefore, an electromagnetic wave is a transverse wave. However, unlike a mechanical transverse wave, which can only travel through matter, an electromagnetic transverse wave can travel through empty space. When waves travel through matter, they lose some energy to the matter as they pass through it. But when waves travel through space, no energy is lost. Therefore, electromagnetic waves don't get weaker as they travel. However, the energy is "diluted" as it travels farther from its source because it spreads out over an ever-larger area.

Electromagnetic Wave Interactions

When electromagnetic waves strike matter, they may interact with it in the same ways that mechanical waves interact with matter. Electromagnetic waves may:

- reflect, or bounce back from a surface;
- refract, or bend when entering a new medium;
- diffract, or spread out around obstacles.

Electromagnetic waves may also be absorbed by matter and converted to other forms of energy. Microwaves are a familiar example. When microwaves strike food in a microwave oven, they are absorbed and converted to thermal energy, which heats the food.

Sources of Electromagnetic Waves

The most important source of electromagnetic waves on Earth is the sun. Electromagnetic waves travel from the sun to Earth across space and provide virtually all the energy that supports life on our planet. Many other sources of electromagnetic waves depend on technology. Radio waves, microwaves, and X rays are examples. We use these electromagnetic waves for communications, cooking, medicine, and many other purposes.

Summary

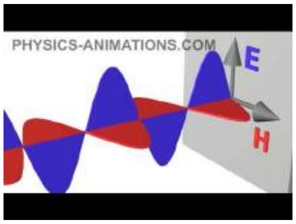
- Electromagnetic waves are waves that consist of vibrating electric and magnetic fields. They transfer energy through matter or across space. The transfer of energy by electromagnetic waves is called electromagnetic radiation.
- The electric and magnetic fields of an electromagnetic wave are areas of electric or magnetic force. The fields can exert force over objects at a distance.
- An electromagnetic wave begins when an electrically charged particle vibrates. This causes a vibrating electric field, which in turn creates a vibrating magnetic field. The two vibrating fields together form an electromagnetic wave.
- An electromagnetic wave is a transverse wave that can travel across space as well as through matter. When it travels through space, it doesn't lose energy to a medium as a mechanical wave does.
- When electromagnetic waves strike matter, they may be reflected, refracted, or diffracted. Or they may be absorbed by matter and converted to other forms of energy.
- The most important source of electromagnetic waves on Earth is the sun. Many other sources of electromagnetic waves depend on technology.

Vocabulary

- **electromagnetic radiation** : Transfer of energy by electromagnetic waves across space or through matter.
- **electromagnetic wave** : Transverse wave consisting of vibrating electric and magnetic fields that can travel across space.

Practice

Watch the electromagnetic wave animation at the following URL, and then answer the questions below. http://www.youtube.com/watch?v=4CtnUETLIFs



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1. Identify the vibrating electric and magnetic fields of the wave.
2. Describe the direction in which the wave is traveling.

Review

1. What is an electromagnetic wave?
2. Define electromagnetic radiation.
3. Describe the electric and magnetic fields of an electromagnetic wave.
4. How does an electromagnetic wave begin? How does it travel?
5. Compare and contrast electromagnetic and mechanical transverse waves.
6. List three sources of electromagnetic waves on Earth.

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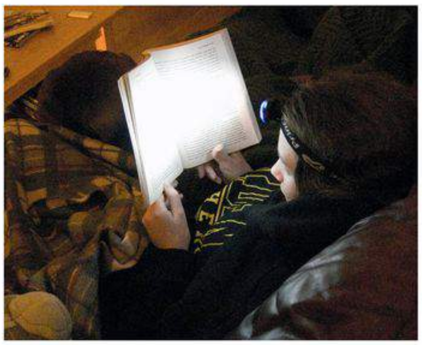
CONCEPT 2 Properties of Electromagnetic Waves

- State the speed of light.
- Describe wavelengths and frequencies of electromagnetic waves.
- Relate wave frequency to wave energy.
- Show how to calculate wavelength or wave frequency if the other value is known.



What do these two photos have in common? They both represent electromagnetic waves. These are waves that consist of vibrating electric and magnetic fields. They transmit energy through matter or across space. Some electromagnetic waves are generally harmless. The light we use to see is a good example. Other electromagnetic waves can be very harmful and care must be taken to avoid too much exposure to them. X rays are a familiar example. Why do electromagnetic waves vary in these ways? It depends on their properties. Like other waves, electromagnetic waves have properties of speed, wavelength, and frequency.

Speed of Electromagnetic Waves

All electromagnetic waves travel at the same speed through empty space. That speed, called the **speed of light** , is about 300 million meters per second (3.0×10^8 m/s). Nothing else in the universe is known to travel this fast. The sun is about 150 million kilometers (93 million miles) from Earth, but it takes electromagnetic radiation only 8 minutes to reach Earth from the sun. If you could move that fast, you would be able to travel around Earth 7.5 times in just 1 second! You can learn more about the speed of light at this URL: http://videos.howstuffworks.com/discovery/29407-assignment-discovery-speed-of-light-video.htm

MEDIA

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Wavelength and Frequency of Electromagnetic Waves

Although all electromagnetic waves travel at the same speed across space, they may differ in their wavelengths, frequencies, and energy levels.

- Wavelength is the distance between corresponding points of adjacent waves (see the **Figure** 2.1). Wavelengths of electromagnetic waves range from longer than a soccer field to shorter than the diameter of an atom.
- Wave frequency is the number of waves that pass a fixed point in a given amount of time. Frequencies of electromagnetic waves range from thousands of waves per second to trillions of waves per second.
- The energy of electromagnetic waves depends on their frequency. Low-frequency waves have little energy and are normally harmless. High-frequency waves have a lot of energy and are potentially very harmful.

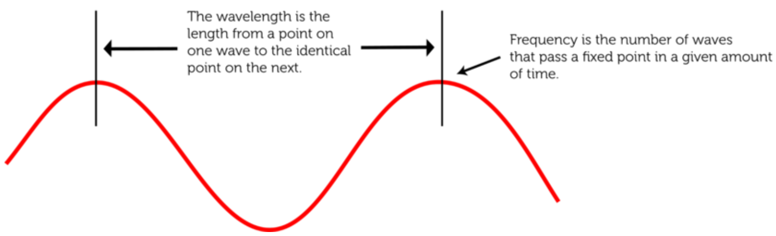


FIGURE 2.1

Q: Which electromagnetic waves do you think have higher frequencies: visible light or X rays?

A: X rays are harmful but visible light is harmless, so you can infer that X rays have higher frequencies than visible light.

Speed, Wavelength, and Frequency

The speed of a wave is a product of its wavelength and frequency. Because all electromagnetic waves travel at the same speed through space, a wave with a shorter wavelength must have a higher frequency, and vice versa. This relationship is represented by the equation:

$$\text{Speed} = \text{Wavelength} \times \text{Frequency}$$

The equation for wave speed can be rewritten as:

$$\text{Frequency} = \frac{\text{Speed}}{\text{Wavelength}} \text{ or Wavelength} = \frac{\text{Speed}}{\text{Frequency}}$$

Therefore, if either wavelength or frequency is known, the missing value can be calculated. Consider an electromagnetic wave that has a wavelength of 3 meters. Its speed, like the speed of all electromagnetic waves, is 3.0×10^8 meters per second. Its frequency can be found by substituting these values into the frequency equation:

$$\text{Frequency} = \frac{3.0 \times 10^8 \text{ m/s}}{3.0 \text{ m}} = 1.0 \times 10^8 \text{ waves/s , or } 1.0 \times 10^8 \text{ Hz}$$

Q: What is the wavelength of an electromagnetic wave that has a frequency of 3.0×10^8 hertz?

A: Use the wavelength equation:

$$\text{Wavelength} = \frac{3.0 \times 10^8 \text{ m/s}}{3.0 \times 10^8 \text{ waves/s}} = 1.0 \text{ m}$$

You can learn more about calculating the frequency and wavelength of electromagnetic waves at these URLs: http://www.youtube.com/watch?v=GwZvtfZRNKk and http://www.youtube.com/watch?v=wjPk108Ua8k

Summary

- All electromagnetic waves travel across space at the speed of light, which is about 300 million meters per second (3.0 x 10 ^{8} m/s).
- Electromagnetic waves vary in wavelength and frequency. Longer wavelength electromagnetic waves have lower frequencies, and shorter wavelength waves have higher frequencies. Higher frequency waves have more energy.
- The speed of a wave is a product of its wavelength and frequency. Because the speed of electromagnetic waves through space is constant, the wavelength or frequency of an electromagnetic wave can be calculated if the other value is known.

Vocabulary

- **speed of light** : Speed at which all electromagnetic waves travel through space, which is 3.0×10^{8} m/s.

Practice

Use the calculator at the following URL to find the frequency and energy of electromagnetic waves with different wavelengths. Use at least eight values for wavelength. Record and make a table of the results. http://www.1728.org/freqwave.htm

Review

1. What is the speed of light across space?
2. Describe the range of wavelengths and frequencies of electromagnetic waves.
3. How is the energy of an electromagnetic wave related to its frequency?
4. If the frequency of an electromagnetic wave is 6.0×10^{8} Hz, what is its wavelength?

References

1. Christopher Auyeung. . CC BY-NC 3.0

CONCEPT 3 Electromagnetic Spectrum

- Describe electromagnetic radiation and its properties.
- Give an overview of the electromagnetic spectrum.

It's a warm sunny Saturday, and Michael and Lavar have a big day planned. They're going to ride across town to meet their friends and then go to the zoo. The boys may not realize it, but they will be bombarded by electromagnetic radiation as they ride their bikes and walk around the zoo grounds. The only kinds of radiation they can detect are visible light, which allows them to see, and infrared light, which they feel as warmth on their skin.

Q: Besides visible light and infrared light, what other kinds of electromagnetic radiation will the boys be exposed to in sunlight?

A: Sunlight consists of all the different kinds of electromagnetic radiation, from harmless radio waves to deadly gamma rays. Fortunately, Earth's atmosphere prevents most of the harmful radiation from reaching Earth's surface. You can read about the different kinds of electromagnetic radiation in this article.

Electromagnetic Radiation

Electromagnetic radiation is energy that travels in waves across space as well as through matter. Most of the electromagnetic radiation on Earth comes from the sun. Like other waves, electromagnetic waves are characterized by certain wavelengths and wave frequencies. Wavelength is the distance between two corresponding points on adjacent waves. Wave frequency is the number of waves that pass a fixed point in a given amount of time. Electromagnetic waves with shorter wavelengths have higher frequencies and more energy.

A Spectrum of Electromagnetic Waves

Visible light and infrared light are just a small part of the full range of electromagnetic radiation, which is called the **electromagnetic spectrum** . You can see the waves of the electromagnetic spectrum in the **Figure** 3.1 . At the top of the diagram, the wavelengths of the waves are given. Also included are objects that are about the same size as the corresponding wavelengths. The frequencies and energy levels of the waves are shown at the bottom of the diagram. Some sources of the waves are also given. For a video introduction to the electromagnetic spectrum, go to this URL: http://www.youtube.com/watch?NR=1&feature=endscreen&v=cfXzwh3KadE

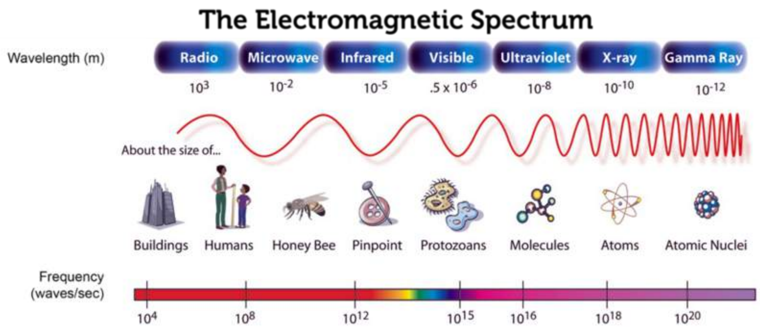


FIGURE 3.1

- On the left side of the electromagnetic spectrum diagram are radio waves and microwaves. Radio waves have the longest wavelengths and lowest frequencies of all electromagnetic waves. They also have the least amount of energy.
- On the right side of the diagram are X rays and gamma rays. They have the shortest wavelengths and highest frequencies of all electromagnetic waves. They also have the most energy.
- Between these two extremes are waves that are commonly called light. Light includes infrared light, visible light, and ultraviolet light. The wavelengths, frequencies, and energy levels of light fall in between those of radio waves on the left and X rays and gamma rays on the right.

Q: Which type of light has the longest wavelengths?

A: Infrared light has the longest wavelengths.

Q: What sources of infrared light are shown in the diagram?

A: The sources in the diagram are people and light bulbs, but all living things and most other objects give off infrared light.

Summary

- Electromagnetic radiation travels in waves through space or matter. Electromagnetic waves with shorter wavelengths have higher frequencies and more energy.
- The full range of electromagnetic radiation is called the electromagnetic spectrum. From longest to shortest wavelengths, it includes radio waves, microwaves, infrared light, visible light, ultraviolet light, X rays, and gamma rays.

Vocabulary

- **electromagnetic spectrum** : Full range of wavelengths of electromagnetic waves, from radio waves to gamma rays.

Practice

At the first URL below, read about electromagnetic waves with different frequencies. Then use the information to complete the table at the second URL. http://www.darvill.clara.net/emag/index.htm and http://www.darvill.clara.net/emag/gcseemag.pdf

Review

1. Describe the relationship between the wavelength and frequency of electromagnetic waves.
2. What is the electromagnetic spectrum?
3. Which electromagnetic waves have the longest wavelengths?
4. Identify a source of microwaves.
5. Which type of light has the highest frequencies?
6. Explain why gamma rays are the most dangerous of all electromagnetic waves.

References

1. NASA. . public domain

CONCEPT 4 Law of Reflection

- Define reflection and image.
- Compare and contrast regular and diffuse reflection.
- State the law of reflection.

These dancers are practicing in front of a mirror so they can see how they look as they performs. They're watching their image in the mirror as they dance. What is an image, and how does it get "inside" a mirror? In this article, you'll find out.

Reflected Light and Images

Reflection is one of several ways that light can interact with matter. Light reflects off surfaces such as mirrors that do not transmit or absorb light. When light is reflected from a smooth surface, it may form an image. An **image** is a copy of an object that is formed by reflected (or refracted) light.

Q: Is an image an actual object? If not, what is it?

A: No, an image isn't an actual object. It is focused rays of light that make a copy of an object, like a picture projected on a screen.

Regular and Diffuse Reflection

If a surface is extremely smooth, as it is in a mirror, then the image formed by reflection is sharp and clear. This is called regular reflection (also called specular reflection). However, if the surface is even slightly rough or bumpy, an image may not form, or if there is an image, it is blurry or fuzzy. This is called diffuse reflection.

Q: Look at the boats and their images in the **Figure** 4.1 . Which one represents regular reflection, and which one represents diffuse reflection?

FIGURE 4.1

A: Reflection of the boat on the left is regular reflection. The water is smooth and the image is sharp and clear. Reflection of the boat on the right is diffuse reflection. The water has ripples and the image is blurry and wavy.

In the **Figure** 4.2 , you can see how both types of reflection occur. Waves of light are represented by arrows called rays. Rays that strike the surface are referred to as incident rays, and rays that reflect off the surface are known as reflected rays. In regular reflection, all the rays are reflected in the same direction. This explains why regular reflection forms a clear image. In diffuse reflection, the rays are reflected in many different directions. This is why diffuse reflection forms, at best, a blurry image. You can see animations of both types of reflection at this URL: http://toolboxes.flexiblelearning.net.au/demosites/series5/508/Laboratory/StudyNotes/snReflectionMirrors.htm

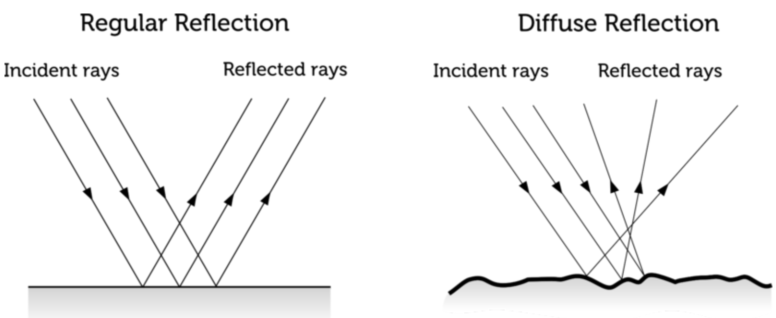


FIGURE 4.2

Law of Reflection

One thing is true of both regular and diffuse reflection. The angle at which the reflected rays leave the surface is equal to the angle at which the incident rays strike the surface. This is known as the **law of reflection** . The law is illustrated in the **Figure** 4.3 and also in this animation: http://www.physicsclassroom.com/mmedia/optics/lr.cfm

Summary

- Reflection is one of several ways that light can interact with matter. When light is reflected from a smooth surface, it may form an image. An image is a copy of an object that is formed by reflected (or refracted) light.
- Regular reflection occurs when light reflects off a very smooth surface and forms a clear image. Diffuse reflection occurs when light reflects off a rough surface and forms a blurry image or no image at all.
- According to the law of reflection, the angle at which light rays reflect off a surface is equal to the angle at which the incident rays strike the surface.

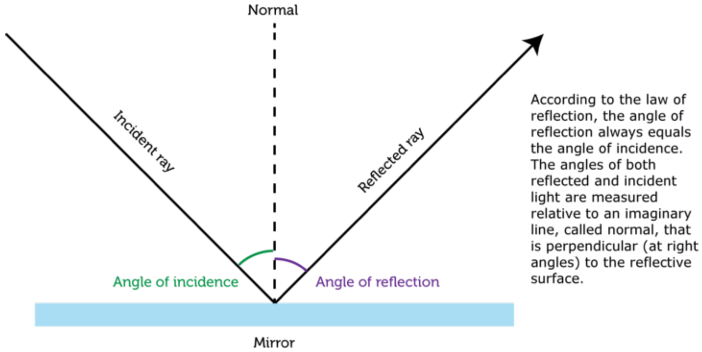


FIGURE 4.3

Vocabulary

- **image** : Copy of an object that is formed by reflected or refracted light.
- **law of reflection** : Law stating that the angle at which reflected rays of light bounce off a surface is equal to the angle at which the incident rays strike the surface.
- **reflection** : Bouncing back of waves from a barrier they cannot pass through.

Practice

At the following URL, review the law of reflection and watch the animation. Then fill in the blanks in the sentence below. http://www.physicsclassroom.com/mmedia/optics/lr.cfm

1. When a ray of light strikes a plane mirror, the light ray __________ off the mirror.
2. Reflection involves a change in __________ of a light ray.
3. The angle of incidence equals the angle between the incident ray and ________.
4. The angle of __________ equals the angle of incidence.
5. The normal line is __________ to the mirror.

Review

1. What is an image?
2. Identify the object and the image in the **Figure** 4.4 . Which type of reflection formed the image: regular reflection or diffuse reflection? How do you know?

FIGURE 4.4

3. What is the law of reflection?
4. Label the angle of incidence and the angle of reflection in the **Figure** 4.5 .

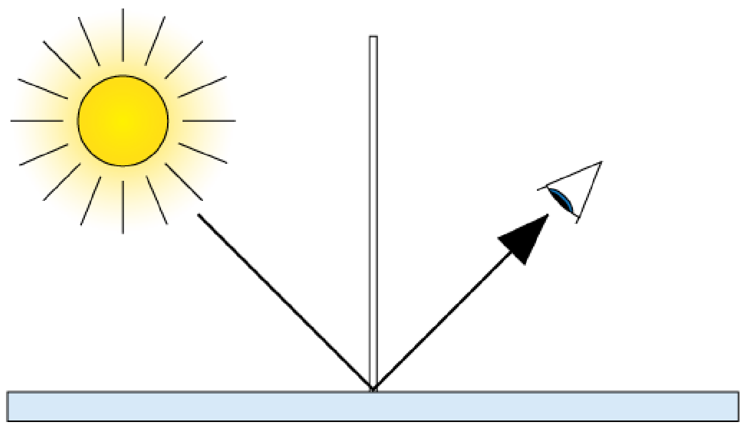


FIGURE 4.5

References

1. Left: Kenneth Baruch; Right: Damian Gadal. . CC BY 2.0
2. Joy Sheng. . CC BY-NC 3.0
3. Christopher Auyeung. . CC BY-NC 3.0
4. Mike Baird. . CC BY 2.0
5. Zachary Wilson. . CC-BY-NC-SA 3.0

CONCEPT 5 Refraction of Light

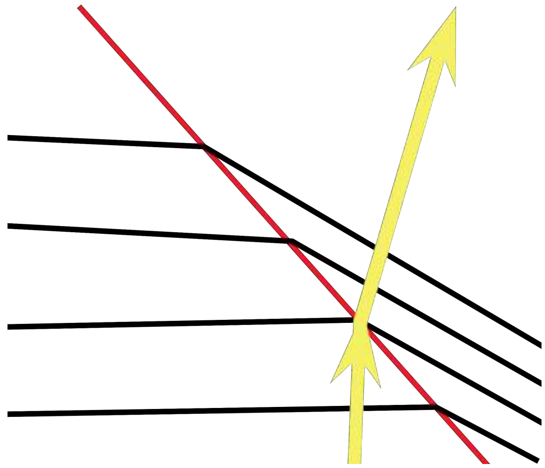
- Define refraction.
- Given data about the optical density of the media, predict whether the light will bend toward the normal or away from the normal.
- State Snell's Law and solve refraction problems using it.
- Solve problems using the relationship between the index of refraction and the velocity of light in the media.
- Explain effects caused by the refraction of light.

When a light ray passes at an angle through the boundary between optically different media, the light does not travel in a straight line. The pencil in the glass of liquid shown above is a normal straight pencil. The light that travels from the pencil through the liquid, through the glass, and into the air is bent differently than light from the portion of the pencil that is not in the liquid. Your eye assumes the light from both portions of the pencil moved in a straight line, but the two portions of the pencil do not appear to be lined up. Your eye thinks the pencil is broken.

Refraction of Light

The speed of light is different in different media. If the speed of light is slower in a particular medium, that medium is said to be more **optically dense**. When a wave front enters a new medium at an angle, it will change directions. If the light is entering a more optically dense medium, the light bends toward the normal line. If the light is entering a less optically dense medium, the light will bend away from the normal line. Remember that the normal line is the line perpendicular to the medium interface.

In the sketch below, light wave fronts are moving upward from the bottom of the page and encounter a boundary into a more optically dense medium. The light waves bend toward the normal line. Because the right end of the wave fronts enter the new medium first, they slow down first. When the right side of the wave front is moving more slowly that the left side, the wave front will change directions.



When light is traveling from air into another medium, **Snell's Law** states the relationship between the angle of incidence and angle of refraction is

$$n = \frac{\sin\theta_i}{\sin\theta_r}$$

where θ_i is the angle of incidence, θ_r is the angle of refraction, and n is the ratio of the two sines and is called the **index of refraction** . Snell's Law may be stated that a ray of light bends in such a way that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.

The index of refraction is also related to the relative speeds of light in a vacuum and in the medium.

$$n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in the medium}}$$

When a ray of light is traveling from medium into another medium, Snell's Law can be written as $n_i \sin\theta_i = n_r \sin\theta_r$.

Table of Indices of Refraction

TABLE 5.1:

Medium	n
Vacuum	1.00
Air	1.0003
Water	1.36
Ethanol	1.36
Crown Glass	1.52
Quartz	1.54
Flint Glass	1.61
Diamond	2.42

Example Problem: A ray of light traveling through air is incident upon a slab of Flint glass at an angle of 40.0° . What is the angle of refraction?

Solution: $n = \frac{\sin\theta_i}{\sin\theta_r}$ so $\sin\theta_r = \frac{\sin\theta_i}{n} = \frac{0.643}{1.61} = 0.399$

The angle of refraction $= sin^{-1}.399 = 23.5°$

Example Problem: What is the speed of light in a diamond?

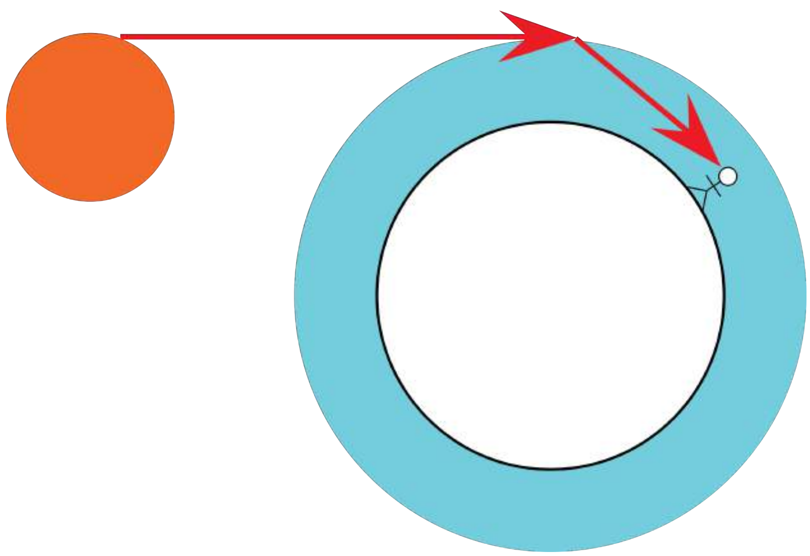
Solution: speed of light in diamond $= \frac{\text{speed of light in a vacuum}}{n} = \frac{3.00 \times 10^8\ m/s}{2.42}$

speed of light in diamond $= 1.24 \times 10^8\ m/s$

Effects of Refraction

Bending the Sun's Rays

Because air is slightly more optically dense than a vacuum, when sunlight passes from the vacuum of space into our atmosphere, it bends slightly towards the normal. When the sun is below the horizon and thus not visible on a direct line, the light path will bend slightly and thus make the sun visible by refraction. Observers can see the sun before it actually comes up over the horizon, or after it sets.



Mirages

In the Figure below, the sun shines on the road, heating the air just above the road. The difference in density between the hot air over the road and the surrounding air causes the hot air to refract light that passes through it. When you look at the road, you see a **mirage** . What appears to be water on the road is actually light coming from the sky that has been refracted as it passes through the hot air above the road. This phenomenon is common on hot roads and in the desert.

Summary

- The speed of light is different in different media.
- When a wave front enters a new medium at an angle, it will change directions. If the light is entering a more optically dense medium, the light bends toward the normal line. If the light is entering a less optically dense medium, the light will bend away from the normal line.

FIGURE 5.1

- When light is traveling from air into another medium, Snell's Law states that $n = \frac{\sin \theta_i}{\sin \theta_r}$.
- The index of refraction is also related to the relative speeds of light in a vacuum and in the medium. $n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in the medium}}$
- When a ray of light is traveling from one medium into another medium, Snell's Law can be written as $n_i \sin \theta_i = n_r \sin \theta_r$.

Practice

Use the video on refraction to answer the questions below.

http://video.mit.edu/watch/mit-physics-demo-refraction-a-total-internal-reflection-12044/

MEDIA

Click image to the left for more content.

1. What happens to the path of a light beam when it enters a new medium at an angle?
2. Light moving from air into water is bent ___________ the normal.
3. Light moving from water into air is bent ___________ the normal.

Review

1. Light moving through air is incident on a piece of crown glass at an angle of 45° . What is the angle of refraction?
2. A ray of light passes from air into water at an incident angle of 60.0° . Find the angle of refraction.
3. Light passes from water into a block of transparent plastic. The angle of incidence from the water is 31° and the angle of refraction in the block is 27° . What is the index of refraction for the plastic?
4. The index of refraction of water is 1.36. What is the speed of light in water?

5. If the speed of light in a piece of plastic is $2.00 \times 10^8 \ m/s$, what is the index of refraction for the plastic?

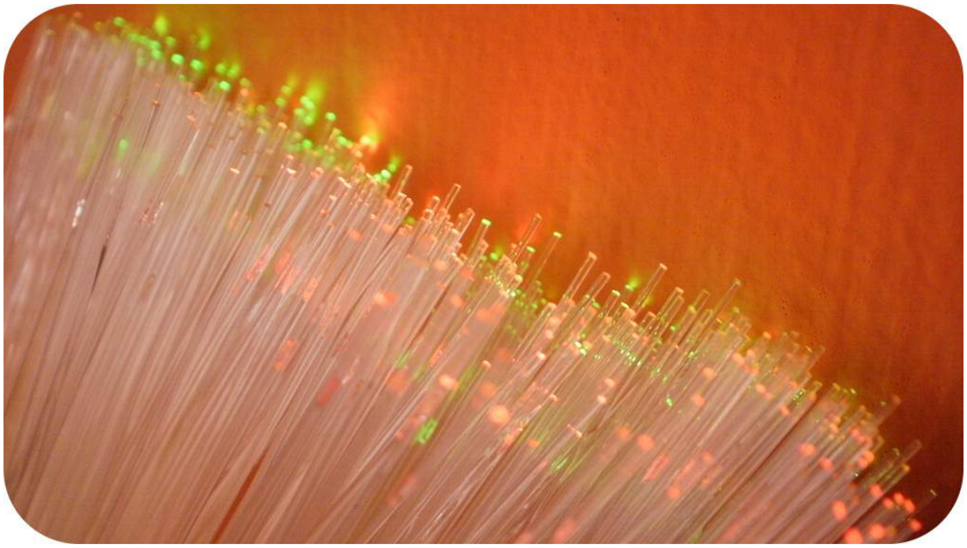
- **refraction :** The change of direction of a ray of light or sound in passing obliquely from one medium into another in which its wave velocity is different.
- **optically dense:** Refers to the ability of a material to slow the light waves. The greater the optical density of a material, the greater the slowing effect.
- **Snell's Law:** For a light ray incident on the interface of two media, the sine of the angle of incidence times the index of refraction of the first medium is equal to the sine of the angle of refraction times the index of refraction of the second medium.
- **index of refraction:** The ratio of the speed of light in a vacuum to the speed of light in a medium under consideration.
- **mirage:** An optical phenomenon that creates the illusion of water, often with inverted reflections of distant objects, and results from the refraction of light by alternate layers of hot and cool air.

References

1. Image copyright leonello calvetti, 2013. http://www.shutterstock.com . Used under license from Shutterstock.com
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
4. Michael Gil (Flickr: MSVG). http://www.flickr.com/photos/msvg/5994891327/ . CC-BY 2.0

CONCEPT 6 Total Internal Reflection

- Describe total internal reflection.
- Use the critical angle to determine when total internal reflection will occur.

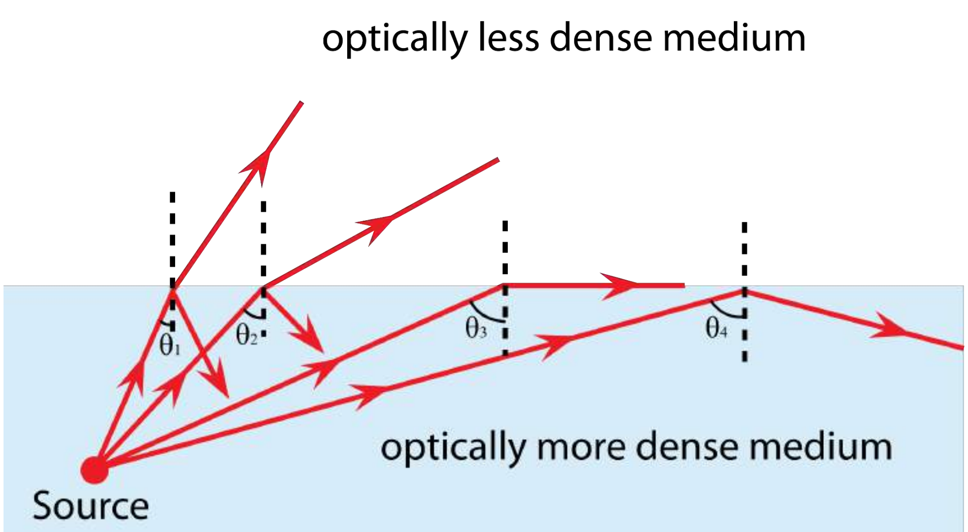


Total internal reflection allows the light to travel down the optical fiber and not pass through the sides of the tube. The light continuously reflects from the inside of the tube and eventually comes out the end. Optical fibers make interesting lamps but they are also used to transport telephone and television signals.

Total Internal Reflection

We already know that when light passes from one medium into a second medium where the index of refraction is smaller, the light refracts away from the normal.

In the image below, the light rays are passing into an optically less dense medium; therefore, the rays bend away from the normal. As the angle of incidence increases, the light ray bends even further away from the normal. Eventually, the angle of incidence will become large enough that the angle of refraction equals 90° , meaning the light ray will not enter the new medium at all.



Consider a ray of light passing from water into air. The index of refraction for air is 1.00 and for water is 1.36. Using Snell's Law, $n_i \sin\theta_i = n_r \sin\theta_r$, and allowing the angle of refraction to be 90° , we can solve for the angle of incidence which would cause the light ray to stay in the old medium.

$n_i \sin\theta_i = n_r \sin\theta_r$

$(1.36)(\sin\theta_i) = (1.00)(\sin 90^\circ)$

$\sin\theta_i = 0.735 \ \ and \ \ \theta_i = 47^\circ$

This result tells us that when light is passing from water into air, if the angle of incidence exceeds 47° , the light ray will not enter the new medium. The light ray will be completely reflected back into the original medium. This is called **total internal reflection** . The minimum angle of incidence for total internal reflection to occur is called the **critical angle** .

Total internal reflection is the principle behind **fiber optics** . A bundle of fibers made out of glass or plastic only a few micrometers in diameter is called a light pipe since light can be transmitted along it with almost no loss. Light passing down the fibers makes glancing collisions with the walls so that total internal reflection occurs.

Summary

- When light passes from one medium into a second medium with a smaller index of refraction, the light refracts away from the normal.
- If the angle of incidence becomes large enough that the angle of refraction equals 90° , the light ray will not enter the new medium with the smaller angle of refraction.
- Total internal reflection means the light ray will not enter the new medium but will be completely reflected back into the original medium.

Practice

MEDIA
Click image to the left for more content.

http://www.youtube.com/watch?v=hBQ8fh_Fp04

In this video, a hole is drilled in the side of a plastic bottle filled with liquid. An arc of liquid shoots out through the hole. A laser pointer is aimed through the opposite side of the bottle so that the light also exits through the hole. The stream of liquid acts like an optical fiber and the light undergoes total internal reflection as it follows the stream of liquid. As the amount of liquid in the bottle decreases, the arc of the stream of liquid changes and the direction of the light follows the stream of liquid. Toward the end, the light beam is shining almost 90° from the direction of the laser pointer.

The following video discusses total internal reflection. Use this resource to answer the questions that follow.

http://www.khanacademy.org/science/physics/waves-and-optics/v/total-internal-reflection#

MEDIA

Click image to the left for more content.

1. What phenomenon occurs when the light does not enter the new medium and remains in the old medium?
2. When does this phenomenon occur?

Review

1. Find the critical angle for light passing from diamond into air, given $n_{\text{diamond}} = 2.42$
2. When two swimmers are under water in a swimming pool, it is possible for the interface between the water and the air to act as a mirror, allowing the swimmers to see images of each other if they look up at the underside of the surface. Explain this phenomenon.
3. Robert shines a laser beam through a slab of plastic and onto the interface between the slab of plastic and the air on the other side. The index of refraction for the plastic is 1.62. If the angle of incidence in the plastic is 54° , will the laser beam pass out of the plastic into the air?

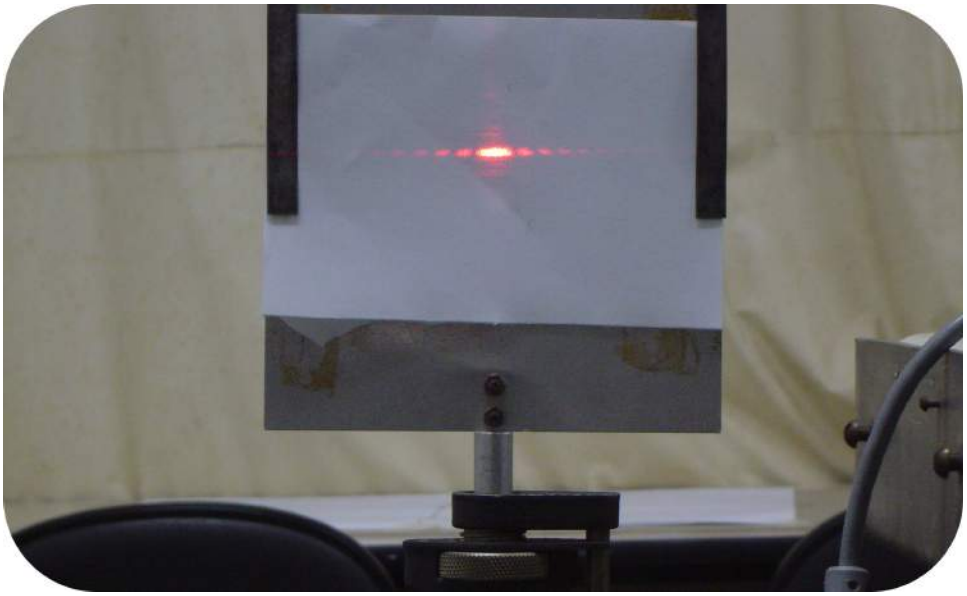
- **total internal reflection:** When light is passing from a medium of higher index of refraction into a medium of lower index of refraction is completely reflected by the boundary between the two media.
- **critical angle:** The smallest angle of incidence at which a light ray passing from one medium to another less refractive medium will be totally reflected from the boundary between the two.
- **fiber optics:** The science or technology of light transmission through very fine, flexible glass or plastic fibers without energy loss making use of the principle of total internal reflection.

References

1. Roshan Nikam (Flickr: roshan1286). http://www.flickr.com/photos/31916678@N07/4753800195/ . CC-BY 2.0
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

CONCEPT 7 Single Slit Interference

- Explain how single slit diffraction patterns occur.
- Use single slit diffraction patterns to calculate wavelength.

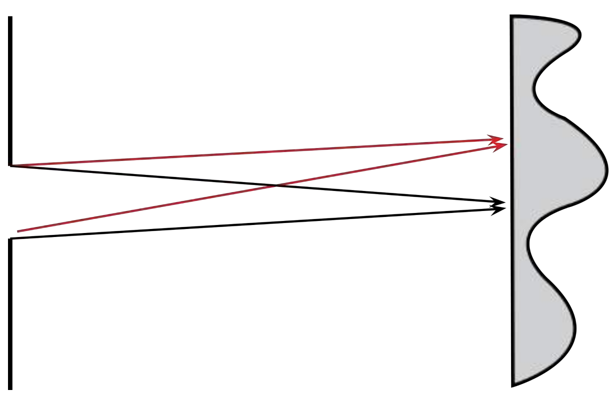


Though it looks like a double slit interference pattern, the pattern on the screen are actually the results of light diffracting through a single slit with the ensuing interference.

Single Slit Interference

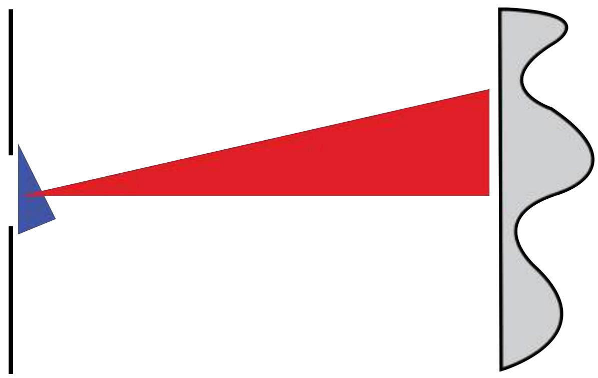
Interference patterns are produced not only by double slits but also by single slits, otherwise known as **single slit interference** . In the case of a single slit, the particles of medium at both corners of the slit act as point sources, producing circular waves from both edges. These circular waves move across to the back wall and interfere in the same way that interference patterns were produced by double slits.

In the sketch at below, the black lines intersect at the center of the pattern on the back wall. This center point is equidistanct from both edges of the slit. Therefore, the waves striking at this position will be **in phase;** that is, the waves will produce constructive interference. Also shown in the sketch, just above the central bright spot where the red lines intersect, is a position where destructive interference occurs. One of these red lines is one-half wavelength longer than the other, causing the two waves to hit the wall **out of phase** and undergo destructive interference. A dark bank appears at this position.



Just as in double slit interference, a pair of similar triangles can be constructed in the interference pattern. The pertinent values from these triangles are the width of the slit, w, the wavelength, λ, the distance from the central bright spot to the first dark band, x, and the distance from the center of the slit to back wall, L. The relationship of these four values is

$\frac{\lambda}{w} = \frac{x}{L}$ *or* $\lambda = \frac{wx}{L}$.



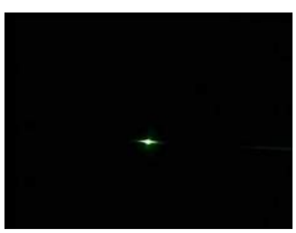
Example Problem: Monochromatic light of wavelength 605 nm falls on a slit of width 0.095 mm. The slit is located 85 cm from a screen. How far is the center of the central bright band to the first dark band?

Solution: $x = \frac{\lambda L}{w} = \frac{(6.05\times10^{-7}\ m)(0.85\ m)}{(9.5\times10^{-5}\ m)} = 0.0054\ m$

Summary

- Interference patterns can also be produced by single slits.
- In the case of a single slit, the particles of medium at both corners of the slit act as point sources, and produce circular waves from both edges.
- The wavelength can be determined by this equation: $\frac{\lambda}{w} = \frac{x}{L}$ *or* $\lambda = \frac{wx}{L}$.

Practice



MEDIA

Click image to the left for more content.

http://www.youtube.com/watch?v=_HO7LJDcqos

Follow up questions.

1. The interference pattern appears as the slit becomes ___________ (wider, thinner).

Review

1. The same set up is used for two different single slit diffraction experiments. In one of the experiments, yellow light is used, and in the other experiment, green light is used. Green light has a shorter wavelength than yellow light. Which of the following statements is true?

(a) The two experiments will have the same distance between the central bright band and the first dark band.
(b) The green light experiment will have a greater distance between the central bright band and the first dark band.
(c) The yellow light experiment will have a greater distance between the central bright band and the first dark band.

2. Why are the edges of shadows often fuzzy?

   (a) Interference occurs on the wall on which the shadow is falling.
   (b) Light diffracts around the edges of the object casting the shadow.
   (c) The edges of the object casting the shadow is fuzzy.
   (d) Light naturally spreads out.

3. Monochromatic, coherent light passing through a double slit will produce exactly the same interference pattern as when it passes through a single slit.

   (a) True
   (b) False

4. If monochromatic light passes through a 0.050 mm slit and is projected onto a screen 0.70 m away with a distance of 8.00 mm between the central bright band and the first dark band, what is the wavelength of the light?
5. A krypton ion laser with a wavelength of 524.5 nm illuminates a 0.0450 mm wide slit. If the screen is 1.10 m away, what is the distance between the central bright band and the first dark band?
6. Light from a He-Ne laser ($\lambda = 632.8\ nm$) falls on a slit of unknown width. In the pattern formed on a screen 1.15 m away, the first dark band is 7.50 mm from the center of the central bright band. How wide is the slit?

- **single slit interference:** When monochromatic, coherent light falls upon a small single slit it will produce a pattern of bright and dark fringes. These fringes are due to light from one side of the slit interacting (interfering) with light from the other side.

References

1. Luiz Sauerbronn. http://commons.wikimedia.org/wiki/File:Fresnel_Diffraction_experiment_DSC04573.JPG . Public Domain
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

CONCEPT 8 Double Slit Interference

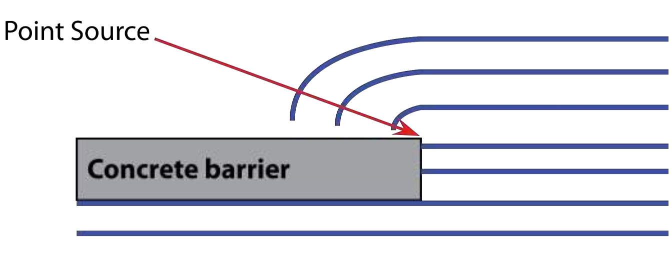
- Define diffraction of light.
- Define wave interference.
- Describe double slit interference patterns.
- Explain Thomas Young's contibutions to physics.
- Calculate the wavelength from a double slit interference pattern.

FIGURE 8.1

When waves strike a small slit in a wall, they create circular wave patterns on the other side of the barrier. This is seen in the image above, where ocean waves create precise circular waves. The circular waves undergo constructive and destructive interference, which generates a regular interference pattern.

Diffraction and Interference

When a series of straight waves strike an impenetrable barrier, the waves stop at the barrier. However, the last particle of the medium at the back corner of the barrier will create circular waves from that point, called the **point source.** This can be seen in the image below. This phenomenon is called **diffraction** , and it occurs in liquid, sound, and light waves. While the waves become circular waves at the point source, they continue as straight waves where the barrier does not interfere with the waves.



Any two waves in the same medium undergo **wave interference** as they pass each other. At the location where the two waves collide, the result is essentially a summation of the two waves. In some places, a wave crest from one source will overlap a wave crest from the other source. Since both waves are lifting the medium, the combined wave crest will be twice as high as the original crests. Nearby, a wave trough will overlap another wave trough and the new trough will be twice as deep as the original. This is called **constructive interference** because the resultant wave is larger than the original waves. Within the interference pattern, the amplitude will be twice the original amplitude. Once the waves pass through each other and are alone again, their amplitudes return to their original values.

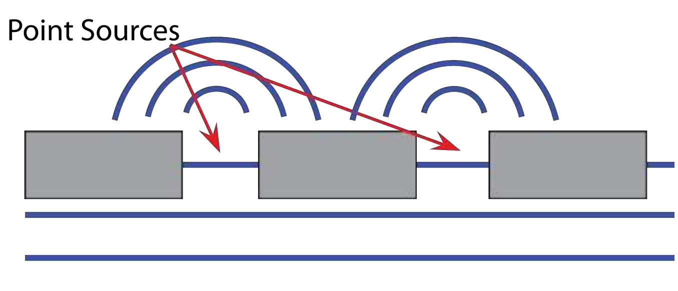
In other parts of the wave pattern, crests from one wave will overlap troughs from another wave. When the two waves have the same amplitude, this interaction causes them to cancel each other out. Instead of a crest or a trough, there is nothing. When this cancellation occurs, it is called **destructive interference** .

FIGURE 8.2

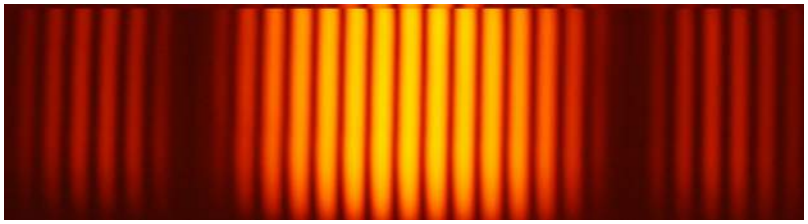
It is easy to see how waves emanating from multiple sources, such as drops of rainwater in still water, create interference patterns. But a single source of waves can create interference patterns with itself as a result of diffraction.

The Double Slit Experiment

A similar situation to the raindrops above occurs when straight waves strike a barrier containing two slits. These waves are cut off everywhere except for where the waves that pass through the two slits. The medium in the slits again acts as a point source to produce circular waves on the far side of the barrier.



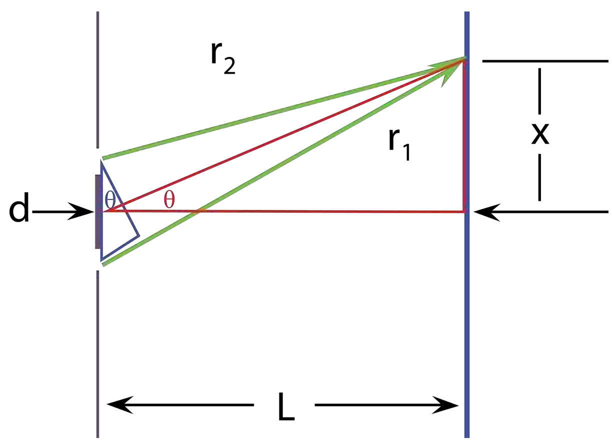
As long as these two circular waves have the same wavelength, they interfere constructively and destructively in a specific pattern. This pattern is called the **wave interference pattern** and is characterized by light and dark bands. The light bands are a result of constructive interference, and the dark bands occur because of destructive interference.



In the early 1800's, light was assumed to be a particle. There was a significant amount of evidence to point to that conclusion, and famous scientist Isaac Newton's calculations all support the particle theory. In 1803, however, Thomas Young performed his famous Double Slit Experiment to prove that light was a wave. Young shined a light onto the side of a sealed box with two slits in it, creating an interference pattern on the inside of the box opposite the slits. As seen above, interference patterns are characterized by alternating bright and dark lines. The bright lines are a result of constructive interference, while the dark lines are a result of destructive interference. By creating this interference pattern, Young proved light is a wave and changed the course of physics.

Calculating Wavelength from Double Slit Pattern

Using the characteristics of the double slit interference pattern, it is possible to calculate the wavelength of light used to produce the interference. To complete this calculation, it is only necessary to measure a few distances. As can be seen below, five distances are measured. In the sketch, L is the distance from the two slits to the back wall where the interference pattern can be seen. d is the distance between the two slits. To understand x, look again at the interference pattern shown above. The middle line, which is the brightest, is called the **central line**. The remaining lines are called **fringes**. The lines on either side of the central line are called the first order fringes, the next lines are called the second order fringes, and so on. x is the distance from the central line to the first order fringe.



r_1 and r_2 are the distances from the slits to the first order fringe. We know that the fringes are a result of constructive interference, and that the fringe is a result of the crest of two waves interfering. If we assume that r_2 is a whole number of wavelengths (confirm for yourself that this is a logical assumption), then r_1 must be one more wavelength. This is because r_1 and r_2 are the distances to the first order fringe. Mathematically, we can let

$r_2 = n\lambda$ and $r_1 = n\lambda + \lambda$, where λ is the wavelength and n is a constant.

Using this relationship, we determine that $r_1 - r_2 = \lambda$.

Looking again at the diagram, the red and blue triangles are similar, which means that the ratios of corresponding sides are the same. The ratio of x to L in the red triangle is equal to the ratio of λ to d in the blue triangle. For proof of this, visit http://www.physicsclassroom.com/class/light/u12l3c.cfm . From this, we can determine that the wavelength is dependent on *x, d,* and *L:*

$\lambda = \frac{xd}{L}$

Example Problem: Monochromatic light falls on two narrow slits that are 0.0190 mm apart. A first order fringe is 21.1 mm from the central line. The screen (back wall) is 0.600 m from the slits. What is the wavelength of the light?

Solution: $\lambda = \frac{xd}{L} = \frac{(0.021\ m)(0.000019\ m)}{(0.600\ m)} = 6.68 \times 10^{-7}\ m$

Summary

- The last particle of medium at the back corner of an impenetrable barrier will act as a point source and produce circular waves.
- Diffraction is the bending of waves around a corner.
- Constructive interference occurs when two wave crests overlap, doubling the wave amplitude at that location.
- Destructive interference occurs when a wave crest overlaps with a trough, causing them to cancel out.
- Light is a wave, and creates an interference pattern in the double slit experiment.
- An interference pattern consists of alternating bright and dark lines; the bright lines are called fringes.
- In a double slit experiment, the wavelength can be calculated using this equation: $\lambda = \frac{xd}{L}$

Practice

http://www.youtube.com/watch?v=AMBcgVlamoU

Follow up questions.

1. When the amplitude of waves add, it is called __________________ interference.
2. When the amplitude of waves subtract, it is called __________________ interference.
3. What do we call the phenomenon of light bending around a corner?

Review

1. Destructive interference in waves occurs when
   (a) two troughs overlap.
   (b) crests and troughs align.
   (c) two crests overlap.
   (d) a crest and a trough overlap.
2. Bright bands in interference patterns result from
   (a) destructive diffraction.
   (b) destructive interference.
   (c) constructive diffraction.
   (d) constructive interference.
3. In a double slit experiment with slits $1.00 \times 10^{-5}\ m$ apart, light casts the first bright band $3.00 \times 10^{-2}\ m$ from the central bright spot. If the screen is 0.650 m away, what is the wavelength of this light?
   (a) 510 nm
   (b) 390 nm
   (c) 430 nm
   (d) 460 nm
4. Violet light falls on two slits separated by $1.90 \times 10^{-5}\ m$. A first order bright line appears 13.2 mm from the central bright spot on a screen 0.600 m from the slits. What is the wavelength of the violet light?
5. Suppose in the previous problem, the light was changed to yellow light with a wavelength of $5.96 \times 10^{-7}\ m$ while the slit separation and distance from screen to slits remained the same. What would be the distance from the central bright spot to the first order line?

6. Light with a wavelength of 6.33×10^{-7} m is used in a double slit experiment. The screen is placed 1.00 m from the slits and the first order line is found 65.5 mm from the central bright spot. What is the separation between the slits?

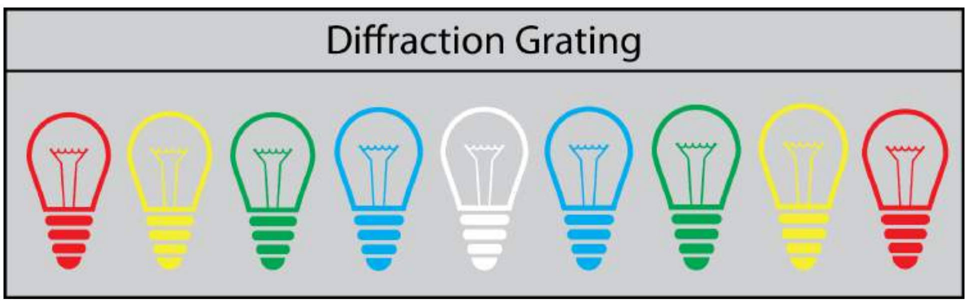
- **diffraction:** Change in the directions and intensities of a group of waves after passing by an obstacle or through an aperture whose size is approximately the same as the wavelength of the waves.
- **monochromatic:** Light having only one wavelength.
- **constructive interference:** The interference of two or more waves of equal frequency and phase, resulting in their mutual reinforcement and producing a single amplitude equal to the sum of the amplitudes of the individual waves.
- **destructive interference:** The interference of two waves of equal frequency and opposite phase, resulting in their cancellation where the negative displacement of one always coincides with the positive displacement of the other.

References

1. . . CC BY-NC-SA
2. . Kathy Shield . CC BY-NC-SA
3. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
4. Pieter Kuiper. http://commons.wikimedia.org/wiki/File:SodiumD_two_double_slits.jpg . Public Domain
5. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

CONCEPT 9 Diffraction Gratings

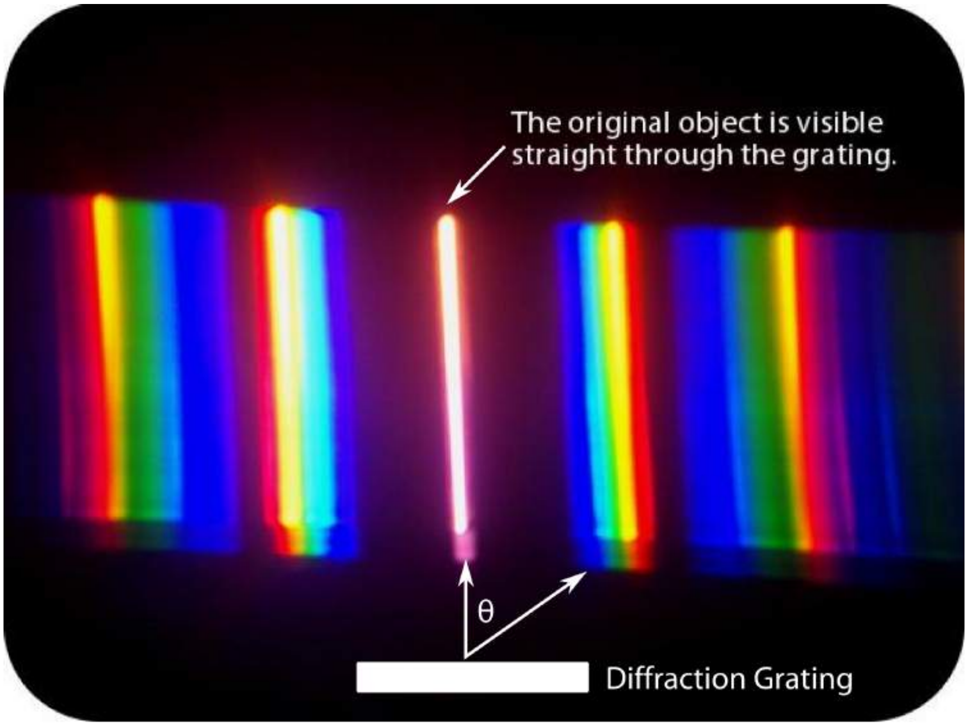
- Understand the structure of a diffraction grating.
- Explain the interference pattern formed by diffraction gratings.
- Use diffraction grating interference patterns to calculate the wavelength of light.



Suppose we had a light bulb that emitted exactly four frequencies of light; one frequency in each of the colors red, yellow, green, and blue. To our eye, this bulb would appear white because the combination of those four colors produces white light. If viewed through a diffraction grating, however, each color of light would be visible. The original white light bulb is visible in the center of the image, and interference causes the light bulb to appear in each color to the left and the right.

Diffraction Gratings

Diffraction gratings are composed of a multitude of slits lined up side by side, not unlike a series of double slits next to each other. They can be made by scratching very fine lines with a diamond point on glass, or by pressing plastic film on glass gratings so that the scratches are replicated. The clear places between the scratches behave as slits similar to the slits in a double slit experiment and the gratings form interference patterns in the same general way that double slits do. With more slits, there are more light waves out of phase with each other, causing more destructive interference. Compared to the interference pattern of a double slit, the diffraction grating interference pattern's colors are spread out further and the dark regions are broader. This allows for more precise wavelength determination than with double slits. The image below shows the diffraction pattern emanating from a white light.



Also in this image is the measurement for θ, which can be used to calculate the wavelength of the original light source. The equation from the double slit experiment can be adjusted slightly to work with diffraction gratings. Where λ is the wavelength of light, d is the distance between the slits on the grating, and θ is the angle between the incident (original) light and the refracted light,

$\lambda = \frac{xd}{L} = d\sin\theta$ (Note that $\frac{x}{L} = \sin\theta$, using the small angle approximation theorem.)

Looking at the equation, $x = \frac{\lambda L}{d}$, it should be apparent that as the distance between the lines on the grating become smaller and smaller, the distance between the images on the screen will become larger and larger. Diffraction gratings are often identified by the number of lines per centimeter; gratings with more lines per centimeter are usually more useful because the greater the number of lines, the smaller the distance between the lines, and the greater the separation of images on the screen.

Example Problem: A good diffraction grating has 2500 lines/cm. What is the distance between two lines on the grating?

Solution: $d = \frac{1}{2500\ cm^{-1}} = 0.00040\ cm$

Example Problem: Using a diffraction grating with a spacing of 0.00040 cm, a red line appears 16.5 cm from the central line on the screen. The screen is 1.00 m from the grating. What is the wavelength of the light?

Solution: $\lambda = \frac{xd}{L} = \frac{(0.165\ m)(4.0\times10^{-6}\ m)}{1.00\ m} = 6.6\times10^{-7}\ m$

Summary

- Diffraction gratings can be made by blocking light from traveling through a translucent medium; the clear places behave as slits similar to the slits in a double slit experiment.
- Diffraction gratings form interference patterns much like double slits, though brighter and with more space between the lines.
- The equation used with double slit experiments to measure wavelength is adjusted slightly to work with diffraction gratings. $\lambda = \frac{xd}{L} = d\sin\theta$

Practice

http://vimeo.com/39495562

Follow up questions.

1. How does a diffraction grating differ from single or double slit?
2. What happens when you increase the number of slits in a diffraction grating?

Review

1. White light is directed toward a diffraction grating and that light passes through the grating, causing its monochromatic bands appear on the screen. Which color will be closest to the central white?
2. Three discrete spectral lines occur at angles of 10.1°, 13.7°, and 14.8° respectively in the first order spectrum. If the grating has 3660 lines/cm, what are the wavelengths of these three colors of light?
3. A 20.0 mm section of diffraction grating has 6000 lines. At what angle will the maximum bright band appear if the wavelength is 589 nm?
4. Laser light is passed through a diffraction grating with 7000 lines/cm. The first order maximum on the screen is 25° away from the central maximum. What is the wavelength of the light?

- **diffraction grating:** A glass, plastic, or polished metal surface having a large number of very fine parallel grooves or slits and used to produce optical spectra by diffraction.

References

1. CK-12 Foundation - Samantha Bacic, using light bulb images copyright Ruslan Klimovich, 2013. http://www.shutterstock.com . Used under license from Shutterstock.com
2. Candace (Flickr: cosmiccandace). http://www.flickr.com/photos/candace/315205005/ . CC-BY 2.0

Concept 10 Wave-Particle Theory

- State the wave-particle theory of electromagnetic radiation.
- Describe a photon.
- Identify evidence that electromagnetic radiation is both a particle and a wave.

What a beautiful sunset! You probably know that sunlight travels in waves through space from the sun to Earth. But do you know what light really is? Is it just energy, or is it something else? In this article you'll find out that light may be more than it seems.

The Question

Electromagnetic radiation, commonly called light, is the transfer of energy by waves called electromagnetic waves. These waves consist of vibrating electric and magnetic fields. Where does electromagnetic energy come from? It is released when electrons return to lower energy levels in atoms. Electromagnetic radiation behaves like continuous waves of energy most of the time. Sometimes, however, electromagnetic radiation seems to behave like discrete, or separate, particles rather than waves. So does electromagnetic radiation consist of waves or particles?

The Debate

This question about the nature of electromagnetic radiation was debated by scientists for more than two centuries, starting in the 1600s. Some scientists argued that electromagnetic radiation consists of particles that shoot around like tiny bullets. Other scientists argued that electromagnetic radiation consists of waves, like sound waves or water waves. Until the early 1900s, most scientists thought that electromagnetic radiation is either one or the other and that scientists on the other side of the argument were simply wrong.

Q: Do you think electromagnetic radiation is a wave or a particle?

A: Here's a hint: it may not be a question of either-or. Keep reading to learn more.

A New Theory

In 1905, the physicist Albert Einstein developed a new theory about electromagnetic radiation. The theory is often called the **wave-particle theory** . It explains how electromagnetic radiation can behave as both a wave and a particle. Einstein argued that when an electron returns to a lower energy level and gives off electromagnetic energy, the energy is released as a discrete "packet" of energy. We now call such a packet of energy a **photon** . According to Einstein, a photon resembles a particle but moves like a wave. You can see this in the **Figure** 10.1 . The theory posits that waves of photons traveling through space or matter make up electromagnetic radiation.

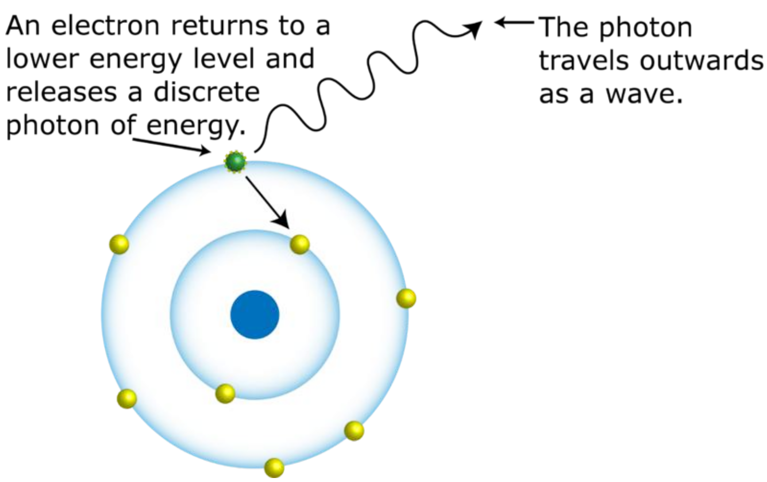


FIGURE 10.1

Energy of a Photon

A photon isn't a fixed amount of energy. Instead, the amount of energy in a photon depends on the frequency of the electromagnetic wave. The frequency of a wave is the number of waves that pass a fixed point in a given amount of time, such as the number of waves per second. In waves with higher frequencies, photons have more energy.

Evidence for the Wave-Particle Theory

After Einstein proposed his theory, evidence was discovered to support it. For example, scientists shone laser light through two slits in a barrier made of a material that blocked light. You can see the setup of this type of experiment in the sketch below. Using a special camera that was very sensitive to light, they took photos of the light that passed through the slits. The photos revealed tiny pinpoints of light passing through the double slits. This seemed to show that light consists of particles. However, if the camera was exposed to the light for a long time, the pinpoints accumulated in bands that resembled interfering waves. Therefore, the experiment showed that light seems to consist of particles that act like waves.

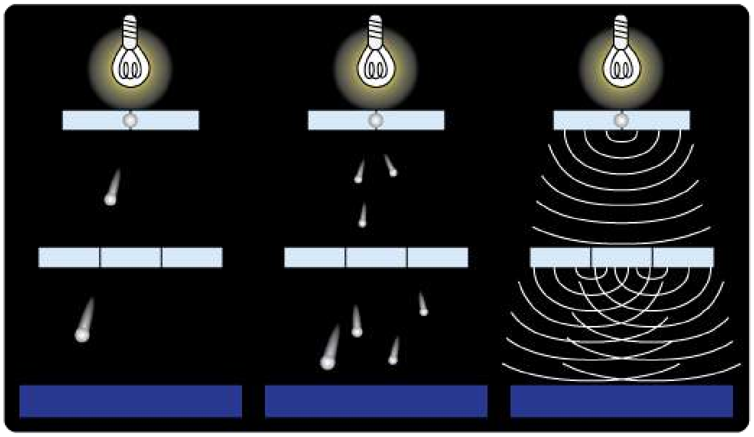


FIGURE 10.2

Summary

- Electromagnetic radiation behaves like waves of energy most of the time, but sometimes it behaves like particles. From the 1600s until the early 1900s, most scientists thought that electromagnetic radiation consists either of particles or of waves but not both.
- In 1905, Albert Einstein proposed the wave-particle theory of electromagnetic radiation. This theory states that electromagnetic energy is released in discrete packets of energy—now called photons—that act like waves.
- After Einstein presented his theory, scientists found evidence to support it. For example, double-slit experiments showed that light consists of tiny particles that create patterns of interference just as waves do.

Vocabulary

- **photon** : Tiny "packet" of electromagnetic radiation that is released when an electron returns to a lower.
- **wave-particle theory** : Theory proposed by Albert Einstein that electromagnetic energy is released in discrete packets of energy (now called photons) that act like waves.

Practice

Watch the animation "Let There Be Light" at the following URL. Then create a timeline of ideas and discoveries about the nature of light. http://www.abc.net.au/science/explore/einstein/lightstory.htm

Review

1. Why did scientists debate the nature of electromagnetic radiation for more than 200 years?
2. State Einstein's wave-particle theory of electromagnetic radiation.
3. What is a photon?
4. After Einstein proposed his wave-particle theory, how did double-slit experiments provide evidence to support the theory?

References

1. Christopher Auyeung. . CC BY-NC 3.0
2. Zachary Wilson. . CC-BY-NC-SA 3.0

CK-12 FlexBook

Physics Unit 12: Static Electricity

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CONCEPT 1 Static Electricity and Static Discharge

- Describe static electricity.
- Explain static discharge.
- Outline how lightning occurs.

You're a thoughtful visitor, so you wipe your feet on the welcome mat before you reach out to touch the brass knocker on the door. Ouch! A spark suddenly jumps between your hand and the metal, and you feel an electric shock.

Q: Why do you think an electric shock occurs?

A: An electric shock occurs when there is a sudden discharge of static electricity.

What Is Static Electricity?

Static electricity is a buildup of electric charges on objects. Charges build up when negative electrons are transferred from one object to another. The object that gives up electrons becomes positively charged, and the object that accepts the electrons becomes negatively charged. This can happen in several ways.

One way electric charges can build up is through friction between materials that differ in their ability to give up or accept electrons. When you wipe your rubber-soled shoes on the wool mat, for example, electrons rub off the mat onto your shoes. As a result of this transfer of electrons, positive charges build up on the mat and negative charges build up on you.

Once an object becomes electrically charged, it is likely to remain charged until it touches another object or at least comes very close to another object. That's because electric charges cannot travel easily through air, especially if the air is dry.

Q: You're more likely to get a shock in the winter when the air is very dry. Can you explain why?

A: When the air is very dry, electric charges are more likely to build up objects because they cannot travel easily through the dry air. This makes a shock more likely when you touch another object.

Static Discharge

What happens when you have become negatively charged and your hand approaches the metal doorknocker? Your negatively charged hand repels electrons in the metal, so the electrons move to the other side of the knocker. This makes the side of the knocker closest to your hand positively charged. As your negatively charged hand gets very close to the positively charged side of the metal, the air between your hand and the knocker also becomes electrically charged. This allows electrons to suddenly flow from your hand to the knocker. The sudden flow of electrons is **static discharge** . The discharge of electrons is the spark you see and the shock you feel. Watch the animation "John Travoltage" at the following URL to see an example of static electricity and static discharge.

http://www.cabrillo.edu/~jmccullough/Physics/Electric_Forces_Fields.html

How Lightning Occurs

Another example of static discharge, but on a much larger scale, is lightning. You can see how it occurs in the following diagram and animation as you read about it below.

http://micro.magnet.fsu.edu/electromag/java/lightning/index.html

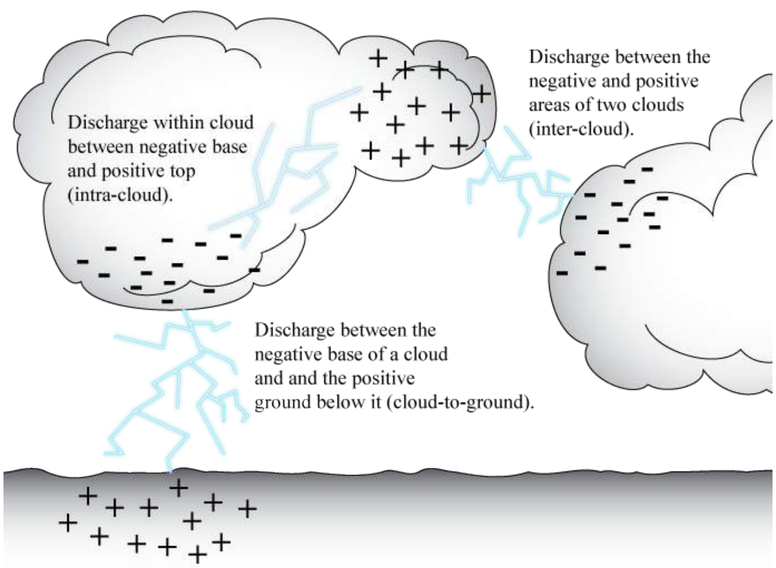


FIGURE 1.1

During a rainstorm, clouds develop regions of positive and negative charge due to the movement of air molecules, water drops, and ice particles. The negative charges are concentrated at the base of the clouds, and the positive charges are concentrated at the top. The negative charges repel electrons on the ground beneath them, so the ground below the clouds becomes positively charged. At first, the atmosphere prevents electrons from flowing away from areas of negative charge and toward areas of positive charge. As more charges build up, however, the air between the oppositely charged areas also becomes charged. When this happens, static electricity is discharged as bolts of lightning.

At the URL below, you can watch an awesome slow-motion lightning strike. Be sure to wait for the real-time lightning strike at the end of the video. You'll be amazed when you realize how much has occurred during that split-second discharge of static electricity.

http://www.youtube.com/watch?v=Y8oN0YFAXWQ&feature=related

MEDIA

Click image to the left for more content.

Summary

- Static electricity is a buildup of electric charges on objects. It occurs when electrons are transferred from one object to another.
- A sudden flow of electrons from one charged object to another is called static discharge.
- Examples of static discharge include lightning and the shock you sometimes feel when you touch another object.

Vocabulary

- **static discharge** : Sudden flow of electrons from an object that has a buildup of charges.
- **static electricity** : Buildup of charges on an object that occurs through induction.

Practice

Watch the video at the following URL. Then answer the discussion questions. Read the background essay if you need help with any of the questions. http://www.teachersdomain.org/resource/phy03.sci.phys.mfe.zsnap/

Review

1. What is static electricity?
2. How does static discharge occur?
3. Explain why a bolt of lightning is like the spark you might see when you touch a metal object and get a shock.

References

1. Zachary Wilson. . CC BY-NC 3.0

Concept 2 Electric Charge and Electric Force

- Define electric charge.
- Describe electric forces between charged particles.

A lightning bolt is like the spark that gives you a shock when you touch a metal doorknob. Of course, the lightning bolt is on a *much* larger scale. But both the lightning bolt and spark are a sudden transfer of electric charge.

Introducing Electric Charge

Electric charge is a physical property of particles or objects that causes them to attract or repel each other without touching. All electric charge is based on the protons and electrons in atoms. A proton has a positive electric charge, and an electron has a negative electric charge. In the **Figure** 2.1 , you can see that positively charged protons (+) are located in the nucleus of the atom, while negatively charged electrons (-) move around the nucleus.

Electric Force

When it comes to electric charges, opposites attract, so positive and negative particles attract each other. You can see this in the diagram below. This attraction explains why negative electrons keep moving around the positive nucleus of the atom. Like charges, on the other hand, repel each other, so two positive or two negative charges push apart. This is also shown in the diagram. The attraction or repulsion between charged particles is called **electric force** . The strength of electric force depends on the amount of electric charge on the particles and the distance between

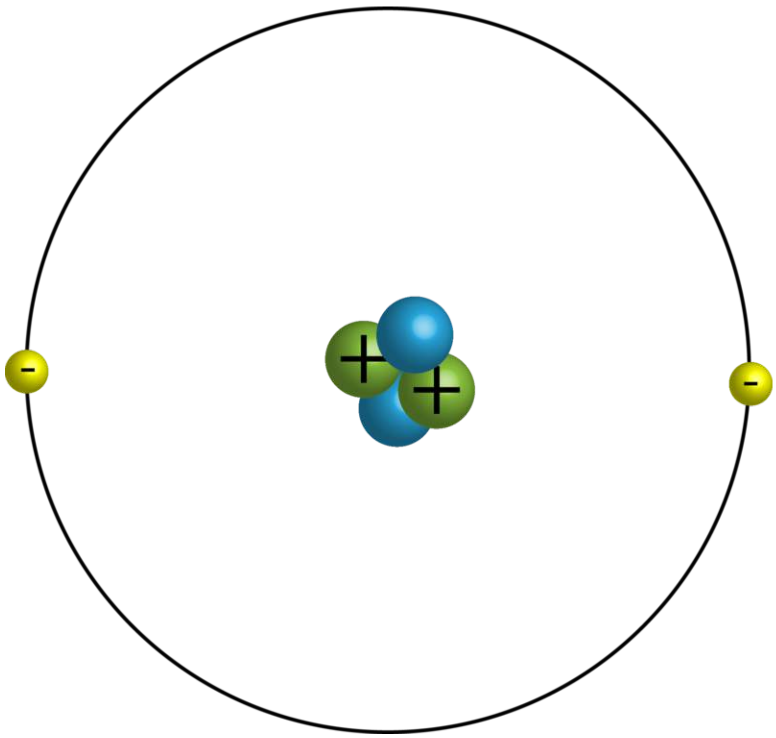


FIGURE 2.1

them. Larger charges or shorter distances result in greater force. You can experiment with electric force with the animation at the following URL. http://www.colorado.edu/physics/2000/waves_particles/wavpart2.html

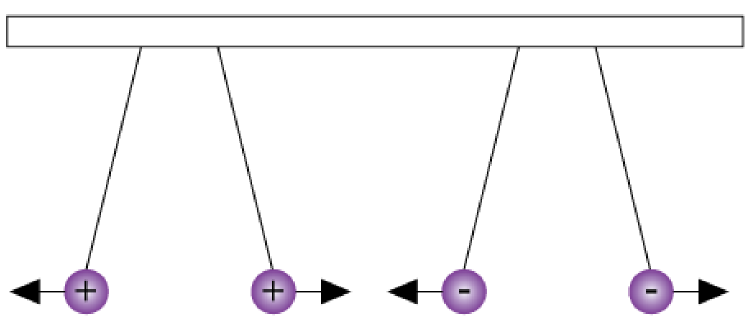


FIGURE 2.2

Q: How do positive protons stay close together inside the nucleus of the atom if like charges repel each other?

A: Other, stronger forces in the nucleus hold the protons together.

Summary

- Electric charge is a physical property of particles or objects that causes them to attract or repel each other without touching.
- Particles that have opposite charges attract each other. Particles that have like charges repel each other. The force of attraction or repulsion is called electric force.

Vocabulary

- **electric charge** : Physical property of particles or objects that causes them to attract or repel each other without touching; may be positive or negative.
- **electric force** : Force of attraction or repulsion between charged particles.

Practice

Read the first four boxes of text at the following URL. Then write a concise paragraph explaining why direction E is the correct answer to the quick quiz. http://www.physics.wisc.edu/undergrads/courses/208-f07/Lectures/lect6.pdf

Review

1. What is electric charge?
2. Make a simple table summarizing electric forces between charged particles.

References

1. Christopher Auyeung. . CC BY-NC 3.0
2. Zachary Wilson. . CC BY-NC 3.0

Concept 3 Transfer of Electric Charge

- Describe how the transfer of electrons changes the charge of matter.
- Relate the transfer of electrons to the law of conservation of charge.
- Compare and contrast three ways that electric charge can be transferred.

Why is this girl's hair standing straight up? She is touching a device called a van de Graaff generator. The dome on top of the device has a negative electric charge. When the girl places her hand on the dome, she becomes negatively charged as well—right down to the tip of each hair! You can see a video demonstrating a van de Graff generator at this URL: http://www.youtube.com/watch?v=SREXQWAIDJk

MEDIA
Click image to the left for more content.

Q: Why is the man's hair standing on end?

A: All of the hairs have all become negatively charged, and like charges repel each other. Therefore, the hairs are pushing away from each other, causing them to stand on end.

Transferring Electrons

The man pictured above became negatively charged because electrons flowed from the van de Graaff generator to him . Whenever electrons are transferred between objects, neutral matter becomes charged. This occurs even with individual atoms. Atoms are neutral in electric charge because they have the same number of negative electrons as positive protons. However, if atoms lose or gain electrons, they become charged particles called ions. You can see how this happens in the **Figure** 3.1 . When an atom loses electrons, it becomes a positively charged ion, or cation. When an atom gains electrons, it becomes a negative charged ion, or anion.

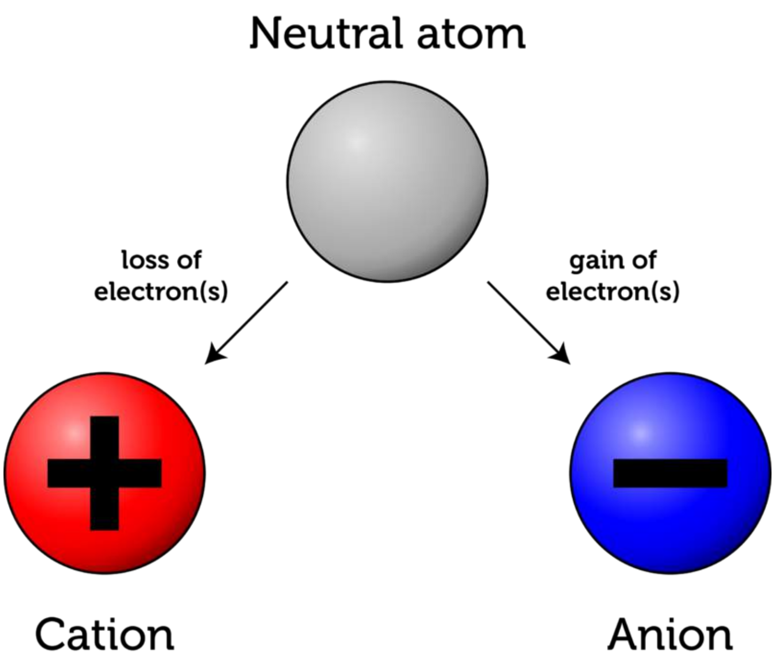


FIGURE 3.1

Conservation of Charge

Like the formation of ions, the formation of charged matter in general depends on the transfer of electrons, either between two materials or within a material. Three ways this can occur are referred to as conduction, polarization, and friction. All three ways are described below. However, regardless of how electrons are transferred, the total charge always remains the same. Electrons move, but they aren't destroyed. This is the **law of conservation of charge** .

Conduction

The transfer of electrons from the van de Graaff generator to the man is an example of conduction. Conduction occurs when there is direct contact between materials that differ in their ability to give up or accept electrons. A van de Graff generator produces a negative charge on its dome, so it tends to give up electrons. Human hands are positively charged, so they tend to accept electrons. Therefore, electrons flow from the dome to the man's hand when they are in contact.

You don't need a van de Graaff generator for conduction to take place. It may occur when you walk across a wool carpet in rubber-soled shoes. Wool tends to give up electrons and rubber tends to accept them. Therefore, the carpet transfers electrons to your shoes each time you put down your foot. The transfer of electrons results in you becoming negatively charged and the carpet becoming positively charged.

Polarization

Assume that you have walked across a wool carpet in rubber-soled shoes and become negatively charged. If you then reach out to touch a metal doorknob, electrons in the neutral metal will be repelled and move away from your hand before you even touch the knob. In this way, one end of the doorknob becomes positively charged and the other end becomes negatively charged. This is called polarization. Polarization occurs whenever electrons within a neutral object move because of the electric field of a nearby charged object. It occurs without direct contact between the two objects. The **Figure** 3.2 models how polarization occurs.

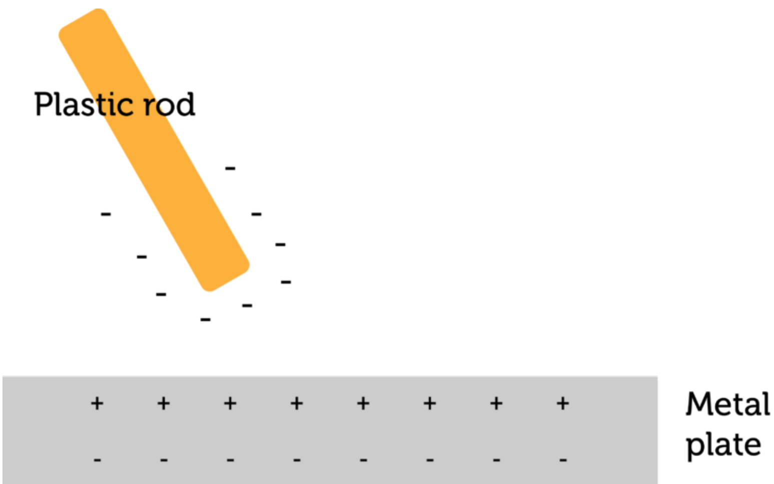


FIGURE 3.2

Q: What happens when the negatively charged plastic rod in the diagram is placed close to the neutral metal plate?

A: Electrons in the plate are repelled by the positive charges in the rod. The electrons move away from the rod, causing one side of the plate to become positively charged and the other side to become negatively charged.

Friction

Did you ever rub an inflated balloon against your hair? You can see what happens in the **Figure** 3.3 . Friction between the balloon and hair cause electrons from the hair to "rub off" on the balloon. That's because a balloon attracts electrons more strongly than hair does. After the transfer of electrons, the balloon becomes negatively charged and the hair becomes positively charged. The individual hairs push away from each other and stand on end because like charges repel each other. The balloon and the hair attract each other because opposite charges attract.

Electrons are transferred in this way whenever there is friction between materials that differ in their ability to give up or accept electrons. Watch the animation "Balloons and Static Electricity" at the following URL to see how electrons are transferred by friction between a sweater and a balloon. http://www.cabrillo.edu/~jmccullough/Physics/Electric_Forces_Fields.html

Q: If you rub a balloon against a wall, it may stick to the wall. Explain why.

A: Electrons are transferred from the wall to the balloon, making the balloon negatively charged and the wall positively charged. The balloon sticks to the wall because opposite charges attract.

Summary

- Whenever electrons are transferred between objects, neutral matter becomes charged. For example, when atoms lose or gain electrons they become charged particles called ions.

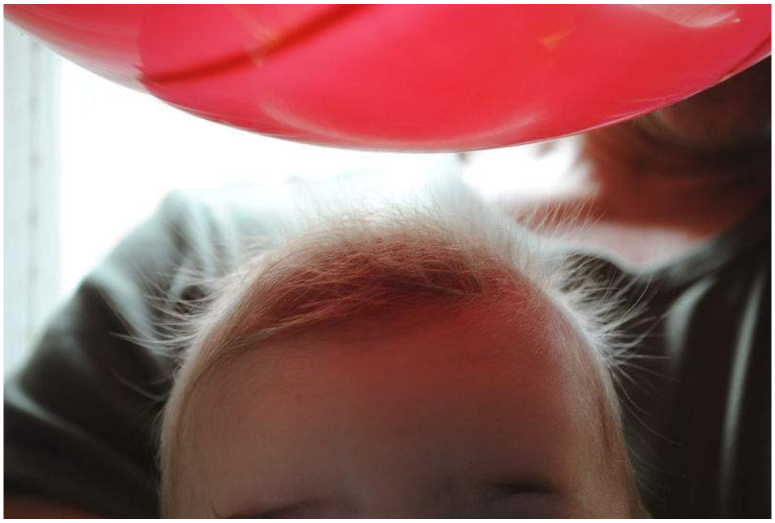


FIGURE 3.3

- Three ways electrons can be transferred are conduction, friction, and polarization. In each case, the total charge remains the same. This is the law of conservation of charge.
- Conduction occurs when there is direct contact between materials that differ in their ability to give up or accept electrons.
- Polarization is the movement of electrons within a neutral object due to the electric field of a nearby charged object. It occurs without direct contact between the two objects.
- Electrons are transferred whenever there is friction between materials that differ in their ability to give up or accept electrons.

Vocabulary

- **law of conservation of charge** : Law stating that charges are not destroyed when they are transferred between two materials or within a material, so the total charge remains the same.

Practice

At the following URL, review how charges are transferred through friction. Watch the animation and read the list of more-positive to less-positive materials. Then answer the questions below.

http://www.regentsprep.org/regents/physics/phys03/atribo/default.htm

1. If you rub glass with a piece of plastic wrap, will the glass become positively or negatively charged?
2. Assume that after you pet your dog with very dry hands, you touch a metal doorknob and get a shock. Is electric charge transferred from your hand to the doorknob or the other way around?

Review

1. How is charge transferred by a van de Graaff generator?
2. Compare and contrast the formation of cations and anions.
3. State the law of conservation of charge.
4. Explain how conduction and polarization occur, using the example of walking across a wool carpet in rubber-soled shoes and then reaching out to touch a metal doorknob.

5. Predict what will happen to the charges of a plastic comb and a piece of tissue paper if you rub the tissue paper on the comb. (*Hint* : Plastic tends to accept electrons and tissue paper tends to give up electrons.)

References

1. Christopher Auyeung. . CC BY-NC 3.0
2. Christopher Auyeung. . CC BY-NC 3.0
3. Flickr:olga.palma. . CC BY 2.0

CONCEPT 4 Forces on Charged Objects

- Describe the changes that occur in the sub-atomic arrangement in matter when charged.
- Describe how to charge an object.
- Define conductors and insulators.
- Understand the difference between conduction and induction.
- Summarize the forces between charged objects.

Lightning is the discharge of static electricity that has built up on clouds. Every year, the earth experiences an average of 25 million lightning strikes. Lightning bolts travel at speeds up to 60,000 miles per second, and can reach temperatures of 50,000°F, which is five times the temperature of the surface of the sun. The energy contained in a single lightning strike could light a 100 Watt light bulb 24 hours per day for 90 days.

Forces on Charged Objects

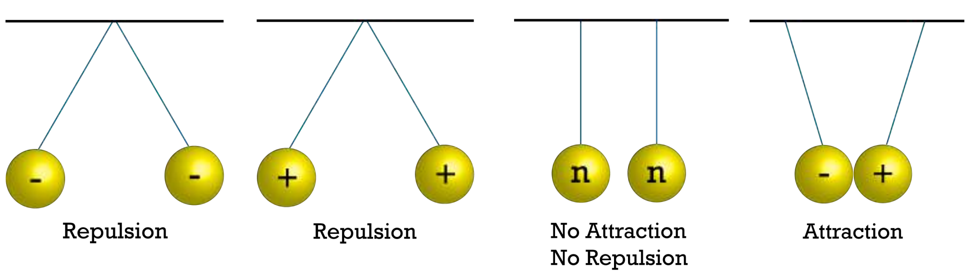
Electric charges exist within the atom. At the turn of the 20th century, J. J. Thomson and Ernest Rutherford determined that atoms contain very light-weight negatively charged particles called **electrons** and more massive, positively charged particles called **protons** . The protons are lodged in the **nucleus** of the atoms, along with the neutrally charged particles called **neutrons,** while the electrons surround the nucleus. When the number of electrons in the electron cloud and the number of protons in the nucleus are equal, the object is said to be **neutral** .

Changes to the nucleus of an atom require tremendous amounts of energy, so protons are not easily gained or lost by atoms. Electrons, on the other hand, are held fairly loosely and can often be removed quite easily. When an object loses some electrons, the remaining object is now positively charged because it has an excess of protons. The electrons may either remain free or may attach to another object. In that case, the extra electrons cause that object to become negatively charged. Atoms that have lost electrons and become positively charged are called **positive ions,** and atoms that have gained electrons and become negatively charged are called **negative ions** .

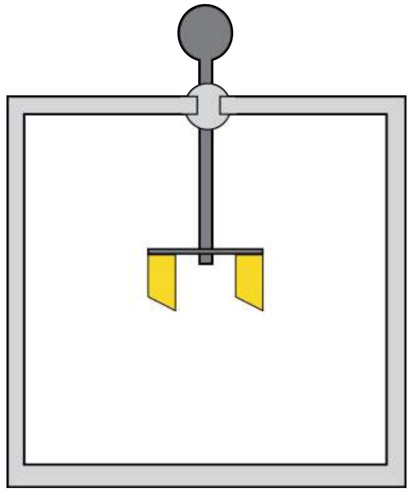
Electrons can be removed from some objects using friction, simply by rubbing one substance against another substance. There are many examples of objects becoming charged by friction, including a rubber comb through hair, and a balloon on a sweater. In both these instances, the electrons move from the second object to the first, causing the first object to become negatively charged and the second one positively charged. Friction between the tires on a moving car and the road cause the tires to become charged, and wind causes friction between clouds and air which causes clouds to become charged and can result in tremendous bolts of lightning.

A common method of producing charge in the lab is to rub cat or rabbit fur against stiff rubber, producing a negative charge on the rubber rod. If you hold a rubber rod on one end and rub only the tip of the other end with a fur, you will find that only the tip becomes charged. The electrons you add to the tip of the rod remain where you put them instead of moving around on the rod. Rubber is an **insulator.** Insulators are substances that do not allow electrons to move through them. Glass, dry wood, most plastics, cloth, and dry air are common insulators. Materials that allow electrons to flow freely are called **conductors.** Metals have at least one electron that can move around freely, and all metals are conductors.

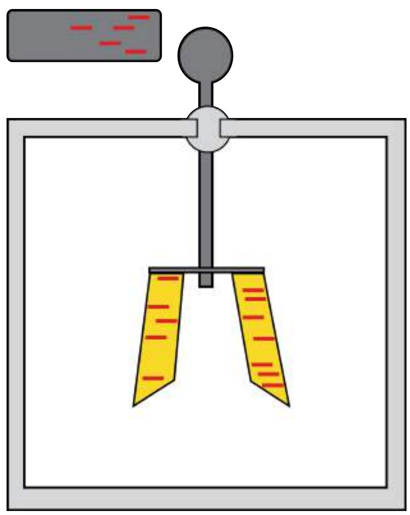
Forces are exerted on charged objects by other charged objects. You've probably heard the saying "opposites attract," which is true in regards to charged particles. Opposite charges attract each other, while like charges repulse each other. This can be seen in the image below. When two negatively charged objects are brought near each other, a repulsive force is produced. When two positively charged objects are brought near each other, a similar repulsive force is produced. When a negatively charged object is brought near a positively charged object, an attractive force is produced. Neutral objects have no influence on each other.



A laboratory instrument used to analyze and test for static charge is called an **electroscope.** Seen below, an electroscope consists of a metal knob connected by a metal stem to two very lightweight pieces of metal called leaves, shown in yellow. The leaves are enclosed in a box to eliminate stray air currents.



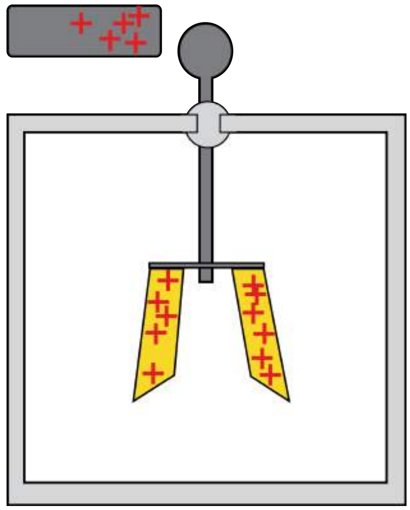
When a negatively charged object is brought near the knob of a neutral electroscope, the negative charge repels the electrons in the knob, and those electrons move down the stem into the leaves. Excess electrons flow from the rod into the ball, and then downwards making both leaves negatively charged. Since both leaves are negatively charged, they repel each other. When the rod is removed, the electroscope will remain charged because of the extra electrons added to it.



Conversely, if the rod is brought near the knob but doesn't touch it, the electroscope will appear the same while the rod is near. That is, the negative charge in the rod repels the electrons in the ball, causing them to travel down to the leaves. The leaves will separate while the rod is nearby. No extra electrons were added to the electroscope, meaning that the electrons in the electroscope will redistribute when the negatively charged rod is taken away. The leaves return to neutral, and they stop repelling each other. If the rod touches the knob, the electroscope leaves are permanently charged but if the rod is brought near but does not touch the knob, the electroscope leaves are only temporarily charged.

If the leaves are permanently charged and the rod removed, the electroscope can then be used to determine the type of unknown charge on an object. If the electroscope has been permanently negatively charged, and a negatively charge object is brought near the knob, the leaves will separate even further, showing the new object has the same charge as the leaves. If a positively charged object is brought near a negatively charged electroscope, it will attract some of the excess electrons up the stem and out of the leaves, causing the leaves to come slightly together.

Similar to the results of a negatively charged rod, if a positively charged rod is brought near the knob of a neutral electroscope, it will attract some electrons up from the leaves onto the knob. That process causes both of the leaves to be positively charged (excess protons), and the leaves will diverge. If the positively charged rob is actually touched to the knob, the rob will remove some electrons and then when the rob is removed, the electroscope will remain positively charged. This is a permanent positive charge.



Charging an object by touching it with another charged object is called **charging by conduction.** By bringing a charged object into contact with an uncharged object, some electrons will migrate to even out the charge on both objects. Charging by conduction gives the previously uncharged object a permanent charge. An uncharged object can also be charged using a method called **charging by induction** . This process allows a change in charge without actually touching the charged and uncharged objects to each other. Imagine a negatively charged rod held near the knob, but not touching. If we place a finger on the knob, some of the electrons will escape into our body, instead of down the stem and into the leaves. When both our finger and the negatively charged rod are removed, the previously uncharged electroscope now has a slight positive charge. It was charged by induction. Notice that charging by induction causes the newly charged object to have the opposite charge as the originally charged object, while charging by conduction gives them both the same charge.

Summary

- Electric charges exist with the atom.
- Atoms contain light-weight, loosely held, negatively charged particles called electrons and heavier, tightly-held, positvely charged particles called protons.
- When the number of electrons and the number of protons are equal, the object is neutral.
- The loss of electrons gives an ion a positive charge, while the gain of electrons gives it a negative charge.
- Materials that allow electrons to flow freely are called conductors, while those that do not are called insulators.
- Opposite charges attract, and like charges repel.
- Charging an object by touching it with another charged object is called charging by conduction.

Practice

The following video shows a young woman placing her hands on a Van de Graf generator which then gives her a static charge. Use this resource to answer the two questions that follow.

http://www.youtube.com/watch?v=87DqbdqBx8U

MEDIA
Click image to the left for more content.

1. What happens to her hair when she touches a ground?
2. What happens to her hair when she steps off the platform?

This video shows the static charge from the Van de Graf generator.

http://www.youtube.com/watch?v=prgu6AvauuI

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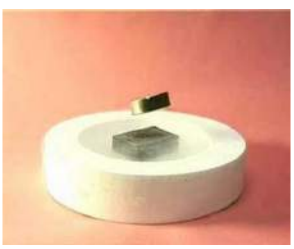
Click image to the left for more content.

This video demonstrates superconductivity that occurs at extremely low temperatures.

http://www.youtube.com/watch?feature=player_embedded&v=nWTSzBWEsms



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Additional Practice Questions:

1. When a glass rod is rubbed with a silk cloth and the rod becomes positively charged,
   (a) electrons are removed from the rod.
   (b) protons are added to the silk.
   (c) protons are removed from the silk.
   (d) the silk remains neutral.
2. Electric charge is
   (a) found only in a conductor.
   (b) found only in insulators.
   (c) conserved.
   (d) not conserved.
3. When two objects are rubbed together and they become oppositely charged, they are said to be charge by
   (a) conduction.
   (b) induction.

   (c) friction.
   (d) grounding.

4. Two objects each carry a charge and they attract. What do you know about the charge of each object?
   (a) They are both charged positively.
   (b) They have opposite charged from each other.
   (c) They are both charged negatively.
   (d) Any of the above are possible.
5. A material that easily allows the flow of electric charge through it is called a(n)
   (a) insulator.
   (b) conductor.
   (c) semiconductor.
   (d) heat sink.
6. What is the most common way of acquiring a positive static electrical charge?
   (a) by losing electrons
   (b) by gain protons
   (c) by losing protons
   (d) by gaining electrons
   (e) by switching positions of electrons and protons in the atom

Review

1. How does friction generate static electricity?
   (a) Friction heats the materials, thus causing electricity.
   (b) Rubbing materials together displaces atoms, causing sparks to fly.
   (c) Rubbing materials together can strip electrons off atoms, causing one material to become positive and the other to become negative.
   (d) Rubbing materials together causes neutrons and electrons to trade places.
   (e) None of the above.
2. What electrical charge does an electron have?
   (a) A negative charge.
   (b) A positive charge.
   (c) A neutral charge.
   (d) May be any of the above.
   (e) None of the above.
3. What happens when opposite charges get close to each other?
   (a) They repel each other.
   (b) They attract each other.
   (c) Nothing happens.
   (d) They attract surrounding objects.
   (e) They repel surrounding objects.
4. What is an electrical conductor?
   (a) A material that allows electrons to travel through it freely.
   (b) A material that doesn't allow electrons to travel through it freely.
   (c) A material that melts at low temperature.
   (d) A material that creates free electrons.
   (e) None of the above.
5. Which of the following is a good insulator of electricity?

(a) Copper
(b) Iron
(c) Rubber
(d) Salt water
(e) None of these.

- **electrons:** A fundamental sub-atomic particle, meaning it cannot be broken into smaller particles. Electrons are found in the "electron cloud" surrounding an atomic nucleus, or they may break free and exist as a free electron.
- **protons:** A stable, positively charged, sub-atomic particle, found in atomic nuclei in numbers equal to the atomic number of the element.
- **neutral:** A neutral particle, object, or system is one that has a net electric charge of zero.
- **conductors:** Materials through which electric charge can pass.
- **insulator:** Substances that block or retard the flow of electrical current or charge.
- **positive ions:** An atom or a group of atoms that has acquired a net positive charge by losing one or more electrons.
- **negative ions:** An atom or a group of atoms that has acquired a net negative charge by gaining one or more electrons.
- **ions:** An atom or a group of atoms that has acquired a net electric charge by gaining or losing one or more electrons.
- **electroscope:** An instrument used to detect the presence and sign of an electric charge by the mutual attraction or repulsion of metal foils.
- **charging by conduction:** Involves the contact of a charged object to a neutral object.
- **charging by induction:** A method used to charge an object without actually touching the object to any other charged object.

References

1. Courtesy of NOAA Photo Library, NOAA Central Library; OAR/ERL/National Severe Storms Laboratory (NSSL). http://www.photolib.noaa.gov/htmls/nssl0016.htm . Public Domain
2. Sweater: Image copyright Sibiryanka, 2013; Balloon: Image copyright simpleman, 2013. http://www.shutterstock.com . Used under licenses from Shutterstock.com
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CONCEPT 5 Coulomb's Law

- State Coulomb's Law.
- Describe how electric force varies with charge and separation of charge.
- State the SI unit of charge.
- Solve problems using Coulomb's Law.

Electric cars are becoming more popular. One large advantage for electric cars is the low cost of operation, which may become an ever bigger advantage as gas prices climb. Energy costs for electric cars average about one-third of the cost for gasoline engine cars, but they can only travel about 200 miles per charge at this point. These cars run using the science of electrical charges and forces.

Coulomb's Law

The questions regarding the relationship between the electrical force, the size of the charge, and the separation between the charges were solved by Charles Coulomb in 1785. He determined that electrical force between two charges is directly related to the size of the charges and inversely proportional to the distance between the charges. This is known as **Coulomb's Law.**

$F_e = \frac{Kq_1q_2}{d^2}$

In this equation, q_1 and q_2 are the two charges, d is the distance between the two charges, and K is a constant of proportionality. F_e is the **electric force**, which occurs as a result of interactions between two charged particles. For the purpose of calculating electric forces, we assume all charge is a **point charge**, in which the entire charge of the particle is located in a massless point.

The SI unit of charge is the coulomb, C, which is the charge of 6.25×10^{18} electrons. The charge on a single electron is $1.60 \times 10^{-19}\ C$. The charge on a single electron is known as the **elementary charge**. The charge on a proton is the same magnitude but opposite in sign. When the charges are measured in coulombs, the distance in meters, and the force in Newtons, the constant K is $9.0 \times 10^9\ N \cdot m^2/C^2$.

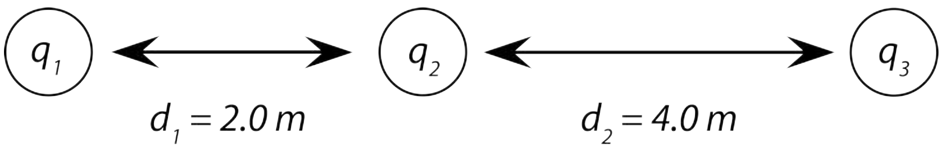
The electrical force, like all forces, is a vector quantity. If the two charges being considered are both positive or both negative, the sign of the electrical force is positive and this force is repulsive. If the two charges are opposite in sign, the force will have a negative sign and the force is attractive.

Example Problem: Object A has a positive charge of $6.0 \times 10^{-6}\ C$. Object B has a positive charge of $3.0 \times 10^{-6}\ C$. If the distance between A and B is 0.030 m, what is the force on A?

Solution: $F_e = \frac{Kq_1q_2}{d^2} = \frac{(9.0\times10^9\ N\cdot m^2/C^2)(6.0\times10^{-6}\ C)(3.0\times10^{-6}\ C)}{(0.030\ m)^2} = 180\ N$

The positive sign of the force indicates the force is repulsive. This makes sense, because both objects have a positive charge.

Example Problem: In the sketch below, the charges are $q_1 = 10.0 \times 10^{-6}\ C$, $q_2 = 2.0 \times 10^{-6}\ C$, and $q_3 = -6.0 \times 10^{-6}\ C$. Calculate the total force on q_2 .



Solution: $F_e = \frac{Kq_1q_2}{d^2} = \frac{(9.0\times10^9\ N\cdot m^2/C^2)(10.0\times10^{-6}\ C)(2.0\times10^{-6}\ C)}{(2.0\ m)^2} = 0.045\ N$ (towards q_3)

$F_e = \frac{Kq_2q_3}{d^2} = \frac{(9.0\times10^9\ N\cdot m^2/C^2)(2.0\times10^{-6}\ C)(-6.0\times10^{-6}\ C)}{(4.0\ m)^2} = -0.007\ N$ (towards q_3)

Since the two forces act in the same direction, their absolute values can be added together; the total force on q_2 is 0.052 N towards q_3 .

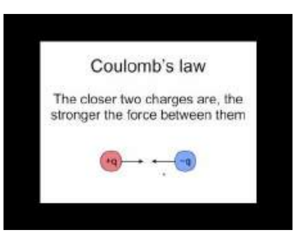
Summary

- Coulomb determined that electrical force between two charges is directly related to the size of the charges and inversely proportional to the distance between the charges: $F_e = \frac{Kq_1q_2}{d^2}$
- The SI unit of charge is the coulomb, C , which is the charge of 6.25×10^{18} electrons.
- The charge on a single electron is $1.60 \times 10^{-19}\ C$ and is known as the elementary charge.
- The electrical force is a vector quantity that is positive in repulsion and negative in attraction.

Practice

The following video covers Coulomb's Law. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=rYjo774UpHI



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Click image to the left for more content.

1. What happens when like charges are placed near each other?
2. What happens when opposite charged are placed near each other?
3. What happens to the force of attraction if the charges are placed closer together?

Practice problems on Coulomb's Law.

http://physics.info/coulomb/problems.shtml

Review

1. Suppose that two point charges, each with a charge of +1.00 C, are separated by a distance of 1.0 m:
   (a) Will the charges attract or repel?
   (b) What is the magnitude of the force between them?
   (c) If the distance between them is doubled, what does the force become?
2. What is the electrical force between two balloons, each having 5.00 C of charge, that are 0.300 m apart?
3. Two spheres are charged with the same charge of -0.0025 C and are separated by a distance of 8.00 m. What is the electrical force between them?
4. A red foam ball and a blue foam ball are 4.00 m apart. The blue ball has a charge of 0.000337 C and is attracting the red ball with a force of 626 N. What is the charge on the red ball?

- **Coulomb's Law:** States the force of attraction or repulsion acting along a straight line between two electric charges is directly proportional to the product of the charges and inversely to the square of the distance between them.

References

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CONCEPT 6 The Electric Field

- Define an electric field.
- Solve problems relating to field, force, and charge.

A plasma globe, such as the one pictured above, is filled with a mixture of noble gases and has a high-voltage electrode at the center. The swirling lines are electric discharge lines that connect from the inner electrode to the outer glass insulator. When a hand is placed on the surface of the globe, all the electric discharge travels directly to that hand.

The Electric Field

Coulomb's Law gives us the formula to calculate the force exerted on a charge by another charge. On some occasions, however, a test charge suffers an electrical force with no apparent cause. That is, as observers, we cannot see or detect the original charge creating the electrical force. Michael Faraday dealt with this problem by developing the concept of an **electric field.** According to Faraday, a charge creates an electric field about it in all directions. If a second charge is placed at some point in the field, the second charge interacts with the field and experiences an electrical force. Thus, the interaction we observe is between the test charge and the field and a second particle at some distance is no longer necessary.

The strength of the electric field is determined point by point and can only be identified by the presence of test charge. When a positive test charge, $q\ t$, is placed in an electric field, the field exerts a force on the charge. The field strength can be measured by dividing the force by the charge of the test charge. Electric field strength is given the symbol E and its unit is Newtons/coulomb.

$$E = \frac{F_{onq_t}}{q_t}$$

The test charge can be moved from location to location within the electric field until the entire electric field has been mapped in terms of **electric field intensity.**

Example Problem: A positive test charge of 2.0×10^{-5} C is placed in an electric field. The force on the test charge is 0.60 N. What is the electric field intensity at the location of the test charge?

Solution: $E = \frac{F}{q} = \frac{0.60\ \mathbf{N}}{2.0 \times 10^{-5}\ \mathbf{C}} = 3.0 \times 10^{4}$ **N/C**

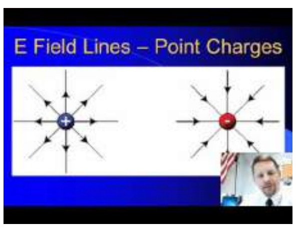
Summary

- An electric field surrounds every charge and acts on other charges in the vicinity.
- The strength of the electric field is given by the symbol E , and has the unit of Newtons/coulomb .
- The equation for electric field intensity is $E = \frac{F}{q}$.

Practice

The following video covers electric fields. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=lpb94QF0_mM



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1. What does it mean when a force is called a non-contact force?
2. What symbol is used to represent electric field strength?
3. What is the relationship between the direction of the electric field and the direction of the electric force?

Review

1. The weight of a proton is 1.64×10^{-26} N . The charge on a proton is $+1.60 \times 10^{-19}$ C . If a proton is placed in a uniform electric field so that the electric force on the proton just balances its weight, what is the magnitude and direction of the field?
2. A negative charge of 2.0×10^{-8} C experiences a force of 0.060 N to the right in an electric field. What is the magnitude and direction of the field?
3. A positive charge of 5.0×10^{-4} C is in an electric field that exerts a force of 2.5×10^{-4} N on it. What is the magnitude of the electric field at the location of the charge?
4. If you determined the electric field intensity in a field using a test charge of 1.0×10^{-6} C and then repeated the process with a test charge of 2.0×10^{-6} C , would the forces on the charges be the same? Would you find the value for E ?
5. A 0.16 C charge and a 0.04 C charge are separated by a distance of 3.0 m. At what position between the two charges would a test charge experience an electric field intensity of zero?

- **electric field:** A region of space characterized by the existence of a force that is generated by an electric charge.
- **electric field intensity:** Electric field intensity or field strength is described as the ratio of force to the amount of test charge.

References

1. User:Slimsdizz/Wikipedia. http://commons.wikimedia.org/wiki/File:Glass_plasma_globe.jpg . Public Domain

CONCEPT 7 Electric Potential

Objectives

The student will:

- Understand how to solve problems using electric potential energy.
- Understand how to solve problems using voltage differences.
- Understand how to solve problems in a uniform electric field.

Vocabulary

- **electric potential:** Energy per unit charge.
- **electric potential difference:** The difference in electric potential between two points within an electric field.
- **voltage:** The amount of work done by the electric field per unit charge in moving a charge between two points in the electric field $\Delta V = \frac{W}{q}$, also known as a change in potential energy.

Introduction

In order to draw an analogy between **gravitational potential energy** and **electrical potential energy** , we liken the electric field E to the gravitational acceleration g and the mass m of a particle to the charge q of a particle. Of course, g is assumed constant (uniform) when we remain close to the surface of the Earth. As of yet, we have not encountered an example of a uniform electric field E .

But that won't stop us from making a prediction!

Since the gravitational potential energy of a mass m in a uniform gravitational field is $PE_{gravity} = (mg)h$, we predict the electric potential energy $(PE_{electric})$ of a charge (q) in a constant electric field is $PE_{electric} = (qE)h$.

Furthermore, the **electric potential** (the energy per unit charge) can be defined as $V_e = \frac{PE_{electric}}{q}$ (The subscript "e" will be dropped from now on.) We will discuss electric potential later.

It must be understood that, just as with gravity, the electric potential energy and the electric potential are measured at the same point. If a point charge q has electric potential energy PE_{x_1} at point x_1 , the electric potential at x_1 is V_{x_1}

$$PE_{x_1} = qV_{x_1} \rightarrow V_{x_1} = \frac{PE_{x_1}}{q}$$

Again, only differences in electric potential and electric potential energy are meaningful. That is, ΔPE or $\Delta V \rightarrow V_f - V_i = \frac{PE_f}{q} - \frac{PE_i}{q}$.

The unit of electric potential is called the volt and from the definition above we see that the volt is equivalent to $\frac{Joules}{Coulomb} \rightarrow V = \frac{J}{C}$.

Electric Potential Difference

The **electric potential difference** is the difference in electric potential between two points within an electric field. For example, a 1.5-volt battery has a potential difference of 1.5 volts (written 1.5 V) between its positive and negative terminals.

Parallel Plate Conductors: A uniform Electric Field

The equation $E = k\frac{q}{r^2}$ for the electric field holds for point charges or for a charge distribution that effectively acts as a point charge. It turns out, however, that if opposite charges are placed on two parallel conducting plates, the electric field between the plates is more or less uniform as long as the distance between the plates is much smaller than the dimensions of the plates. The plates can be charged by connecting them to the positive and negative terminals of a battery.

A battery contains a substance (called an electrolyte) which causes two dissimilar metals to acquire opposite charges. The two dissimilar metals form the positive and negative terminals of the battery. If a metal plate is connected to the positive terminal of the battery, and another metal plate is connected to the negative terminal of the battery, and the two plates brought closely together, a parallel plate arrangement (parallel-plate conductors) can be constructed with a uniform electric field between the plates (seen edge on) in **Figure** below . We will see later that parallel plate conductors are also referred to as capacitors .

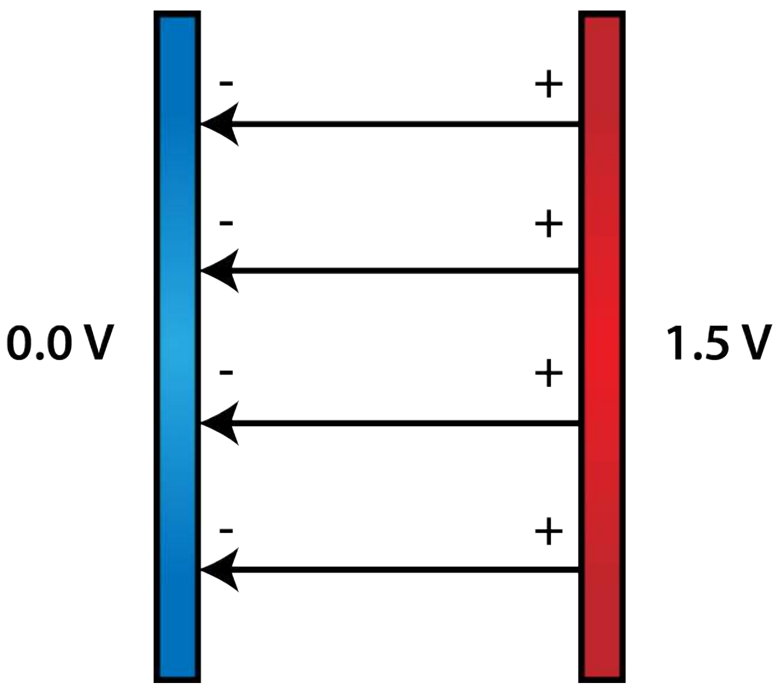


FIGURE 7.1
Parallel plates.

Just as in the case of the battery, one of the plates of the parallel-plate conductor will be at a higher potential than the other plate. Think, for example, of a standard AA battery with a **voltage** rating of 1.5V, **Figure** below .

See the link below to learn more about how a battery works.

http://phet.colorado.edu/en/simulation/battery-voltage

FIGURE 7.2
volt battery

Electrical Potential Energy

In our gravitational analogy, the energy that a charge possesses at the plate with the higher potential is analogous to the energy a mass possesses above the ground. Additionally, now that we have found a way to create a uniform electric field, we have an analog to a uniform gravitational field.

If a positive charge $+q$ is placed at the positive plate in **Figure** below , it will be repelled by the positive charges on the plate and move toward the negative plate. (Think of $+q$ as the object m falling toward the ground.)

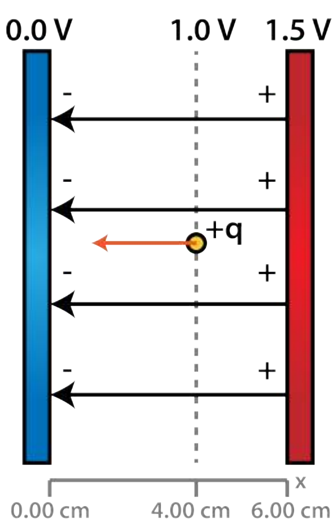


FIGURE 7.3
A positive charge moving toward the negative plate

What is the force acting on the $+q$ charge? Recall that the Coulomb force on a charge placed in an electrostatic field is $F = qE$. The work that the electric field does on the charge is equal to the negative change in the potential energy of the charge, just as in the gravitational case. We can find an expression for the electric potential energy by finding the work that is done on the charge. Recall that $W = F\Delta x$. We write

$$W_{field} = F\Delta x = (qE)\Delta x = -\Delta PE \rightarrow$$
$$qE(x_f - x_i) = -\Delta PE \rightarrow$$
$$qEx_f - qEx_i = -\Delta PE$$

The expression for the electric potential energy is thus: $PE_{electrical} = qEx$.

Recall that the equation for the gravitational potential energy is $PE_{gravitational} = mgh$.

We can compare the terms in the gravitational and electrical cases as follows:

$$m \rightarrow q$$
$$g \rightarrow E$$
$$h \rightarrow x$$

Thus, we see that our prediction for the equation of electric potential energy stated in the introduction of the lesson, was correct!

Check Your Understanding

1a. The electrical potential at the negative plate in **Figure** above is defined as zero volts. What is the electrical potential energy of a charge $+q = 15.0\mu C$ at the positive plate if the electric field between the plates is $25.0\frac{N}{C}$?

The positive plate has position $6.00cm = 6.00 \times 10^{-2}\ m$ according to **Figure** above .

Answer : $PE_{positive\ plate} = qEx = (15.0 \times 10^{-6}\ C)\left(25.0\frac{N}{C}\right)(6.00 \times 10^{-2}\ m) = 3.75 \times 10^{-4}\ J$

1b. What is the change in the electrical potential energy ΔPE of the charge $+q = 15.0\mu C$ if its potential changes from 1.5 V to 1.0 V ?

Answer : Just as in the case of a change in gravitational potential energy, the charge must lose potential energy, since it gains kinetic energy.

The charge moves from the position $x_i = 6.00 \times 10^{-2}\ m\ (1.5\ V)$ to the position $x_i = 4.00 \times 10^{-2}\ m\ (1.0\ V)$.

$$\Delta PE = qEx_f - qEx_i = qE(x_f - x_i) =$$
$$(15.0 \times 10^{-6}\ C)\left(25.0\frac{N}{C}\right)(4.00 \times 10^{-2}\ m - 6.00 \times 10^{-2}\ m) = -7.50 \times 10^{-6}\ J.$$

1c. What is the work done on the charge by the electric field?

Answer:

$$W_{field} = -\Delta PE = -(-7.50 \times 10^{6}) = 7.50 \times 10^{6}\ J$$

Notice that the electric field does positive work on the charge, since the electric force and the displacement of the charge have the same direction.

We should recall a very important point: It is only the change in potential energy that is meaningful, whether we are discussing the gravitational potential energy or the electrical potential energy.

2. An electron placed at the negative plate of a parallel-plate conductor will move toward the positive plate. The potential energy of the electron:

A. Decreases

B. Increases

C. Remains the same.

Answer : The correct answer is A. The electron is repelled by the negative charges of the conducting plate and therefore gains kinetic energy. Just as an object that is dropped gains kinetic energy and loses potential energy, so does the electron. Recall our discussion of the conservation of energy. As long as the total energy remains conserved, the sum of the initial kinetic and potential energies must equal the sum of the final kinetic and potential energies:

$$KE_i + PE_i = KE_f + PE_f \rightarrow \Delta KE = -\Delta PE$$

The gain in kinetic energy occurs due to the loss in potential energy.

In order for the charges of the same sign to be brought together, as in the example above, positive work must be done by an external force against the electrostatic repulsion between the charges. The work increases the potential energy stored in the electric field. When the charges are released, the potential energy of the field is converted into the kinetic energies of the charges. The link below may be helpful in learning more about the work done upon charges in electric fields.

http://www.youtube.com/watch?v=elJUghWSVh4

Electric Potential Difference in a Uniform Electric Field

In working with the change in potential energy above, we wrote the equation

$\Delta PE = qEx_f - qEx_i = qE(x_f - x_i) \rightarrow$ Let us call this Equation A.

Recall that the electric potential was defined at a specific point $V_{x_1} = \frac{PE_{x_1}}{q}$.

We therefore see that $PE_{x_1} = qV_{x_1} \rightarrow$ Let us call this Equation B.

Comparing Equation A and Equation B, we see that the electric potential can be expressed as $V_{x_1} = Ex_1$. If the electric potential is defined as $V = 0$ at $x = 0$, then the potential at any point in the electric field is $V = Ex$. (Assuming that vector E is directed along the $x-$ axis).

Note: It is common to write $V = Ed$, where V is understood to mean the voltage (or potential difference) between the plates of a parallel-plate conductor, and d is the distance between the plates.

Check Your Understanding

Verify that the potential difference between the plates in **Figure** above is 1.5 V . Recall that the electric field is $E = 25.0\frac{N}{C}$.

Answer : $V = Ed = \left(25\frac{N}{C}\right)(6.00 \times 10^{-2}\ m - 0.00m) = 1.5\ V$

Work

We state again:

1. The electric potential is defined as the energy per unit charge $\rightarrow V_{x_1} = \frac{PE_{x_1}}{q}$.

2. The electric potential difference (the voltage) is $V_f - V_i = \frac{PE_f}{q} - \frac{PE_i}{q}$
3. An arbitrary reference level must be established for zero potential (just as in the case of gravitational potential energy).
4. The units of electric potential and electric potential difference are $\frac{J}{C}$ since $V_{x_1} = \frac{PE_{x_1}}{q}$.

It is often useful to express the voltage in terms of the work done on a charge.

From $V_f - V_i = \frac{PE_f}{q} - \frac{PE_i}{q}$, we have $PE_f - PE_i = q(V_f - V_i) \rightarrow \Delta PE = q(V_f - V_i)$.

But the work done on a charge by the field is $W_{field} = -\Delta PE$.

Combining $\Delta PE = q(V_f - V_i)$ and $W_{field} = -\Delta PE$ gives $W_{field} = -q(V_f - V_i)$.

An external force that does work on a charge in an electric field exerts a force in the opposite direction to the field (just as the external force acting on a spring acts opposite to the spring force). The work that an external force does is therefore $W_{external\ force} = q(V_f - V_i)$.

The voltage can be thought of as the amount of work done by the electric field per unit charge in moving a charge between two points in the electric field $\Delta V = \frac{W}{q}$.

We often refer to a change in potential as simply "the voltage."

In computing the work, it is often easier to ignore the sign in the equation and simply see if the force and displacement on the charge are in the same direction (positive work) or opposite to one another (negative work). Recall that the force and displacement need not be in the same direction or oppositely directed. In general, work is expressed as $W = Fx\cos\theta$

http://www.youtube.com/watch?v=F1p3fgbDnkY

Other Units for the Electric Field

The electric field has units $\frac{N}{C}$, since $\overrightarrow{E} = \frac{\overrightarrow{F}}{q}$. But the electric field has been also defined using the scalar equation $V = Ex$. Transposing terms, the electric field is $E = \frac{V}{x}$. So the units of the electric field can be also expressed as volts per meter $\frac{volts}{meter} \rightarrow \frac{V}{m}$.

If we compare the units for the electric field $\frac{N}{C}$ and $\frac{V}{m}$, we see that a $(N * m)$ is equivalent to a $(C * V)$. A Joule can therefore be expressed as a Coulomb-Volt. Recall that work, measured in Joules, is the product of charge and voltage $W = q\Delta V$.

Illustrative Example 16.2.1

All questions refer to **Figure** above .

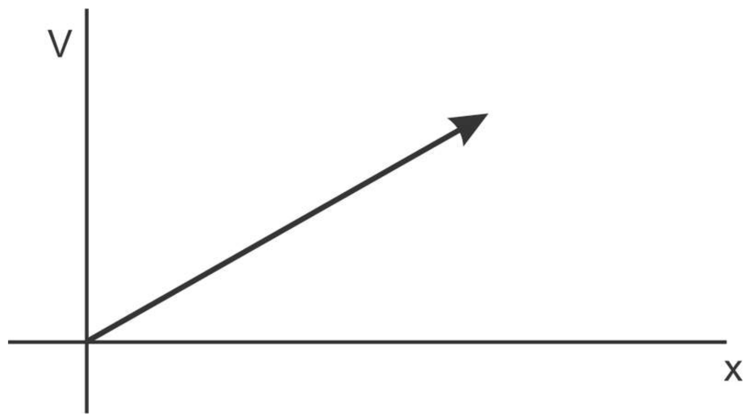
a. What is the potential at $x = 2.0\ cm$ in **Figure** below ? The electric field is $E = 25.0\frac{N}{C}$.

Answer :

The potential V varies directly with the position x between the plates $(V = Ex)$. Thus, $V = 25.0\frac{N}{C}(2.0 \times 10^{-2} m) = 0.50\ V$.

b. Sketch a graph showing the relationship between the potential and the position.

Answer:



c. How much work is done by an external force F moving a $-2.0 \times 10^{-6}\ C$ charge from the positive plate to the negative plate?

Answer: An external force must pull the charge away from the positive plate so the force will be in the same direction as the displacement.

$$W = q(V_f - V_i) = (-2.0 \times 10^{-6} C)(0.00V - 1.50V) = 3.0 \times 10^{-6}\ J$$

d. What is the magnitude of the Coulomb force acting on the charge?

Answer:

$$W = F\times \rightarrow 3.0 \times 10^{-6}\ J = F(6.00 \times 10^{-2}\ m)$$
$$F = \frac{3.0 \times 10^{-6}\ J}{6.00 \times 10^{-2}\ m} = 5.0 \times 10^{-5}\ N$$

Illustrative Example 16.2.2

a. A particle of mass m of $2.00 \times 10^{-5}\ kg$ is has a charge q of $+3.00 \times 10^{-3}\ C$. If the particle is released from the positive plate of a parallel-plate conductor with an electric field E of $1.30 \times 10^5\ \frac{N}{C}$, determine the acceleration of the particle, see **Figure** below .

Answer :

If we ignore gravity, the only force acting on the particle is the electric force $F = qE$. Using Newton's Second Law, the net force on the particle is equal to $\sum F = ma \rightarrow qE = ma$.

The acceleration is $a = \frac{Eq}{m} = \frac{\left(1.30\times 10^5\ \frac{V}{m}\right)(3.00\times 10^{-3}\ C)}{2.00\times 10^{-5}\ kg} = 1.95 \times 10^7\ \frac{V*C}{kg*m}$.

b. Show that the units $\frac{V*C}{kg*m}$ are equivalent to the units $\frac{m}{s^2}$.

Answer :

$$\frac{V*C}{kg*m} = \frac{J}{kg*m} = \frac{N*m}{kg*m} = \frac{N}{kg} = \frac{kg*\frac{m}{s^2}}{kg} = \frac{m}{s^2}$$

c. The plates have separation of 8.00 mm. Determine the velocity of the particle when it reaches the negative plate

Answer :

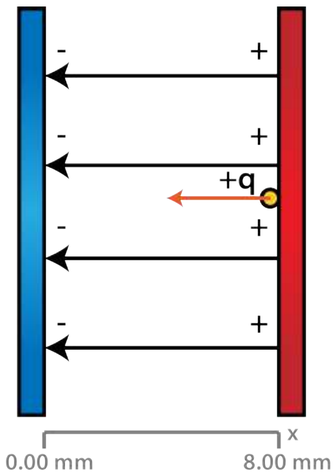


FIGURE 7.4
Illustrative Example 16.3.2

This is a kinematics problem, where the displacement and acceleration are known and the velocity is to be found. Recall the equation $v_f^2 = v_i^2 + 2a\Delta x$.

$$\rightarrow v_f^2 = 0 + 2\left(1.95 \times 10^7 \ \frac{m}{s^2}\right)(8.00 \times 10^{-3} \ m) = 312,000\frac{m^2}{s^2}$$
$$v = 558.6 \rightarrow 5.59 \times 10^2 \ \frac{m}{s}.$$

d. What is the potential difference between the plates?

Answer :

$$V = Ex = \left(1.30 \times 10^5 \ \frac{V}{m}\right)(8.00 \times 10^{-3} \ m) = 1.04 \times 10^3 \ V$$

e. How much work has the field done on the particle as it moved from one plate to the other?

$$W = q\Delta V = (3.00 \times 10^{-3} \ C)(1.04 \times 10^3 \ V) = 3.12 \ J$$

Illustrative Example 16.2.3

An electron is accelerated from rest through a potential difference of 30,000 V. The mass of the electron is $9.11 \times 10^{-31} \ kg$ and the charge of the electron is $1.60 \times 10^{-19} \ C$. Find its velocity.

Answer : Recall that the Work-Energy Principle states that $W = \Delta KE$.

$$W = \Delta KE$$
$$W = q\Delta V$$
$$\Delta KE = q\Delta V$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = q\Delta V$$
$$v_i = 0 \rightarrow v_f^2 = \frac{2q\Delta V}{m} \rightarrow v_f = \sqrt{\frac{2q\Delta V}{m}} \rightarrow$$
$$v_f = \sqrt{\frac{2(1.60 \times 10^{-19}\ C)(3.00 \times 10^4\ V)}{9.11 \times 10^{-31}\ kg}} = 1.026 \times 10^8 \rightarrow 1.03 \times 10^8 \frac{m}{s}$$

Illustrative Example 16.2.4

What magnitude of an electric field is required to balance the gravitational force acting on an electron in **Figure** below ?

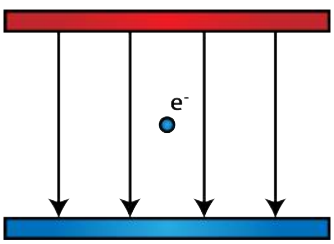
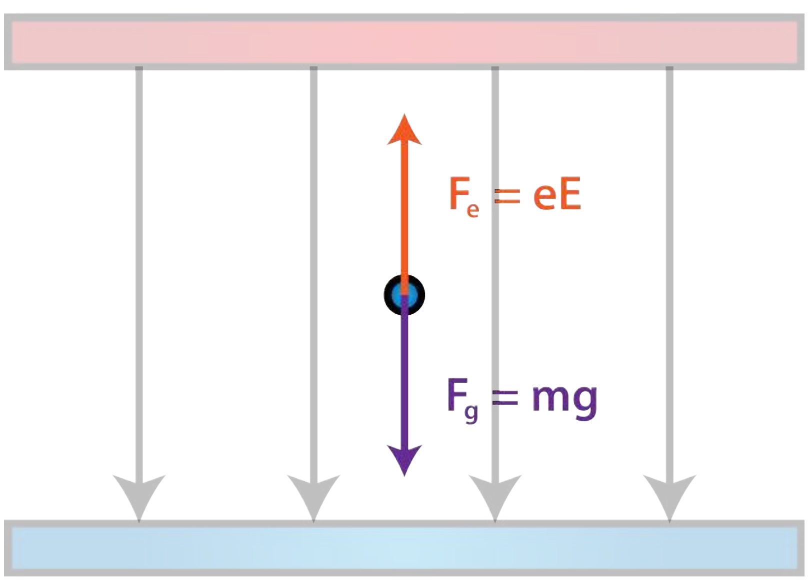


FIGURE 7.5

Illustrative Example 16.2.4-An electron suspended in an electric field.

Answer :

Draw a Free-Body-Diagram (FBD) of the situation. The electrostatic force that acts on the electron points upward and the gravitational force that acts upon on the electron points downward. The electron is suspended motionless (or moves with a constant velocity) when the net force on the electron is zero.



The net force on the electron must be zero, thus

$$\begin{aligned}
\sum F = 0 \rightarrow eF = mg \rightarrow F = \frac{mg}{e} \\
but\ F = eE \rightarrow \frac{mg}{e} = eE \rightarrow E = \frac{mg}{e^2} \rightarrow \\
E = \frac{(9.11 \times 10^{-31}\ kg)\left(9.81 \frac{m}{s^2}\right)}{(1.60 \times 10^{-19}\ C)^2} = 3.49 \times 10^8\ \frac{V}{m}
\end{aligned}$$

http://www.youtube.com/watch?v=wT9AsY79f1k

The Electron-Volt

It is often convenient when dealing with small particles such as electrons, protons, and ions to express the energy of these particles with a smaller unit of measure. The electron-volt is defined as the change in potential energy that an electron acquires when moving through a potential difference of 1 V, or equivalently, its change in kinetic energy after moving through a potential difference of 1 V .

That is, $PE = eV = (1.60 \times 10^{-19}\ C)(1.00\ V) = 1.60 \times 10^{-19}\ J$.

The energy $1.60 \times 10^{-19}\ J$ is defined as one electron-volt. We write one-electron-volt as $1\ eV = 1.60 \times 10^{-19}\ J$.

Check Your Understanding

1. What is the change in kinetic energy KE when an electron is released at the negative plate of a parallel plate conductor with a potential difference of 3,500 V? Express your answer in eV.

Answer : The electron is repelled at the negative plate and therefore gains kinetic energy (and loses potential energy). The change in KE is positive and equal to

$$\begin{aligned}
\frac{1\ eV}{1\ V} = \frac{x\ eV}{3,500\ V} \rightarrow x = 3,500\ eV \\
\Delta KE = 3,500\ eV
\end{aligned}$$

It is simplest to think that for every one volt of potential difference the particle experiences, it gains (or loses) 1 eV .

2. An alpha-particle (the nucleus of a helium atom) is fired toward the positive plate of a parallel plate conductor and passes through a potential difference of 1,500 V. What is the change in its kinetic energy? Express your answer in eV.

Answer :

Protons are the only charges inside the nucleus of an atom and so the alpha particle must be positively charged. A helium nucleus contains two protons (and two neutrons) with a total charge of $2(1.60 \times 10^{-19}\ C)$.

The alpha particle must slow down due to the electrostatic repulsion from the positive plate. It must, therefore, lose kinetic energy and gain potential energy.

Each proton loses 1,500 eV of kinetic energy. $\Delta KE = -3,000\ eV$.

3. An electron and a proton both gain kinetic energy of 1 eV .

True or False: Their speeds must be the same, since they both gained the same amount of energy.

Answer : False. The mass of a proton is nearly 2000 times greater than the mass of an electron. Remember that kinetic energy depends on both the speed and mass of an object. Therefore, the final speed of the electron will be much greater.

Illustrative Example 16.2.5

a. An electron and a proton both gain kinetic energy of 1 eV. What is the ratio of the electron's speed to the proton's speed?

Answer :

As discussed above, though both particles gain the same kinetic energy, their speeds will not be the same, since they have different masses. The mass of the proton is nearly 2000 times as great as the electron's so:

$$\frac{KE_e}{KE_p} = \frac{\frac{1}{2}m_e v_e^2}{\frac{1}{2}m_p v_p^2} = 1 \rightarrow m_e v_e^2 = m_p v_p^2 \rightarrow \frac{v_e^2}{v_p^2} = \frac{m_p}{m_e} = \frac{2000m_e}{m_e} = 2,000 \rightarrow$$
$$\frac{v_e}{v_p} = \sqrt{2,000} = 44.7 \rightarrow 45$$

The electron will move about 45 times faster than the proton.

b. What is the speed of a proton which has a kinetic energy of 37 MeV ?

The mass of a proton is $1.67 \times 10^{-27}\ kg$.

Answer :

Because the electron-volts are a very small unit, they are typically expressed in KeV (1000 electron-volts) and MeV (one million electron-volts). The electron-volt is a convenient unit of measure but it is not an SI unit. In order to find the velocity of a particle if its energy is given in units of eV , we must convert back into Joules.

$$37\ MeV = (37 \times 10^6)(1.60 \times 10^{-19}\ J) = 5.92 \times 10^{-14}\ J$$
$$\frac{1}{2}m_p v^2 = 5.92 \times 10^{-14}\ J \rightarrow \frac{1}{2}(1.67 \times 10^{-27}\ kg)v^2 = 5.92 \times 10^{-14}\ J \rightarrow$$
$$v = 8.4 \times 10^6\ \frac{m}{s}$$

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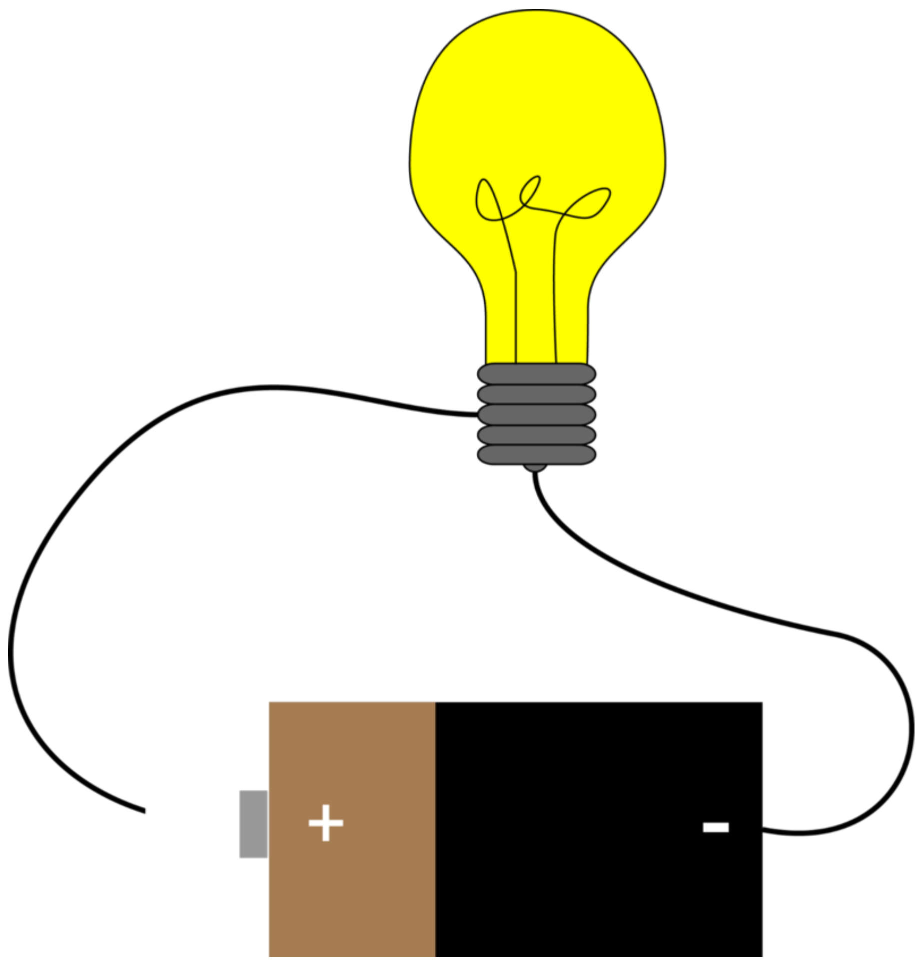
CONCEPT 1 Electric Circuits

- Define electric circuit.
- Describe the parts of an electric circuit.
- Show how to represent a simple electric circuit with a circuit diagram.



Jose made this sketch of a battery and light bulb for science class. If this were a real set up, the light bulb wouldn't work. The problem is the loose wire on the left. It must be connected to the positive terminal of the battery in order for the bulb to light up.

Q: Why does the light bulb need to be connected to both battery terminals?

A: Electric current can flow through a wire only if it forms a closed loop. Charges must have an unbroken path to follow between the positively and negatively charged parts of the voltage source, in this case, the battery.

Electric Circuit Basics

A closed loop through which current can flow is called an **electric circuit** . In homes in the U.S., most electric circuits have a voltage of 120 volts. The amount of current (amps) a circuit carries depends on the number and power of electrical devices connected to the circuit. Home circuits generally have a safe upper limit of about 20 or 30 amps.

Parts of an Electric Circuit

All electric circuits have at least two parts: a voltage source and a conductor. They may have other parts as well, such as light bulbs and switches, as in the simple circuit seen in the **Figure** 1.1 . To see an animation of a circuit like this one, go to: http://www.rkm.com.au/animations/animation-electrical-circuit.html

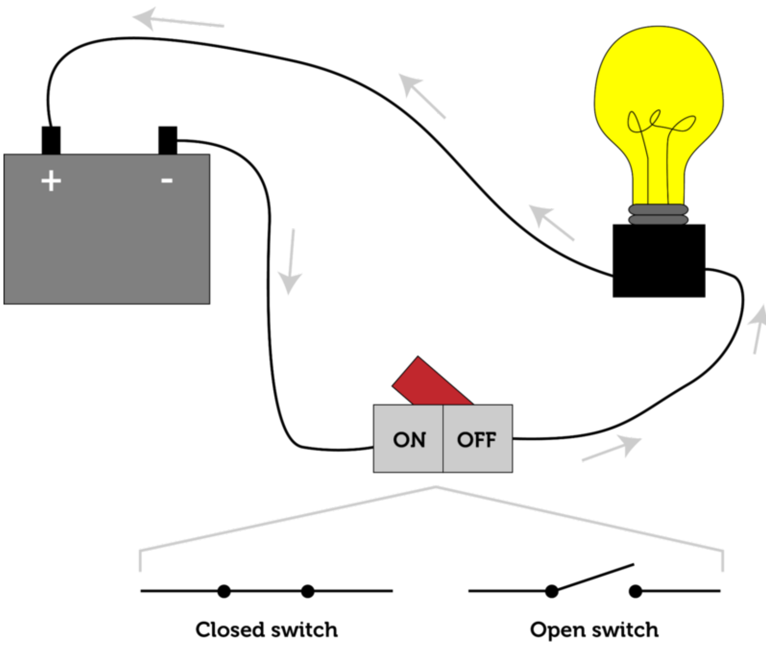


FIGURE 1.1

- The voltage source of this simple circuit is a battery. In a home circuit, the source of voltage is an electric power plant, which may supply electric current to many homes and businesses in a community or even to many communities.
- The conductor in most circuits consists of one or more wires. The conductor must form a closed loop from the source of voltage and back again. In the circuit above, the wires are connected to both terminals of the battery, so they form a closed loop.
- Most circuits have devices such as light bulbs that convert electrical energy to other forms of energy. In the case of a light bulb, electrical energy is converted to light and thermal energy.
- Many circuits have switches to control the flow of current. When the switch is turned on, the circuit is closed and current can flow through it. When the switch is turned off, the circuit is open and current cannot flow through it.

Circuit Diagrams

When a contractor builds a new home, she uses a set of plans called blueprints that show her how to build the house. The blueprints include circuit diagrams. The diagrams show how the wiring and other electrical components are to be installed in order to supply current to appliances, lights, and other electric devices. You can see an example of a very simple circuit in the **Figure** 1.2 . Different parts of the circuit are represented by standard circuit symbols. An ammeter measures the flow of current through the circuit, and a voltmeter measures the voltage. A resistor is any device that converts some of the electricity to other forms of energy. For example, a resistor might be a light bulb or doorbell.

The circuit diagram on the right represents the circuit drawing on the left. Below are some of the standard symbols used in circuit diagrams.

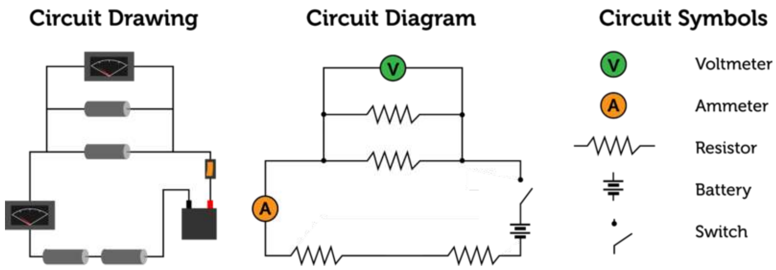


FIGURE 1.2

Q: Only one of the circuit symbols above must be included in every circuit. Which symbol is it?

A: The battery symbol (or a symbol for some other voltage source) must be included in every circuit. Without a source of voltage, there is no electric current.

Summary

- An electric circuit is a closed loop through which current can flow.
- All electric circuits must have a voltage source, such as a battery, and a conductor, which is usually wire. They may have one or more electric devices as well.
- An electric circuit can be represented by a circuit diagram, which uses standard symbols to represent the parts of the circuit.

Vocabulary

- **electric circuit** : Closed loop through which current can flow.

Practice

Take the electric circuit quiz at the following URL. Be sure to have your answers corrected. Try the quiz again if any of your answers are incorrect. http://www.myschoolhouse.com/courses/O/1/68.asp

Review

1. What is an electric circuit?
2. Which two parts must all electric circuits contain?
3. Sketch a simple circuit that includes a battery, switch, and light bulb. Then make a circuit diagram to represent your circuit, using standard circuit symbols.

References

1. Christopher Auyeung. . CC BY-NC 3.0
2. Christopher Auyeung. . CC BY-NC 3.0

CONCEPT 2 Electric Current

Objectives

The student will:

- Understand how electric current is defined
- Solve problems involving electric current

Vocabulary

- **electric current:** A flow of charges under the influence of an electric field, such as between the terminals of a battery. The rate $I = \frac{\Delta Q}{\Delta t}$ at which charges flow within a conducting wire past any point in the wire.

Introduction

The term **electrical current** is familiar to most people. Many electrical devices have electrical specifications printed on them. **Figure** below shows a typical AC adapter ("plug") with its "specs." Can you guess what the terms 5 VDC and 500 mA printed on the adapter mean?

FIGURE 2.1
An electrical plug.

Electric Current

An electric current is a flow of charges under the influence of an electric field. A flow of charges can be established, for instance, between the terminals of a battery, as in **Figure** below . The rate $I = \frac{\Delta Q}{\Delta t}$ at which charges flow within a conducting wire past any point in the wire is defined as the electric current.

The unit of current is $\frac{coulombs}{second}$ which is called the *ampere* or *amp* (named for the French physicist Andre'-Marie Ampere, 1775-1836), **Figure** below .

The symbol A is used to represent the ampere. A rate of one coulomb per second is equivalent to one ampere: $\frac{1C}{1s} = 1A$

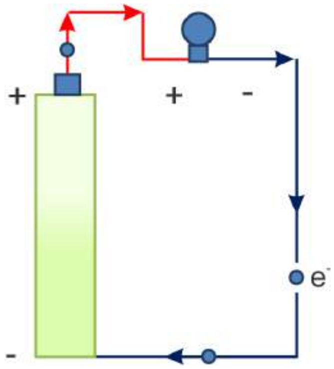


FIGURE 2.2

Figure above shows a flow of electrons (e^-) from the positive terminal of a battery through a lightbulb to the negative terminal of a battery.

FIGURE 2.3
Andre'-Marie Ampere

One ampere is a very large current. The current of 1 A can easily kill a person. In fact, about 0.20 A can kill rather easily. Even relatively small voltage can produce these currents, which is why care must always be taken when dealing with all electrical appliances and any electrical device that is plugged into a wall outlet. A typical 12-V car battery can also be dangerous. Under the right circumstances, it does not take a huge voltage to cause deadly currents.

It is common to express current in milliamperes 1 mA $= 10^{-3}$ A , or microamperes 1 ¯A $= 10^{-6}$ A .

http://demonstrations.wolfram.com/ElectricCurrent/

Illustrative Example 17.4.1

A total of 7.9×10^{12} electrons move past a point in a conducting wire every 1.45 s. What is the average current in the wire?

Answer :

The total charge moving past the point is the product of the electric charge of an electron and the number of electrons moving past the point. The total charge is: $\rightarrow Q = \left(1.6 \times 10^{-19} \frac{C}{electron}\right)(7.9 \times 10^{12}\ electrons) = 12.6 \times 10^{-7} \rightarrow 13 \times 10^{-7}\ C$

The current is $I = \frac{\Delta Q}{\Delta t} = \frac{12.6 \times 10^{-7}}{1.45\ s} = 8.69 \times 10^{-7}\ A \rightarrow 0.87\ \mu A$.

References

1. Ray Dehler (Flickr: raybdbomb). http://www.flickr.com/photos/raybdbomb/2200741209/ . CC-BY 2.0
2. CK-12 Foundation - Ira Nirenberg. . CC-BY-NC-SA 3.0
3. . http://commons.wikimedia.org/wiki/File:Andre-marie-ampere2.jpg . public domain

Concept 3 Electric Resistance

- Define resistance and identify the SI unit for resistance.
- List factors that affect resistance.
- Explain why resistance can be a help or a hindrance.

These athletes are playing rugby, a game that is similar to American football. The players in red and blue are trying to stop the player in orange and black from running across the field with the ball. They are resisting his forward motion. This example of resistance in rugby is a little like resistance in physics.

What Is Resistance?

In physics, **resistance** is opposition to the flow of electric charges in an electric current as it travels through matter. The SI unit for resistance is the ohm. Resistance occurs because moving electrons in current bump into atoms of matter. Resistance reduces the amount of electrical energy that is transferred through matter. That's because some of the electrical energy is absorbed by the atoms and changed to other forms of energy, such as heat.

Q: In the rugby analogy to resistance in physics, what do the players on each team represent?

A:

Factors that Affect Resistance

How much resistance a material has depends on several factors: the type of material, its width, its length, and its temperature.

- All materials have some resistance, but certain materials resist the flow of electric current more or less than other materials do. Materials such as plastics have high resistance to electric current. They are called electric insulators. Materials such as metals have low resistance to electric current. They are called electric conductors.

- A wide wire has less resistance than a narrow wire of the same material. Electricity flowing through a wire is like water flowing through a hose. More water can flow through a wide hose than a narrow hose. In a similar way, more current can flow through a wide wire than a narrow wire.
- A longer wire has more resistance than a shorter wire. Current must travel farther through a longer wire, so there are more chances for it to collide with particles of matter.
- A cooler wire has less resistance than a warmer wire. Cooler particles have less kinetic energy, so they move more slowly. Therefore, they are less likely to collide with moving electrons in current. Materials called superconductors have virtually no resistance when they are cooled to extremely low temperatures.

Is Resistance Good or Bad?

Resistance can be helpful or just a drain on electrical energy. If the aim is to transmit electric current through a wire from one place to another, then resistance is a drawback. It reduces the amount of electrical energy that is transmitted because some of the current is absorbed by particles of matter. On the other hand, if the aim is to use electricity to produce heat or light, then resistance is useful. When particles of matter absorb electrical energy, they change it to heat or light. For example, when electric current flows through the tungsten wire inside an incandescent light bulb like the one in the **Figure** 3.1 , the tungsten resists the flow of electric charge. It absorbs electrical energy and converts some of it to light and heat.

FIGURE 3.1
What's wrong with this picture? (Hint: How does current get to the light bulb?)

Q: The tungsten wire inside a light bulb is extremely thin. How does this help it do its job?

A:

Summary

- In physics, resistance is opposition to the flow of electric charges that occurs as electric current travels through matter. The SI unit for resistance is the ohm.
- All materials have resistance. How much resistance a material has depends on the type of material, its width, its length, and its temperature.
- Resistance is a hindrance when a material is being used to transmit electric current. Resistance is helpful when a material is being used to produce heat or light.

Vocabulary

- **resistance** : Opposition to the flow of electric charges that occurs when electric current travels through matter.

Review

1. What is resistance? Name the SI unit for resistance.
2. Explain what causes resistance.
3. Describe properties of a metal wire that would minimize its resistance to electric current.
4. Extend the rugby analogy to explain why a longer wire has greater resistance to electric current.
5. Copper wires have about one-third the resistance of tungsten wires. Why would copper be less suitable than tungsten as a filament in an incandescent light bulb?

References

1. lenetstan. . Used under license from Shutterstock.com

CONCEPT 4 Ohm's Law

- Explain Ohm's law.
- Use Ohm's law to calculate current from voltage and resistance.

Look at the water spraying out of this garden hose. You have to be careful using water around power tools and electric outlets because water can conduct an electric current. But in some ways, water flowing through a hose is like electric current flowing through a wire.

Introducing Ohm's Law

For electric current to flow through a wire, there must be a source of voltage. Voltage is a difference in electric potential energy. As you might have guessed, greater voltage results in more current. As electric current flows through matter, particles of matter resist the moving charges. This is called resistance, and greater resistance results in less current. These relationships between electric current, voltage, and resistance were first demonstrated in the early 1800s by a German scientist named Georg Ohm, so they are referred to as Ohm's law. **Ohm's law** can be represented by the following equation.

$$\text{Current(amps)} = \frac{\text{Voltage(volts)}}{\text{Resistance(ohms)}}$$

Understanding Ohm's Law

Ohm's law may be easier to understand with an analogy. Current flowing through a wire is like water flowing through a hose. Increasing voltage with a higher-volt battery increases the current. This is like opening the tap wider so more water flows through the hose. Increasing resistance reduces the current. This is like stepping on the hose so less water can flow through it. If you still aren't sure about the relationships among current, voltage, and resistance, watch the video at this URL: http://www.youtube.com/watch?v=KvVTh3ak5dQ

Using Ohm's Law to Calculate Current

You can use the equation for current (above) to calculate the amount of current flowing through a circuit when the voltage and resistance are known. Consider an electric wire that is connected to a 12-volt battery. If the wire has a resistance of 2 ohms, how much current is flowing through the wire?

$$\text{Current} = \frac{12 \text{ volts}}{2 \text{ ohms}} = 6 \text{ amps}$$

Q: If a 120-volt voltage source is connected to a wire with 10 ohms of resistance, how much current is flowing through the wire?

A: Substitute these values into the equation for current:

$$\text{Current} = \frac{120 \text{ volts}}{20 \text{ ohms}} = 12 \text{ amps}$$

Summary

- According to Ohm's law, greater voltage results in more current and greater resistance results in less current.
- Ohm's law can be represented by the equation:'

$$\text{Current(amps)} = \frac{\text{Voltage(volts)}}{\text{Resistance(ohms)}}$$

- This equation can be used to calculate current when voltage and resistance are known.

Vocabulary

- **Ohm's law** : Law stating that current increases as voltage increases or resistance decreases.

Practice

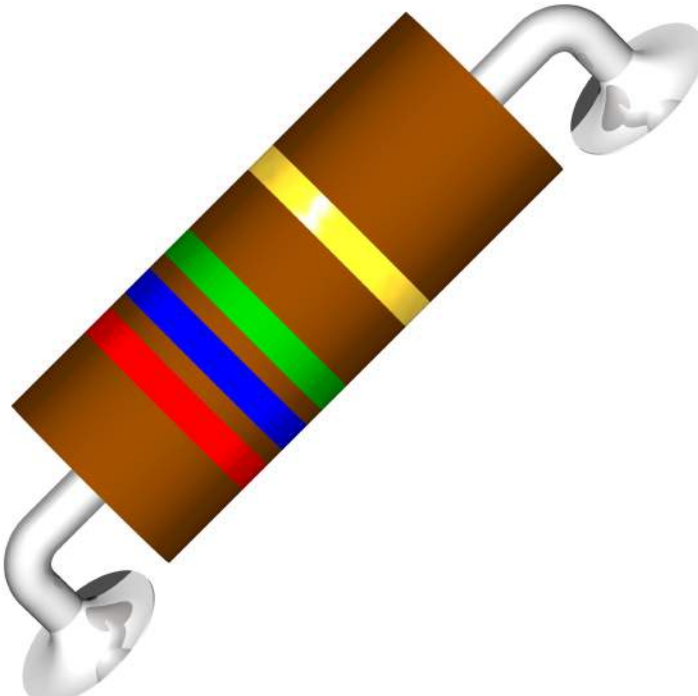
Review Ohm's law and how to calculate current at the following URL. Then try to solve the two problems at the bottom of the Web page. Be sure to check your answers against the correct solutions. http://www.grc.nasa.gov/WWW/k-12/Sample_Projects/Ohms_Law/ohmslaw.html

Review

1. State Ohm's law.
2. An electric appliance is connected by wires to a 240-volt source of voltage. If the combined resistance of the appliance and wires is 12 ohms, how much current is flowing through the circuit?

CONCEPT 5 Resistance and Ohm's Law

- Define resistance.
- Understand the unit for resistance: ohms.
- Use Ohm's Law to solve problems involving current, potential difference, and resistance.



The bands of color on a resistor are a code that indicates the magnitude of the resistance of the resistor. There are four color bands identified by letter: A, B, C, and D, with a gap between the C and D bands so that you know which end is A. This particular resistor has a red A band, blue B band, green C band, and gold D band, but the bands can be different colors on different resistors. Based on the colors of the bands, it is possible to identify the type of resistor. the A and B bands represent significant digits; red is 2 and blue is 6. The C band indicates the multiplier, and green indicates 10^{5}. These three together indicate that this particular resistor is a 26,000 Ohm resistor. Finally, the D band indicates the tolerance, in this case 5%, as shown by the gold band. These terms will be explained over the course of this lesson.

Resistance and Ohm's Law

When a potential difference is placed across a metal wire, a large current will flow through the wire. If the same potential difference is placed across a glass rod, almost no current will flow. The property that determines how much current will flow is called the **resistance.** Resistance is measured by finding the ratio of potential difference, V, to current flow, I.

$$R = \frac{V}{I}$$

When given in the form $V = IR$, this formula is known as **Ohm's Law,** after the man that discovered the relationship. The units of resistance can be determined using the units of the other terms in the equation, namely that the potential difference is in volts (J/C) and current in amperes (C/s):

$$R = \frac{\text{volts}}{\text{amperes}} = \frac{\text{joules/coulomb}}{\text{coulombs/second}} = \frac{\text{joules} \cdot \text{seconds}}{\text{coulombs}^2} = ohms$$

The units for resistance have been given the name **ohms** and the abbreviation is the Greek letter omega, Ω. 1.00 Ω is the resistance that will allow 1.00 ampere of current to flow through the resistor when the potential difference is 1.00 volt. Most conductors have a constant resistance regardless of the potential difference; these are said to obey Ohm's Law.

There are two ways to control the current in a circuit. Since the current is directly proportional to the potential difference and inversely proportional to the resistance, you can increase the current in a circuit by increasing the potential or by decreasing the resistance.

Example Problem: A 50.0 V battery maintains current through a 20.0 Ω resistor. What is the current through the resistor?

Solution: $I = \frac{V}{R} = \frac{50.0\ V}{20.0\ \Omega} = 2.50\ amps$

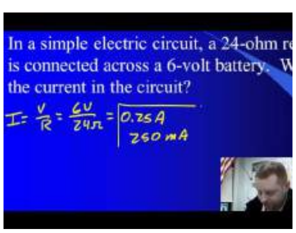
Summary

- Resistance is the property that determines the amount of current flow through a particular material.
- $V = IR$ is known as Ohm's Law.
- The unit for resistance is the ohm, and it has the abbreviation Ω.

Practice

The following video covers Ohm's Law. Use this resource to answer the questions that follow.

http://www.youtube.com/watch?v=uLU4LtG0_hc



MEDIA

Click image to the left for more content.

1. What happens to current flow when voltage is increased?
2. What happens to current flow when resistance is increased?

This website contains instruction and guided practice for Ohm's Law.

http://www.wisc-online.com/Objects/ViewObject.aspx?ID=DCE11904

Review

1. If the potential stays the same and the resistance decreases, what happens to the current?

(a) increase
(b) decrease
(c) stay the same

2. If the resistance stays the same and the potential increases, what happens to the current?

    (a) increase
    (b) decrease
    (c) stay the same

3. How much current can be pushed through a 30.0 Ω resistor by a 12.0 V battery?
4. What voltage is required to push 4.00 A of current through a 32.0 Ω resistor?
5. If a 6.00 volt battery will produce 0.300 A of current in a circuit, what is the resistance in the circuit?

- **resistance:** Opposition of a circuit to the flow of electric current.
- **Ohm's Law:** $V = IR$.
- **Ohms:** A resistance between two points of a conductor when a constant potential difference of 1 volt, applied to these points, produces in the conductor a current of 1 ampere.

References

1. Image copyright Robert Spriggs, 2013. http://www.shutterstock.com . Used under license from Shutterstock.com

CONCEPT 6 Energy Transfer in Electric Circuits

- Explain how devices convert electrical energy to other forms.
- Use $P = I^2R$ and $E = I^2Rt$ for calculations involves energy transfer in electrical circuits.
- Describe the reason for the use of high voltage lines for transmitting electrical energy.
- Define the kilowatt-hour.

Part of the electrical grid, an electrical transmission sub-station receives extremely high current levels, then passes the electrical energy on to as many as 200,000 homes. Approximately 5000 megawatt-hours of energy passes through this particular substation each year.

Energy Transfer in Electric Circuits

Electric power is the energy per unit time converted by an electric circuit into another form of energy. We already know that power through a circuit is equal to the voltage multiplied by the current in a circuit: $P = VI$. It is possible to determine the power dissipated in a single resistor if we combine this expression with Ohm's Law, $V = IR$. This becomes particularly useful in circuits with more than one resistor, to determine the power dissipated in each one. Combining these two equations, we get an expression for electric power that involves only the current and resistance in a circuit.

$$P = I^2R$$

The power dissipated in a resistor is proportional to the square of the current that passes through it and to its resistance.

Electrical energy itself can be expressed as the electrical power multiplied by time:

$E = Pt$

We can incorporate this equation to obtain an equation for electrical energy based on current, resistance, and time. The electrical energy across a resistor is determined to be the current squared multiplied by the resistance and the time.

$E = I^2Rt$

This equation holds true in ideal situations. However, devices used to convert electrical energy into other forms of energy are never 100% efficient. An electric motor is used to convert electrical energy into kinetic energy, but some of the electrical energy in this process is lost to thermal energy. When a lamp converts electrical energy into light energy, some electrical energy is lost to thermal energy.

Example Problem: A heater has a resistance of 25.0 Ω and operates on 120.0 V.

a. How much current is supplied to the resistance?
b. How many joules of energy is provided by the heater in 10.0 s?

Solution:

a. $I = \frac{V}{R} = \frac{120.0\ V}{25.0\ \Omega} = 4.8\ A$
b. $E = I^2Rt = (4.8\ A)^2(25.0\ \Omega)(10.0\ s) = 5760$ joules

Think again about the power grid. When electricity is transmitted over long distances, some amount of energy is lost in overcoming the resistance in the transmission lines. We know the equation for the power dissipated is given by $P = I^2R$. The energy loss can be minimized by choosing the material with the least resistance for power lines, but changing the current also has significant effects. Consider a reduction of the current by a power of ten:

How much power is dissipated when a current of 10.0 A passes through a power line whose resistance is 1.00 Ω? $P = I^2R = (10.0\ A)^2(1.00\ \Omega) = 100.$ Watts

How much power is dissipated when a current of 1.00 A passes through a power line whose resistance is 1.00 Ω? $P = I^2R = (1.00\ A)^2(1.00\ \Omega) = 1.00$ Watts

The power loss is reduced tremendously by reducing the magnitude of the current through the resistance. Power companies must transmit the same amount of energy over the power lines but keep the power loss minimal. They do this by reducing the current. From the equation $P = VI$, we know that the voltage must be increased to keep the same power level.

The Kilowatt-Hour

Even though the companies that supply electrical energy are often called "power" companies, they are actually selling energy. Your electricity bill is based on energy, not power. The amount of energy provided by electric current can be calculated by multiplying the watts (J/s) by seconds to yield joules. The joule, however, is a very small unit of energy and using the joule to state the amount of energy used by a household would require a very large number. For that reason, electric companies measure their energy sales in a large number of joules called a **kilowatt hour** (kWh). A kilowatt hour is exactly as it sounds - the number of kilowatts (1,000 W) transferred per hour.

1.00 kilowatt hour $= (1000\ J/s)(3600\ s) = 3.6 \times 10^6\ J$

Example Problem: A color television uses about 2.0 A when operated on 120 V.

a. How much power does the set use?
b. If the TV is operated for 8.00 hours per day, how much energy in kWh does it use per day?
c. At $0.15 per kWh, what does it cost to run the TV for 30 days?

Solution:

a. $P = VI = (120\ V)(2.0\ A) = 240\ W$
b. $E = \frac{(240\ J/s)(8\ h)(3600\ s/h)}{3.6 \times 10^6\ J/kWh} = 1.92\ kWh$
c. $\text{Cost} = (1.92\ kWh)(30)(\$0.15) = \$8.64$

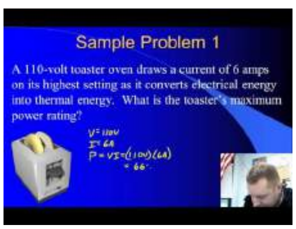
Summary

- Electric power is the energy per unit time converted by an electric circuit into another form of energy.
- The formula for electric power is $P = I^2R$.
- The electric energy transferred to a resistor in a time period is equal to the electric power multiplied by time, $E = Pt$, and can also be calculated using $E = I^2Rt$.
- Electric companies measure their energy sales in a large number of joules called a kilowatt hour (kWh) which is equivalent to $3.6 \times 10^6\ J$.

Practice

The following video is on electrical energy and power. Use this resource to answer the questions that follow.

http://youtu.be/NWcYBvHOiWw



MEDIA

Click image to the left for more content.

1. What is this video about?
2. What is the definition of electrical power?
3. What happens to the electrical energy that is not converted into work?

Instruction and practice problems related to the energy delivered by an electric circuit:

http://www.physicsclassroom.com/Class/circuits/u9l2d.cfm

Review

1. A 2-way light bulb for a 110. V lamp has filament that uses power at a rate of 50.0 W and another filament that uses power at a rate of 100. W. Find the resistance of these two filaments.
2. Find the power dissipation of a 1.5 A lamp operating on a 12 V battery.
3. A high voltage $(4.0 \times 10^5\ V)$ power transmission line delivers electrical energy from a generating station to a substation at a rate of $1.5 \times 10^9\ W$. What is the current in the lines?
4. A toaster oven indicates that it operates at 1500 W on a 110 V circuit. What is the resistance of the oven?

- **electrical energy:** Energy is the ability to do work, so electrical energy is the work done by an electrical circuit.
- **kilowatt hour:** An amount of energy equal to 3.6×10^6 Joules .

References

1. User:Cutajarc/Wikipedia. http://en.wikipedia.org/wiki/File:Melbourne_Terminal_Station.JPG . Public Domain

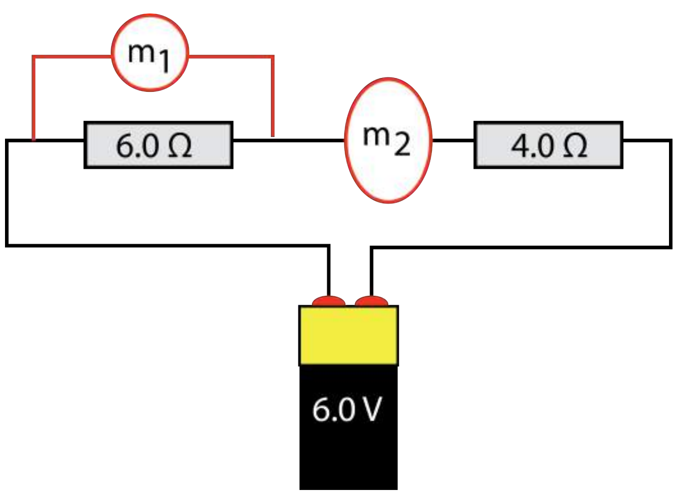
CONCEPT 7 Ammeters and Voltmeters

- Describe the primary difference between the construction of ammeters and voltmeters.
- Describe whether ammeters should be placed in circuits in series or parallel and explain why.
- Describe whether voltmeters should be placed in circuits in series or parallel and explain why.

This photo is of the interior of the control room for a nuclear power plant. Many of the meters are reading information about the water temperature and the nuclear reaction that is occurring, but the majority of the meters are reading data about the electric energy being generated.

Ammeters and Voltmeters

Ammeters and **voltmeters** are cleverly designed for the way they are used. Ammeters measure the current of a circuit, and voltmeters measure the voltage drop across a resistor. It is important in the design and use of these meters that they don't change the circuit in such a way as to influence the readings. While both types of meters are technically resistors, they are specifically designed to make their readings without changing the circuit itself.



Ammeter

An ammeter measures the current traveling through the circuit. They are designed to be connected to the circuit in series, and have an extremely low resistance. If an ammeter were connected in parallel, all of the current would go through the ammeter and very little through any other resistor. As such, it is necessary for the ammeter to be connected in series with the resistors. This allows the ammeter to accurately measure the current flow without causing any disruptions. In the circuit sketched above, the ammeter is m_2.

Voltmeter

In contrast, a voltmeter is designed to be connected to a circuit in parallel, and has a very high resistance. A voltmeter measures the voltage drop across a resistor, and does not need to have the current travel through it to do so. When a voltmeter is placed in parallel with a resistor, all the current continues to travel through the resistor, avoiding the very high resistance of the voltmeter. However, we know that the voltage drop across all resistors in parallel is the same, so connecting a voltmeter in parallel allows it to accurately measure the voltage drop. In the sketch, the voltmeter is m_1.

Summary

- Ammeters measure the current through a resistor.
- Ammeters have low resistances and are placed in the circuit in series.
- Voltmeters measure the voltage drop across a resistor.
- Voltmeters have high resistances and are placed in the circuit in parallel.

Practice

MEDIA

Click image to the left for more content.

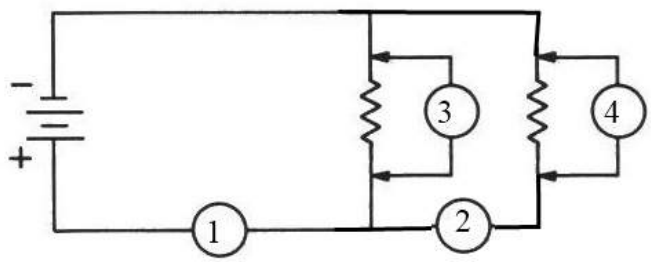
http://www.youtube.com/watch?v=liwan6-w-Pw

In this video, a circuit is established with a power supply, which also has an attached voltmeter, and a lamp (resistor). After the circuit is established, a voltmeter and an ammeter are alternately placed in the circuit.

Follow up questions:

1. What happens when the ammeter is connected in parallel with the lamp?
2. Why do the problems occur when the narrator in the video places the ammeter in parallel with the lamp?

Review



1. In the sketch at above, there are four positions available for the placement of meters. Which position(s) would be appropriate for placement of an ammeter?
   a. 1
   b. 3
   c. 4
   d. All of them.
   e. None of them.
2. Which position(s) would be appropriate for placement of a voltmeter?
   a. 1
   b. 2
   c. 3
   d. All of them.
   e. None of them.
3. Which position could hold an ammeter that would read the total current through the circuit?
   a. 1
   b. 2
   c. 3
   d. 4
   e. None of them.
4. Which position could hold a voltmeter that would read the total voltage drop through the circuit?
   a. 1
   b. 2
   c. 3 or 4
   d. All of them.
   e. None of them.

- **ammeter:** A measuring instrument used to measure the electric current in a circuit.
- **voltmeter:** An instrument used for measuring electrical potential difference between two points in an electric circuit.

References

1. Image copyright rtem, 2013. http://www.shutterstock.com . Used under license from Shutterstock.com
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

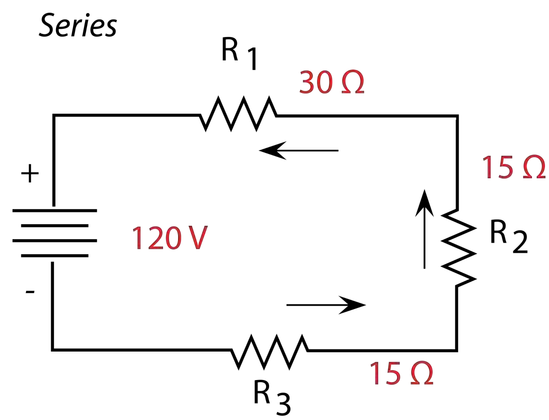
CONCEPT 8 Series Circuits

- Describe a series circuit.
- Understand current as it passes through a series circuit.
- Understand voltage drops in a series circuit.
- Understand resistance in a series circuit with multiple resistors.
- Calculate current, voltage drops, and equivalent resistances for devices connected in a series circuit.

Resistors, including electrical appliances, have a particular current at which they operate most effectively and safely, and excessive current can cause irreparable damage. Therefore, it is important to limit the amount of current that may pass through a particular electrical circuit. There are a number of safety devices used in electrical circuits to limit the current; fuses, circuit breakers, and surge suppressors. When fuses, such as those shown above, are placed in an electrical circuit, all the current must pass through the wire in the fuse.

Series Circuits

Electrical circuits are often modeled by using water in a river. The potential energy of the water is the highest at the source of the river and decreases as the water flows down the river toward the end. When the water reaches the ocean, its potential energy has become zero. The circuit shown above has a similar situation. The current in this circuit is drawn in the direction of the electron flow. It starts at the battery on the left, where electrons leave the negative terminal and travel around the circuit. Since all of the current travels across each resistor, these resistors are said to be in **series** . A series circuit is one in which all of the current must pass through every resistor in the circuit. Returning to the water analogy, there is only one riverbed from the top of the mountain to the ocean.



Consider the series circuit sketched above. This circuit has a voltage drop for the entire circuit of 120 V and has three resistors connected in series. The current in this circuit is drawn in terms of electron flow. The electrons leave the potential difference source at the negative terminal and flow through the three resistors, starting with R_3 . Though they have a small amount of resistance, the resistance of the connecting wires is so small in relation to the resistors that we ignore it. Therefore, we say that there is no voltage drop when the current passes through the connecting wires. The voltage drops occur when the current passes through each of the resistors and the total voltage drop for the entire circuit is equal to the sum of the voltage drops through the three resistors.

$V_T = V_1 + V_2 + V_3$

The current through each of the resistors must be exactly the same because the current in a series circuit is the same everywhere. The current is moving in the entire circuit at the same time.

$I_T = I_1 = I_2 = I_3$

Since the current passes through each resistor, the total resistance in the circuit is equal to the sum of the resistors. In the circuit above, the total resistance is:

$R_T = R_1 + R_2 + R_3 = 30\ \Omega + 15\ \Omega + 15\ \Omega = 60\ \Omega$

Therefore, the total current and the current through each resistor is

$$I = \frac{V}{R} = \frac{120\ V}{60\ \Omega} = 2.0\ A.$$

The individual voltage drops can be calculated using the current through each resistor and each resistor's individual resistance.

$V_1 = I_1 R_1 = (2.0\ A)(30\ \Omega) = 60\ V$

$V_2 = I_2 R_2 = (2.0\ A)(15\ \Omega) = 30\ V$

$V_3 = I_3 R_3 = (2.0\ A)(15\ \Omega) = 30\ V$

Example Problem: Four 15 Ω resistors are connected in series with a 45 V battery. What is the current in the circuit?

Solution: $R_T = 15\ \Omega + 15\ \Omega + 15\ \Omega + 15\ \Omega = 60\ \Omega$

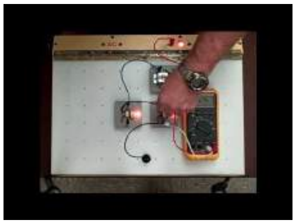
$I = \frac{V}{R} = \frac{45\ V}{60\ \Omega} = 0.75\ A$

Summary

- A series circuit is one in which all of the current must pass through every resistor in the circuit.
- $V_T = V_1 + V_2 + V_3$
- $I_T = I_1 = I_2 = I_3$
- $R_T = R_1 + R_2 + R_3$

Practice

The following video is on series circuits. Use this resource to answer the questions that follow.



MEDIA
Click image to the left for more content.

http://www.youtube.com/watch?v=qO391knBRjE

1. How do the voltage drops across the two light bulbs in the video related to the total voltage drop for the entire circuit?
2. In the video, what was the assumed voltage drop for the connecting wires and the switch?
3. What was the current through the second light bulb as compared to the current through the first light bulb.

Review

1. There are three 20.0 Ω resistors connected in series across a 120 V generator.
   (a) What is the total resistance of the circuit?
   (b) What is the current in the circuit?
   (c) What is the voltage drop across one of the resistors?
2. A 5.00Ω, a 10.0Ω, and a 15.0Ω resistor are connected in a series across a 90.0 V battery.
   (a) What is the equivalent resistance of the circuit?
   (b) What is the current in the circuit?
   (c) What is the voltage drop across the 5.00Ω resistor?
3. A 5.00Ω and a 10.0Ω resistor are connected in series across an unknown voltage. The total current in the circuit is 3.00 A.
   (a) What is the equivalent resistance of the circuit?
   (b) What is the current through the 5.00Ω resistor?
   (c) What is the total voltage drop for the entire circuit?

- **series circuit:** One in which all of the current must pass through every resistor in the circuit.

References

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CONCEPT 9 Resistors in Series

Students will learn how to analyze and solve problems involving circuits with resistors in series.

Students will learn how to analyze and solve problems involving circuits with resistors in series.

Key Equations

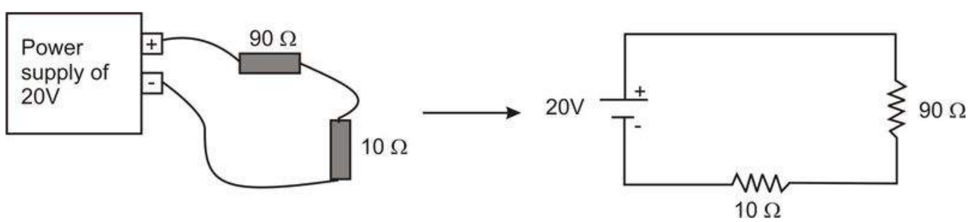
$$R_{total} = R_1 + R_2 + R_3 + \dots$$

Guidance

Resistors in Series: All resistors are connected end to end. There is only one river, so they all receive the same current. But since there is a voltage drop across each resistor, they may all have different voltages across them. The more resistors in series the more rocks in the river, so the less current that flows.

Example 1

A circuit is wired up with two resistors in series.



Both resistors are in the same 'river', so both have the same current flowing through them. Neither resistor has a direct connection to the power supply so neither has 20V across it. But the combined voltages across the individual resistors add up to 20V.

Question: What is the total resistance of the circuit?

Answer: The total resistance is $R_{total} = R_1 + R_2 = 90\ \Omega + 10\ \Omega = 100\ \Omega$

Question: What is the total current coming out of the power supply?

Answer: Use Ohm's Law $(V = IR)$ but solve for current $(I = V/R)$.

$$I_{total} = \frac{V_{total}}{R_{total}} = \frac{20\,V}{100\,\Omega} = 0.20\,A$$

Question: How much power does the power supply dissipate?

Answer: $P = IV$, so the total power equals the total voltage multiplied by the total current. Thus, $P_{total} = I_{total}V_{total} = (0.20\,A)(20V) = 4.0\,W$. So the Power Supply is outputting 4W (i.e. 4 Joules of energy per second).

Question: How much power does each resistor dissipate?

Answer: Each resistor has different voltage across it, but the same current. So, using Ohm's law, convert the power formula into a form that does not depend on voltage.

$$P = IV = I(IR) = I^2R.$$
$$P_{90\,\Omega} = I_{90\,\Omega}^2 R_{90\,\Omega} = (0.2\,A)^2(90\,\Omega) = 3.6\,W$$
$$P_{10\,\Omega} = I_{10\,\Omega}^2 R_{10\,\Omega} = (0.2\,A)^2(10\,\Omega) = 0.4\,W$$

* Note: If you add up the power dissipated by each resistor, it equals the total power outputted, as it should–Energy is always conserved.

Question: How much voltage is there across each resistor?

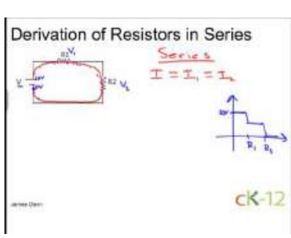
Answer: In order to calculate voltage across a resistor, use Ohm's law.

$$V_{90\,\Omega} = I_{90\,\Omega}R_{90\,\Omega} = (0.2\,A)(90\,\Omega) = 18\,V$$
$$V_{10\,\Omega} = I_{10\,\Omega}R_{10\,\Omega} = (0.2\,A)(10\,\Omega) = 2\,V$$

* Note: If you add up the voltages across the individual resistors you will obtain the total voltage of the circuit, as you should. Further note that with the voltages we can use the original form of the Power equation $(P = IV)$, and we should get the same results as above.

$$P_{90\,\Omega} = I_{90\,\Omega}V_{90\,\Omega} = (18\,V)(0.2\,A) = 3.6\,W$$
$$P_{10\,\Omega} = I_{10\,\Omega}V_{10\,\Omega} = (2.0\,V)(0.2\,A) = 0.4\,W$$

Watch this Explanation



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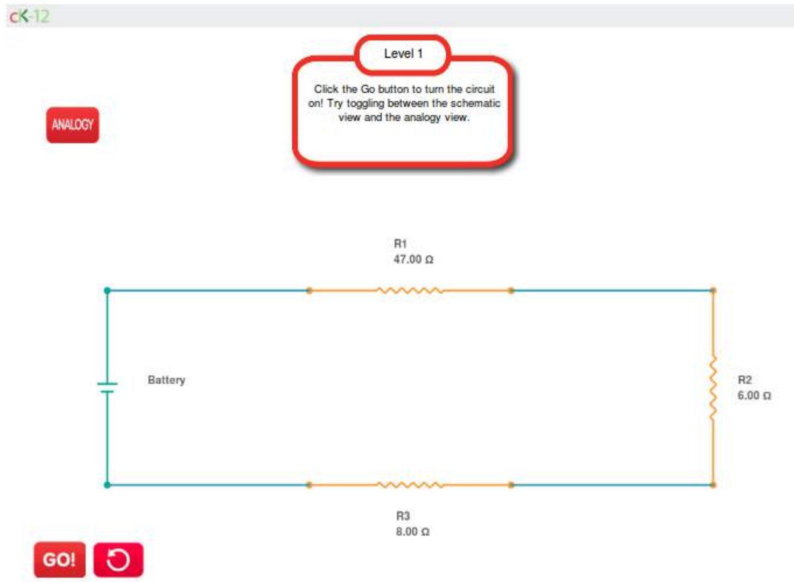
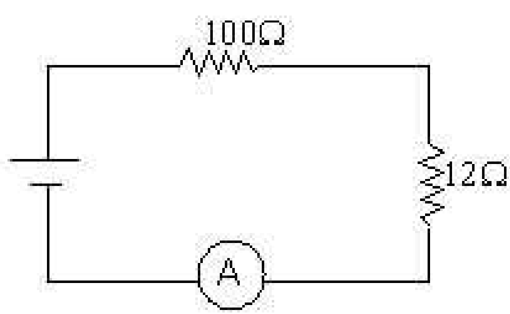
Simulation



- http://simulations.ck12.org/Resistor/

Time for Practice

1. Regarding the circuit below.
   a. If the ammeter reads 2 A , what is the voltage?
   b. How many watts is the power supply supplying?
   c. How many watts are dissipated in each resistor?



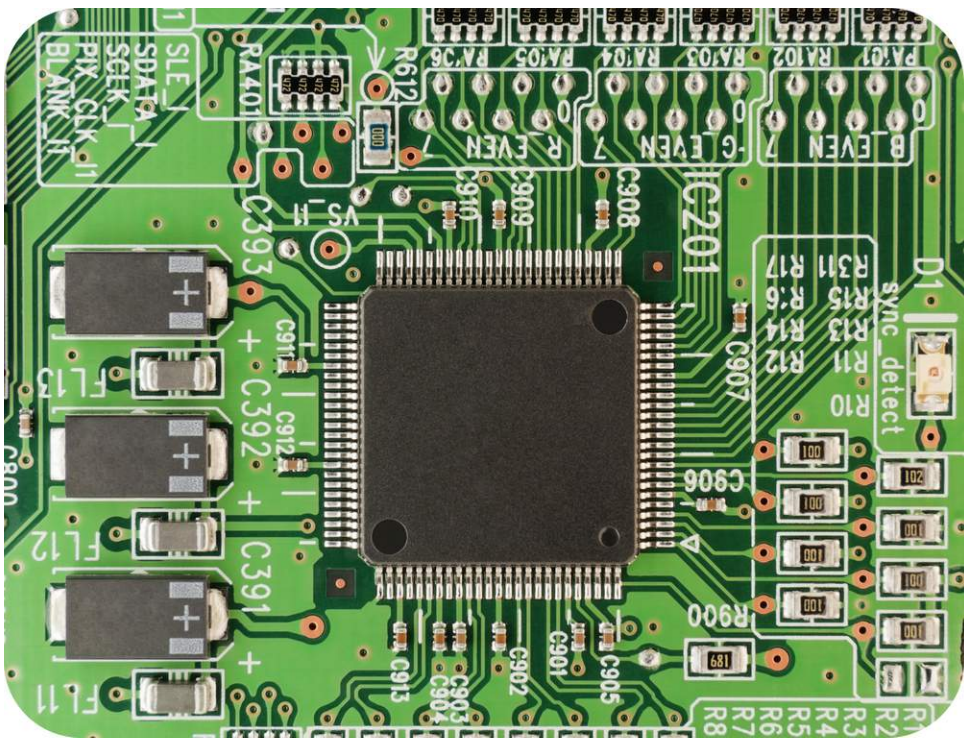
2. Five resistors are wired in series. Their values are 10Ω , 56Ω , 82Ω , 120Ω and 180Ω .
   a. If these resistors are connected to a 6 V battery, what is the current flowing out of the battery?
   b. If these resistors are connected to a 120 V power suppluy, what is the current flowing out of the battery?
   c. In order to increase current in your circuit, which two resistors would you remove?
3. Given the resistors above and a 12 V battery, how could you make a circuit that draws 0.0594 A?

Answers to Selected Problems

1. a. 224 V b. 448 W c. 400 W by 100 Ω and 48 W by 12 Ω
2. a. 0.013 A b. 0.27 A c. 120Ω and 180Ω
3. need about 202Ω of total resistance. So if you wire up the 120Ω and the 82Ω in series, you'll have it.

Concept 10 Parallel Circuits

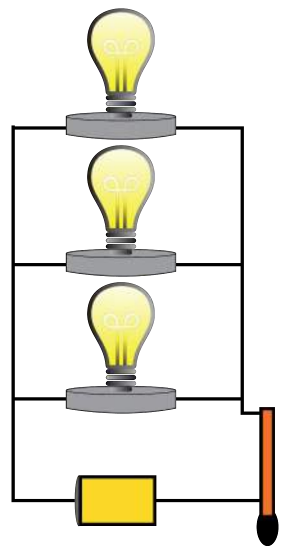
- Describe a parallel circuit.
- Understand current as it passes through a parallel circuit.
- Understand voltage drops in a parallel circuit.
- Understand resistance in a parallel circuit with multiple resistors.
- Calculate voltage drops, currents, and equivalent resistances when devices are connected in a parallel circuit.



Electrical circuits are everywhere: skyscrapers, jumbo jets, arcade games, lights, heating, and security . . . very few complex things work without electrical circuits. Since the late 1970s, electrical circuits have primarily looked like this. The circuits are formed by a thing layer of conducting material deposited on the surface of an insulating board. Individual components are soldered to the interconnecting circuits. Circuit boards are vastly more complicated than the series circuits previously discussed, but operate on many similiar principles.

Parallel Circuits

Parallel circuits are circuits in which the charges leaving the potential source have different paths they can follow to get back to the source. In the sketch below, the current leaves the battery, passes through the orange switch, and then has three different paths available to complete the circuit. Each individual electron in this circuit passes through only one of the light bulbs. After the current passes through the switch, it divides into three pieces and each piece passes through one of the bulbs. The three pieces of current rejoin after the light bulbs and continue in the circuit to the potential source.



In the design of this parallel circuit, each resistor (light bulb) is connected across the battery as if the other two resistors were not present. Remember that the current going through each resistor goes through only the one resistor. Therefore, the voltage drop across each resistor must be equal to the total voltage drop though the circuit.

$V_T = V_1 = V_2 = V_3$

The total current passing through the circuit will be the sum of the individual currents passing through each resistor.

$I_T = I_1 + I_2 + I_3$

If we return to the analogy of a river, a parallel circuit is the same as the river breaking into three streams, which later rejoin to one river again. The amount of water flowing in the river is equal to the sum of the amounts of water flowing in the individual streams.

Ohm's Law applies to resistors in parallel, just as it did to resistors in a series. The current flowing through each resistor is equal to the total voltage drop divided by the resistance in that resistor.

$$I_1 = \frac{V_T}{R_1} \quad \text{and} \quad I_2 = \frac{V_T}{R_2} \quad \text{and} \quad I_3 = \frac{V_T}{R_3}$$

Since $I_T = I_1 + I_2 + I_3$,

then $I_T = \frac{V_T}{R_1} + \frac{V_T}{R_2} + \frac{V_T}{R_3}$,

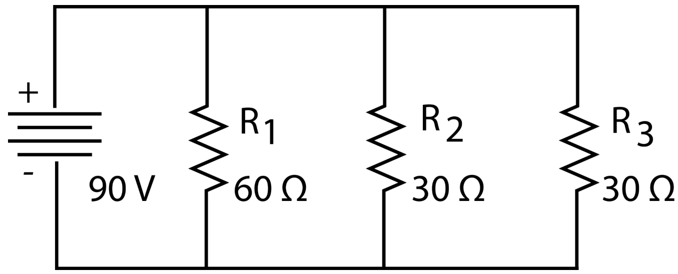
and $\frac{V_T}{R_T} = \frac{V_T}{R_1} + \frac{V_T}{R_2} + \frac{V_T}{R_3}$.

If we divide both sides of the final equation by V_T , we get the relationship between the total resistance of the circuit and the individual parallel resistances in the circuit. The total resistance is sometimes called the **equivalent resistance.**

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Consider the parallel circuit sketched below.

The voltage drop for the entire circuit is 90. V. Therefore, the voltage drop in each of the resistors is also 90. V.



The current through each resistor can be found using the voltage drop and the resistance of that resistor:

$$I_1 = \frac{V_T}{R_1} = \frac{90.\ V}{60.\ \Omega} = 1.5\ A \qquad I_2 = \frac{V_T}{R_2} = \frac{90.\ V}{30.\ \Omega} = 3.0\ A \qquad I_3 = \frac{V_T}{R_3} = \frac{90.\ V}{30.\ \Omega} = 3.0\ A$$

The total current through the circuit would be the sum of the three currents in the individual resistors.

$I_T = I_1 + I_2 = I_3 = 1.5\ A + 3.0\ A + 3.0\ A = 7.5\ A$

The equivalent resistance for this circuit is found using the equation above.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{60.\ \Omega} + \frac{1}{30.\ \Omega} + \frac{1}{30.\ \Omega} = \frac{1}{60.\ \Omega} + \frac{2}{60.\ \Omega} + \frac{2}{60.\ \Omega} = \frac{5}{60.\ \Omega}$$

$R_T = \frac{60.\ \Omega}{5} = 12\ \Omega$

The equivalent resistance for the circuit could also be found by using the total voltage drop and the total current.

$R_T = \frac{V_T}{I_T} = \frac{90.\ \Omega}{7.5\ A} = 12\ \Omega$

Summary

- Parallel electrical circuits have multiple paths the current may take.
- $V_T = V_1 = V_2 = V_3$.
- $I_T = I_1 + I_2 + I_3$.
- $\frac{1}{R_T} = \frac{1}{R_1} = \frac{1}{R_2} + \frac{1}{R_3}$.

Practice

MEDIA

Click image to the left for more content.

http://www.youtube.com/watch?v=apHkG4T6QHM

Follow up questions:

1. Why do the light bulbs glow less brightly when connected across a 120 V source in a series circuit than when connected across the same 120 V source in a parallel circuit?
2. Why do the other bulbs go dark when one bulb is removed in the series circuit but the other bulbs do not go dark when one bulb is removed in the parallel circuit?

Review

1. Three 15.0Ω resistors are connected in parallel and placed across a 30.0 V potential difference.
   (a) What is the equivalent resistance of the parallel circuit?
   (b) What is the total current through the circuit?
   (c) What is the current through a single branch of the circuit?
2. A 12.0Ω and a 15.0Ω resistor are connected in parallel and placed across a 30.0 V potential.
   (a) What is the equivalent resistance of the parallel circuit?
   (b) What is the total current through the circuit?
   (c) What is the current through each branch of the circuit?
3. A 120.0Ω resistor, a 60.0Ω resistor, and a 40.0Ω resistor are connected in parallel and placed across a potential difference of 12.0 V.
   (a) What is the equivalent resistance of the parallel circuit?
   (b) What is the total current through the circuit?
   (c) What is the current through each branch of the circuit?

- **parallel circuit:** A closed electrical circuit in which the current is divided into two or more paths and then returns via a common path to complete the circuit.
- **equivalent resistance:** A single resistance that would cause the same power loss as the actual resistance values distributed throughout a circuit.

References

1. Image copyright vilax, 2013. http://www.shutterstock.com . Used under license from Shutterstock.com
2. Light bulb: Image copyright Snez, 2013; composite created by CK-12 Foundation - Samantha Bacic. http://www.shutterstock.com . Used under license from Shutterstock.com
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CONCEPT 11 Resistors in Parallel

Students will learn how to analyze and solve problems involving circuits with resistors in parallel.

Students will learn how to analyze and solve problems involving circuits with resistors in parallel.

Key Equations

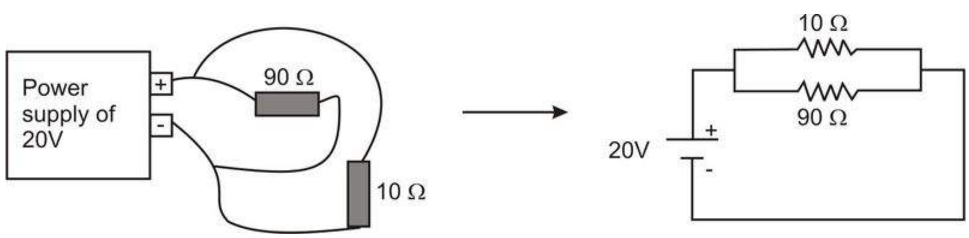
$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Guidance

Resistors in Parallel: All resistors are connected together at both ends. There are many rivers (i.e. The main river branches off into many other rivers), so all resistors receive different amounts of current. But since they are all connected to the same point at both ends they all receive the same voltage.

Example 1

A circuit is wired up with 2 resistors in parallel.



Both resistors are directly connected to the power supply, so both have the same 20V across them. But they are on different 'rivers' so they have different current flowing through them. Lets go through the same questions and answers as with the circuit in series.

Question: What is the total resistance of the circuit?

Answer: The total resistance is $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{90\Omega} + \frac{1}{10\Omega} = \frac{1}{90\Omega} + \frac{9}{90\Omega} = \frac{10}{90\Omega}$ thus, $R_{total} = \frac{90\Omega}{10} = 9\Omega$

* Note: Total resistance for a circuit in parallel will always be smaller than smallest resistor in the circuit.

Question: What is the total current coming out of the power supply?

Answer: Use Ohm's Law $(V = IR)$ but solve for current $(I = V/R)$.

$$I_{total} = \frac{V_{total}}{R_{total}} = \frac{20V}{9\Omega} = 2.2A$$

Question: How much power does the power supply dissipate?

Answer: $P = IV$, so the total power equals the total voltage multiplied by the total current. Thus, $P_{total} = I_{total}V_{total} = (2.2A)(20V) = 44.4W$. So the Power Supply outputs 44W (i.e. 44 Joules of energy per second).

Question: How much power is each resistor dissipating?

Answer: Each resistor has different current across it, but the same voltage. So, using Ohm's law, convert the power formula into a form that does not depend on current. $P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$ Substituted $I = V/R$ into the power formula. $P_{90\Omega} = \frac{V_{90\Omega}^2}{R_{90\Omega}} = \frac{(20V)^2}{90\Omega} = 4.4W; P_{10\Omega} = \frac{V_{10\Omega}^2}{R_{10\Omega}} = \frac{(20V)^2}{10\Omega} = 40W$

* Note: If you add up the power dissipated by each resistor, it equals the total power outputted, as it should–Energy is always conserved.

Question: How much current is flowing through each resistor?

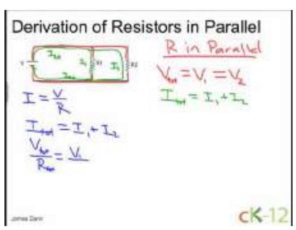
Answer: Use Ohm's law to calculate the current for each resistor.

$$I_{90\Omega} = \frac{V_{90\Omega}}{R_{90\Omega}} = \frac{20V}{90\Omega} = 0.22A \qquad I_{10\Omega} = \frac{V_{10\Omega}}{R_{10\Omega}} = \frac{20V}{10\Omega} = 2.0A$$

Notice that the 10Ω resistor has the most current going through it. It has the least resistance to electricity so this makes sense.

* Note: If you add up the currents of the individual 'rivers' you get the total current of the of the circuit, as you should.

Watch this Explanation



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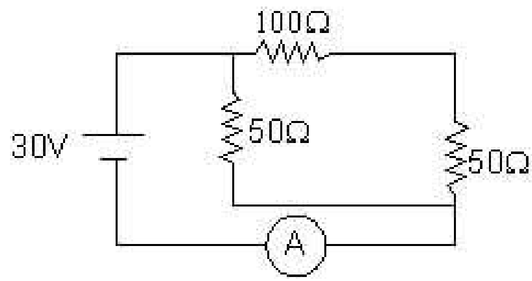
Simulation

- http://simulations.ck12.org/Resistor/

Time for Practice

1. Three 82 Ω resistors and one 12 Ω resistor are wired in parallel with a 9 V battery.
   a. Draw the schematic diagram.
   b. What is the total resistance of the circuit?
2. What does the ammeter read and which resistor is dissipating the most power?



3. Given three resistors, 200 Ω, 300 Ω and 600 Ω and a 120 V power source connect them in a way to heat a container of water as rapidly as possible.
   a. Show the circuit diagram
   b. How many joules of heat are developed after 5 minutes?

Answers to Selected Problems

1. b. 8.3 W
2. 0.8A and the 50 Ω on the left
3. part 2 43200J .

CONCEPT 12 Combined Series-Parallel Circuits

- Solve problems of combined circuits.

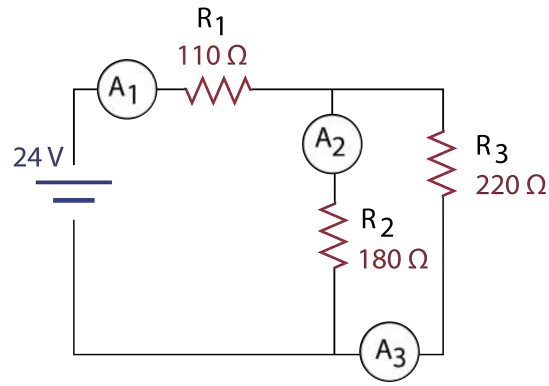
Electrical circuits can become immensely complicated. This circuit is a polynomial plotter, which allows users to plot polynomials and evaluate functions at various x values.

Combined Series-Parallel Circuits

Most circuits are not just a series or parallel circuit; most have resistors in parallel and in series. These circuits are called **combination circuits** . When solving problems with such circuits, use this series of steps.

1. For resistors connected in parallel, calculate the single equivalent resistance that can replace them.
2. For resistors in series, calculate the single equivalent resistance that can replace them.
3. By repeating steps 1 and 2, you can continually reduce the circuit until only a single equivalent resistor remains. Then you can determine the total circuit current. The voltage drops and currents though individual resistors can then be calculated.

Example Problem: In the combination circuit sketched below, find the equivalent resistance for the circuit, find the total current through the circuit, and find the current through each individual resistor.



Solution: We start by simplifying the parallel resistors R_2 and R_3 .

$$\frac{1}{R_{23}} = \frac{1}{180\ \Omega} + \frac{1}{220\ \Omega} = \frac{1}{99\ \Omega}$$
$$R_{23} = 99\ \Omega$$

We then simplify R_1 and R_{23} which are series resistors.

$R_T = R_1 + R_{23} = 110\ \Omega + 99\ \Omega = 209\ \Omega$

We can then find the total current, $I_T = \frac{V_T}{R_T} = \frac{24\ V}{209\ \Omega} = 0.11\ A$

All the current must pass through R_1 , so $I_1 = 0.11\ A$.

The voltage drop through R_1 is $(110\ \Omega)(0.11\ A) = 12.6$ volts .

Therefore, the voltage drop through R_2 and R_3 is 11.4 volts.

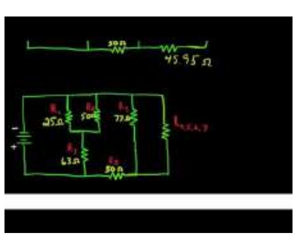
$I_2 = \frac{V_2}{R_2} = \frac{11.4\ V}{180\ \Omega} = 0.063\ A$ and $I_3 = \frac{V_3}{R_3} = \frac{11.4\ V}{220\ \Omega} = 0.052\ A$

Summary

- Combined circuit problems should be solved in steps.

Practice

Video teaching the process of simplifying a circuit that contains both series and parallel parts.



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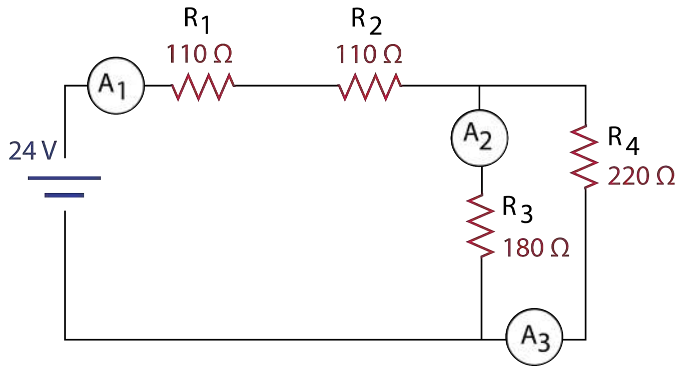
Click image to the left for more content.

http://www.youtube.com/watch?v=In3NF8f-mzg

Follow up questions:

1. In a circuit that contains both series and parallel parts, which parts of the circuit are simplified first?
2. In the circuit drawn below, which resistors should be simplified first?



Review

1. Two 60.0Ω resistors are connected in parallel and this parallel arrangement is then connected in series with a 30.0Ω resistor. The combination is placed across a 120. V potential difference.
   (a) Draw a diagram of the circuit.
   (b) What is the equivalent resistance of the parallel portion of the circuit?
   (c) What is the equivalent resistance for the entire circuit?
   (d) What is the total current in the circuit?
   (e) What is the voltage drop across the 30.0Ω resistor?
   (f) What is the voltage drop across the parallel portion of the circuit?
   (g) What is the current through each resistor?
2. Three 15.0 Ohm resistors are connected in parallel and the combination is then connected in series with a 10.0 Ohm resistor. The entire combination is then placed across a 45.0 V potential difference. Find the equivalent resistance for the entire circuit.

- **combined or combination circuits:** A route for the flow of electricity that has elements of both series and parallel circuits.

References

1. User:Linkaddict/Wikimedia Commons. http://commons.wikimedia.org/wiki/File:Huge_circuit.JPG . Public Domain
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

CK-12 FlexBook

Physics Unit 14: Magnetism

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1	**Properties of Magnets**	**1**
2	**Magnetic Fields**	**6**

CONCEPT 1 Properties of Magnets

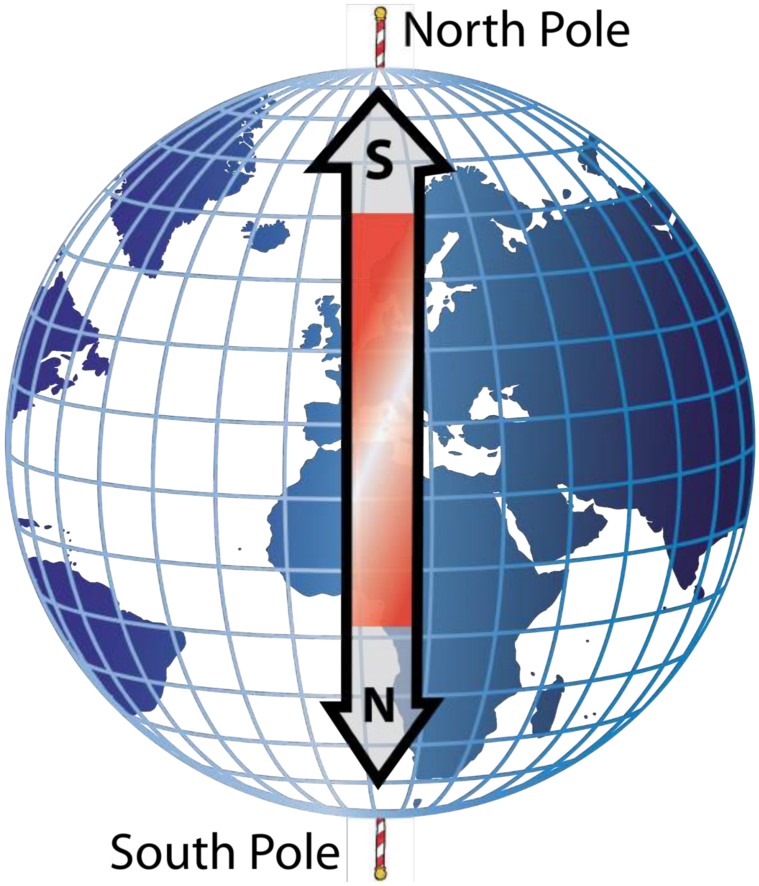
- Describe magnetic fields around permanent magnets.
- Understand ferromagnetism and magnetic domains.
- Describe some properties of magnets.

Some countries are using powerful electromagnets to develop high-speed trains, called maglev, or magnetic levitation, trains. These trains use the repulsive force of magnets to float over a guide way, removing the friction of steel wheels and train tracks. Reducing this friction allows the trains to travel at much higher speeds.

Properties of Magnets

Any **magnet** , regardless of its shape, has two ends called poles where the magnetic effect is strongest. If a magnet is suspended by a fine thread, it is found that one pole of the magnet will always point toward the north. This fact has been made use of in navigation since the eleventh century. The pole of the magnet that seeks the north pole is called the north pole of the magnet, while the opposite side is the south pole.

It is a familiar fact that when two magnets are brought near one another, the magnets exert a force on each other. The magnetic force can be either attractive or repulsive. If two north poles or two south poles are brought near each other, the force will be repulsive. If a north pole is brought near a south pole, the force will be attractive.

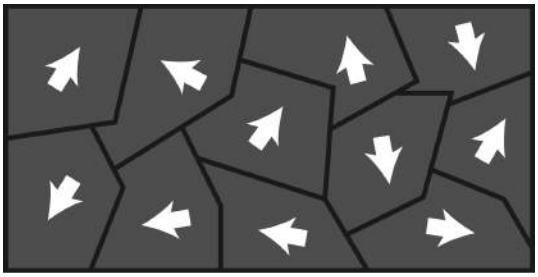


The Earth's geographic north pole (which is close to, but not exactly at the magnetic pole) attracts the north poles of magnets. We know, therefore, that this pole is actually the Earth's magnetic south pole. This can be seen in the image above; the geographic north and south poles are labeled with barber shop poles, and the Earth's **magnetic poles** are indicated with the double-headed arrow.

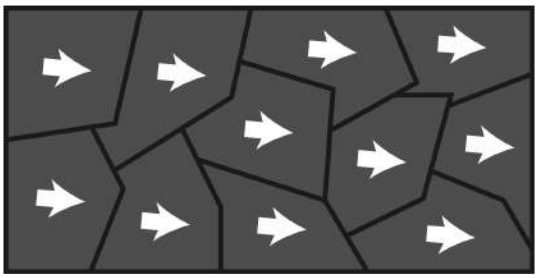
Only iron and few other materials such as cobalt, nickel, and gadolinium show strong magnetic effects. These materials are said to be **ferromagnetic** . Other materials show some slight magnetic effect but it is extremely small and can be detected only with delicate instruments.

Ferromagnetic Domains

Microscopic examination reveals that a magnet is actually made up of tiny regions known as **magnetic domains** , which are about one millimeter in length and width. Each domain acts like a tiny magnet with a north and south pole.



Domains before magnetization



Domains after magnetization

When the ferrous material is not magnetized, the domains are randomly organized so that the north and south poles do not line up and often cancel each other. When the ferrous material is placed in a magnetic field, the domains line up with the magnetic field so that the north poles are all pointed in the same direction and the south poles are all pointed in the opposite direction. In this way, the ferrous material has become a magnet. In many cases, the domains will remain aligned only while the ferrous material is in a strong magnetic field; when the material is removed from the field, the domains return to their previous random organization and the ferrous material loses any magnetic properties. Magnets that have magnetic properties while in the field of another magnet but lose the magnetic properties when removed from the field are called **temporary magnets** . Under certain circumstances, however, the new alignment can be made permanent and the ferrous substance becomes a **permanent magnet** . That is, the ferrous object remains a magnet even when removed from the other magnetic field.

The formation of temporary magnets allows a magnet to attract a non-magnetized piece of iron. You have most likely seen a magnet pick up a paper clip. The presence of the magnet aligns the domains in the iron paper clip and it becomes a temporary magnet. Whichever pole of the magnet is brought near the paper clip will induce magnetic properties in the paper clip that remain as long as the magnet is near.

Permanent magnets lose their magnetic properties when the domains are dislodged from their organized positions and returned to a random jumble. This can occur if the magnet is hammered on or if it is heated strongly.

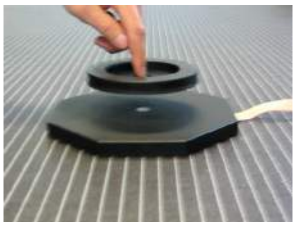
Magnetic Fields

When we were dealing with electrical effects, it was very useful to speak of an electric field that surrounded an electric charge. In the same way, we can imagine a **magnetic field** surrounding a magnetic pole. The force that one magnet exerts on another can be described as the interaction between one magnet and the magnetic field of the other magnet. Magnetic field lines go from the north magnetic pole to the south magnetic pole. We define the magnetic field at any point as a vector (represented by the letter **B**) whose direction is from north to south magnetic poles.

Summary

- Any magnet has two ends called poles where the magnetic effect is strongest.
- The magnetic pole found at the north geographical pole of the earth is a south magnetic pole.
- The force that one magnet exerts on another can be described as the interaction between one magnet and the magnetic field of the other magnet.
- Magnetic field lines go from the north magnetic pole to the south magnetic pole.

Practice



MEDIA

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http://www.darktube.org/watch/crealev-levitating-floating-flying-hovering-bouncing

This video demonstrates magnetic levitation.

1. In the video, one object rests on top of the magnetic field of another. Compare the friction between these two objects to the friction between a saucer and a table the saucer rests on.

Review

1. The earth's magnetic field
   (a) has a north magnetic pole at exactly the same spot as the geographical north pole.
   (b) is what causes compasses to work.
   (c) is what causes electromagnets to work.
   (d) all of these are true.
   (e) none of these are true.
2. A material that can be permanently magnetized is generally said to be
   (a) magnetic.
   (b) electromagnetic.
   (c) ferromagnetic.
   (d) none of these are true.
3. The force between like magnetic poles will be
   (a) repulsive.
   (b) attractive.
   (c) could be repulsive or attractive.
4. Why is a magnet able to attract a non-magnetic piece of iron?
5. If you had two iron rods and noticed that they attract each other, how could you determine if both were magnets or only one was a magnet?

- **magnet:** A body that can attract certain substances, such as iron or steel, as a result of a magnetic field.

- **magnetic pole:** Either of two regions of a magnet, designated north and south, where the magnetic field is strongest. Electromagnetic interactions cause the north poles of magnets to be attracted to the south poles of other magnets, and conversely. The north pole of a magnet is the pole out of which magnetic lines of force point, while the south pole is the pole into which they point.
- **ferromagnetic:** A body or substance having a high susceptibility to magnetization, the strength of which depends on that of the applied magnetizing field, and which may persist after removal of the applied field. This is the kind of magnetism displayed by iron, and is associated with parallel magnetic alignment of adjacent domains.
- **magnetic field:** A field of force surrounding a permanent magnet or a moving charged particle.
- **magnetic domain:** An atom or group of atoms within a material that have some kind of "net" magnetic field.
- **temporary magnet:** A piece of iron that is a magnet while in the presence of another magnetic field but loses its magnetic characteristics when the other field is removed.
- **permanent magnet:** A piece of magnetic material that retains its magnetism after it is removed from a magnetic field.

References

1. User:JakeLM/Wikipedia. http://commons.wikimedia.org/wiki/File:Maglev_june2005.jpg . CC-BY 2.5
2. Globe: Image copyright pdesign, 2013; Poles: Image copyright lineartestpilot, 2013; Composite created by CK-12 Foundation - Samantha Bacic. http://www.shutterstock.com . Used under licenses from Shutterstock.com

CONCEPT 2 Magnetic Fields

Students will learn the idea of magnetic field lines, how they behave in the situation of permanent magnets and current carrying wires and also how to calculate the magnetic field at an arbitrary distance from the wire.

Students will learn the idea of magnetic field lines, how they behave in the situation of permanent magnets and current carrying wires and also how to calculate the magnetic field at an arbitrary distance from the wire.

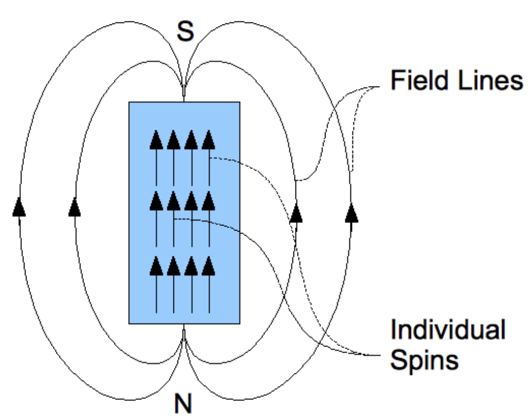
Key Equations

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$$ Magnetic field at a distance r from a current-carrying wire

Where $\mu_0 = 4\pi \times 10^{-7}$ Tm/A Permeability of Vacuum (approximately same for air also)

Guidance

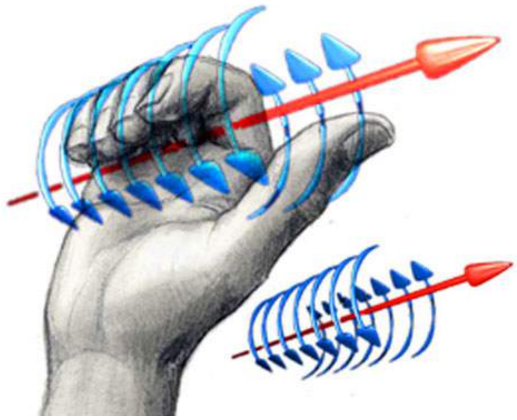
Permanent magnets (like refrigerator magnets) consist of atoms, such as iron, for which the magnetic moments (roughly electron spin) of the electrons are "lined up" all across the atom. This means that their magnetic fields add up, rather than canceling each other out. The net effect is noticeable because so many atoms have lined up. The magnetic field of such a magnet always points from the north pole to the south. The magnetic field of a bar magnet, for example, is illustrated below:



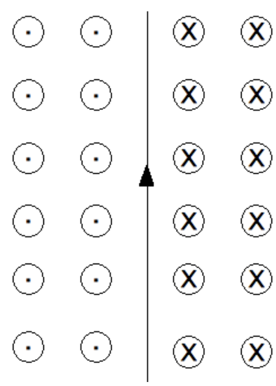
If we were to cut the magnet above in half, it would still have north and south poles; the resulting magnetic field would be qualitatively the same as the one above (but weaker).

Charged particles in motion also generate magnetic fields. The most frequently used example is a current carrying wire, since current is literally moving charged particles. The magnitude of a field generated by a wire depends on distance to the wire and strength of the current (I) (see 'Key Equations' section) :

Meanwhile, its direction can be found using the so called **first right hand rule** : point your thumb in the direction of the current. Then, curl your fingers around the wire. The direction your fingers will point in the same direction as the field. Be sure to use your right hand!



Sometimes, it is necessary to represent such three dimensional fields on a two dimensional sheet of paper. The following example illustrates how this is done.



In the example above, a current is running along a wire towards the top of your page. The magnetic field is circling the current carrying wire in loops which are perpendicular to the page. Where these loops intersect this piece of paper, *we use the symbol* $\odot$ *to represent where the magnetic field is coming* ***out of the page*** *and the symbol* $\otimes$ *to represent where the magnetic field is going* ***into the page***. This convention can be used for all vector quantities: fields, forces, velocities, etc.

Example 1

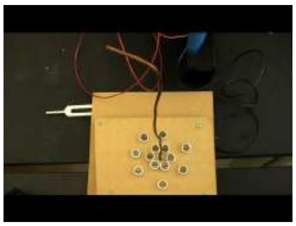
You are standing right next to a current carrying wire and decide to throw your magnetic field sensor some distance perpendicular to the wire. When you go to retrieve your sensor, it shows the magnetic field where it landed to be $4 * 10^{-5}$ T . If you know the wire was carrying 300A, how far did you throw the sensor?

Solution

To solve this problem, we will just use the equation given above and solve for the radius.

$$B = \frac{\mu_o I}{2\pi r}$$
$$r = \frac{\mu_o I}{2\pi B}$$
$$r = \frac{4\pi * 10^{-7}\,\mathrm{Tm/A} * 300\,\mathrm{A}}{2\pi * 4 * 10^{-5}\,\mathrm{T}}$$
$$r = 1.5\,\mathrm{m}$$

Watch this Explanation



MEDIA

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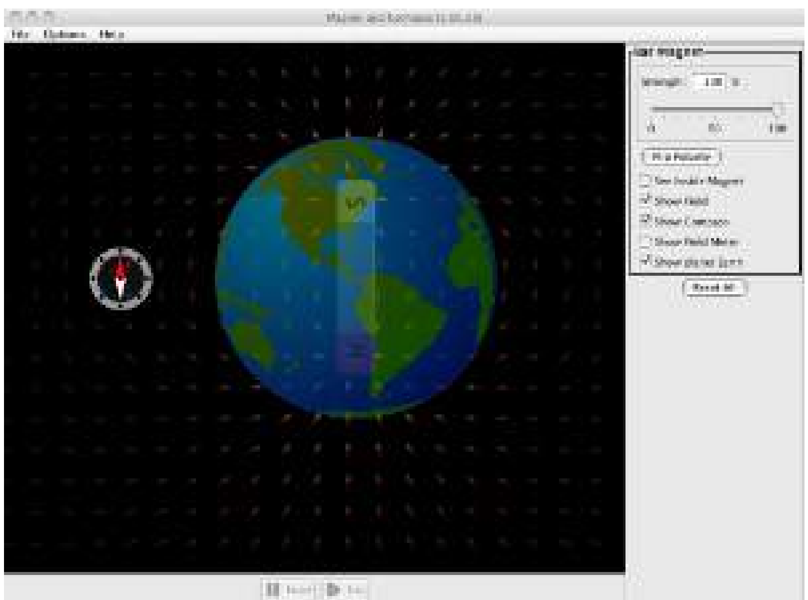
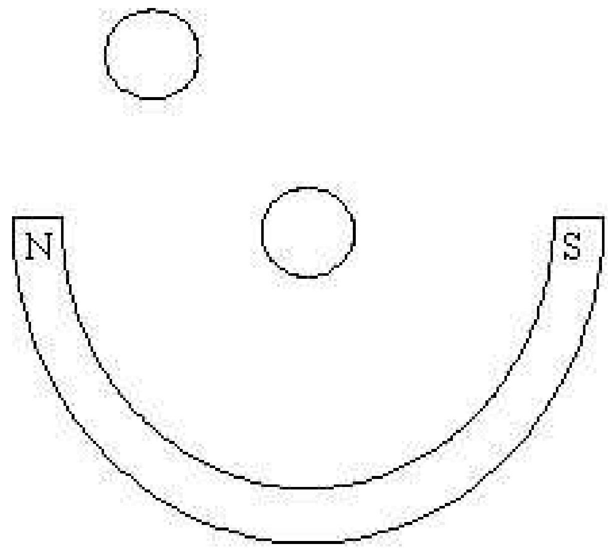
Simulation



Magnet and Compass (PhET Simulation)

Time for Practice

1. Sketch the magnetic field lines for the horseshoe magnet shown here. Then, show the direction in which the two compasses (shown as circles) should point considering their positions. In other words, draw an arrow in the compass that represents North in relation to the compass magnet.



2. Find the magnetic field a distance of 20 cm from a wire that is carrying 3 A of electrical current.
3. In order to measure the current from big power lines the worker simply clamps a device around the wire. This provides safety when dealing with such high currents. The worker simply measures the magnetic field and deduces the current using the laws of physics. Let's say a worker uses such a clamp and the device registers a magnetic field of 0.02 T. The clamp is 0.05 m from the wire. What is the electrical current in the wire?

Answers to Selected Problems

1. Both pointing away from north
2. 3×10^{-6}T
3. 5000 A

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Physics Unit 15: Electromagnetism

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CONCEPT 1 Electromagnetic Induction

- Define electromagnetic induction.
- Explain how electromagnetic induction occurs.
- Describe the current produced by electromagnetic induction.
- Identify ways that electromagnetic induction is used.

The girl on the left in this photo is riding a stationary bike. She's getting exercise, but that's not the real reason she's riding the bike. She's using her muscle power to generate electricity through a process called electromagnetic induction.

What Is Electromagnetic Induction?

Electromagnetic induction is the process of generating electric current with a magnetic field. It occurs whenever a magnetic field and an electric conductor, such as a coil of wire, move relative to one another. As long as the conductor is part of a closed circuit, current will flow through it whenever it crosses lines of force in the magnetic field. One way this can happen is illustrated in the **Figure** 1.1 . The sketch shows a magnet moving through a wire coil. You can watch an animated version of the illustration at this URL: http://jsticca.wordpress.com/2009/09/01/the-magnet-car/

Q: What is another way that a coil of wire and magnet can move relative to one another and generate an electric current?

A: The coil of wire could be moved back and forth over the magnet.

The Current Produced by a Magnet

The device with the pointer in the circuit above is an ammeter. It measures the current that flows through the wire. The faster the magnet or coil moves, the greater the amount of current that is produced. If more turns were added to the coil or a stronger magnet were used, this would produce more current as well.

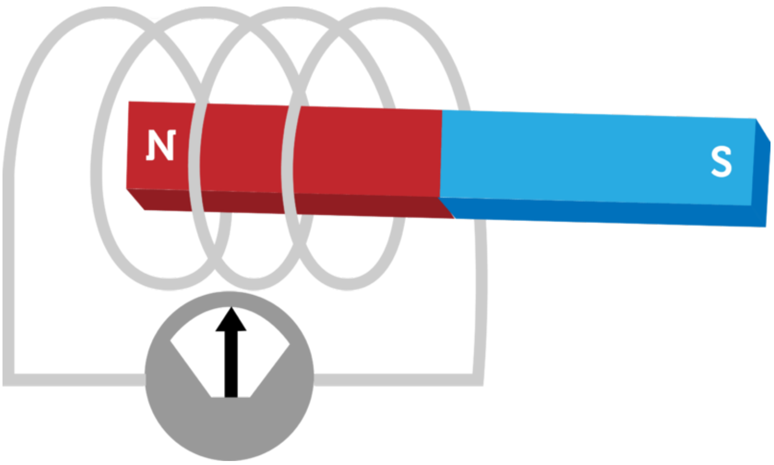


FIGURE 1.1

The **Figure** 1.2 shows the direction of the current that is generated by a moving magnet. If the magnet is moved back and forth repeatedly, the current keeps changing direction. In other words, alternating current (AC) is produced. Alternating current is electric current that keeps reversing direction.

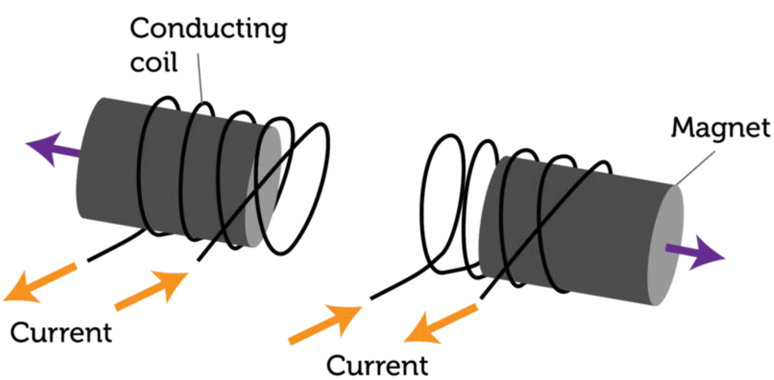


FIGURE 1.2

How Electromagnetic Induction Is Used

Two important devices depend on electromagnetic induction: electric generators and electric transformers. Both devices play critical roles in producing and regulating the electric current we depend on in our daily lives. Electric generators use electromagnetic induction to change kinetic energy to electrical energy. They produce electricity in power plants. Electric transformers use electromagnetic induction to change the voltage of electric current. Some transformers increase voltage and other decrease voltage.

Q: How do you think the girl on the exercise bike in the opening photo is using electromagnetic induction?

A: As she pedals the bike, the kinetic energy of the turning pedals is used to move a conductor through a magnetic field. This generates electric current by electromagnetic induction.

Summary

- Electromagnetic induction is the process of generating electric current with a magnetic field. It occurs whenever a magnetic field and an electric conductor move relative to one another so the conductor crosses lines of force in the magnetic field.
- The current produced by electromagnetic induction is greater when the magnet or coil moves faster, the coil has more turns, or the magnet is stronger. If the magnet or coil is moved back and forth repeatedly, alternating current is produced.
- Electric generators and electric transformers use electromagnetic induction to generate electricity or change the voltage of electric current.

Vocabulary

- **electromagnetic induction** : Process of generating electric current with a changing magnetic field.

Practice

Simulate electromagnetic induction at the following URL. Then answer the questions below. http://micro.magnet.fsu.edu/electromag/java/faraday2/

1. How is electric current created in the simulation? What type of current is it?
2. How is electric current measured in the simulation?
3. What happens when you stop moving the magnet?

Review

1. What is electromagnetic induction? When does it occur?
2. How could you increase the amount of current produced by electromagnetic induction?
3. Explain how a moving magnet and a coil of wire can be used to produce alternating current.
4. List two devices that use electromagnetic induction.

References

1. Christopher Auyeung. . CC-BY-NC-SA 3.0
2. Christopher Auyeung. . CC-BY-NC-SA 3.0

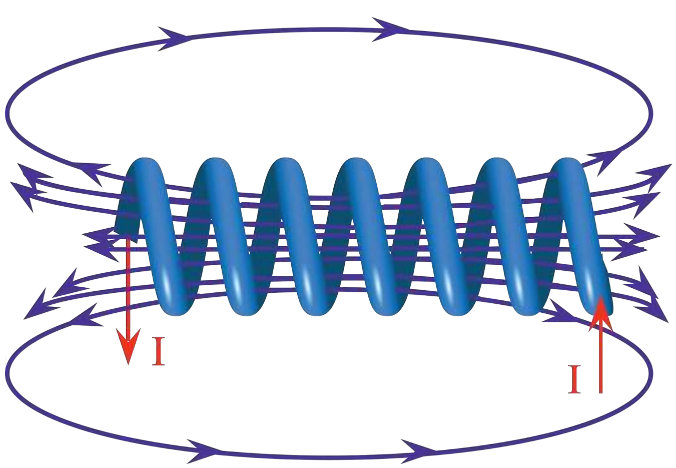
CONCEPT 2 Electromagnets

- Define and describe a solenoid.
- Define and describe an electromagnet.
- Determine the direction of the magnetic field inside a solenoid given the direction of current flow in the coil wire.
- Understand why an electromagnet has a stronger magnetic field than a solenoid.

One of the most famous electric car companies is Tesla, named after Nikola Tesla. These electric cars, and all others, require an electromagnet to run the engine.

Electromagnets

A long coil of wire consisting of many loops of wire and making a complete circuit is called a **solenoid** . The magnetic field within a solenoid can be quite large since it is the sum of the fields due to the current in each individual loop.



The magnetic field around the wire is determined by a hand rule. Since this description doesn't mention electron flow, we must assume that the current indicated by I is conventional current (positive). Therefore, we would use a right hand rule. We grasp a section of wire with our right hand pointing the thumb in the direction of the current flow and our fingers will curl around the wire in the direction of the magnetic field. Therefore, the field points down the cavity in these loops from right to left as shown in the sketch.

If a piece of iron is placed inside the coil of wire, the magnetic field is greatly increased because the domains of the iron are aligned by the magnetic field of the current. The resulting magnetic field is hundreds of time stronger than the field from the current alone. This arrangement is called an **electromagnet** . The picture below shows an electromagnet with an iron bar inside a coil.

Our knowledge of electromagnets developed from a series of observations. In 1820, Hans Oersted discovered that a current-carrying wire produced a magnetic field. Later in the same year, André-Marie Ampere discovered that a coil of wire acted like a permanent magnet and François Arago found that an iron bar could be magnetized by putting it inside coil of current-carrying wire. Finally, William Sturgeon found that leaving the iron bar inside the coil greatly increased the magnetic field.

Two major advantages of electromagnets are that they are extremely strong magnetic fields, and that the magnetic field can be turned on and off. When the current flows through the coil, it is a powerful magnet, but when the current is turned off, the magnetic field essentially disappears.

Electromagnets find use in many practical applications. Electromagnets are used to lift large masses of magnetic materials such as scrap iron, rolls of steel, and auto parts.

The overhead portion of this machine (painted yellow) is a lifting electromagnet. It is lowered to the deck where steel pipe is stored and it picks up a length of pipe and moves it to another machine where it is set upright and lowered into an oil well drill hole.

Electromagnets are essential to the design of the electric generator and electric motor and are also employed in doorbells, circuit breakers, television receivers, loudspeakers, electric dead bolts, car starters, clothes washers, atomic particle accelerators, and electromagnetic brakes and clutches. Electromagnets are commonly used as switches in electrical machines. A recent use for industrial electromagnets is to create **magnetic levitation** systems for bullet trains.

Summary

- A solenoid is a long coil of wire consisting of many loops of wire that makes a complete circuit.
- An electormagnet is a piece of iron inside a solenoid.
- While the magnetic field of a solenoid may be quite large, an electromagnet has a significantly larger magnetic field.
- Electromagnets' magnetic fields can be easily turned off by just halting the current.

Practice

MEDIA

Click image to the left for more content.

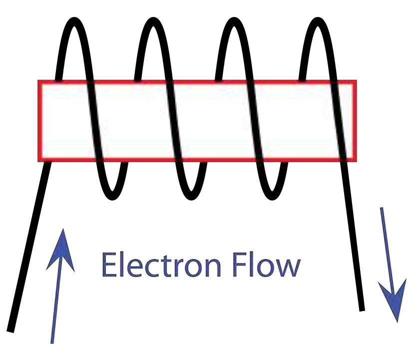
http://www.youtube.com/watch?v=S6oop6RXg9w

Follow up questions.

1. What components are needed to make a homemade electromagnet?
2. What objects were attracted by the electromagnet in the video?

Review

1. Magnetism is always present when electric charges ___________.
2. What happens to the strength of an electromagnet if the number of loops of wire is increased?
3. What happens to the strength of an electromagnet if the current in the wire is increased?
4. Which direction does the magnetic field point in the solenoid sketched here?



- **solenoid:** A current-carrying coil of wire that acts like a magnet when a current passes through it.
- **electromagnet:** A temporary magnet consisting of an iron or steel core wound with a coil of wire, through which a current is passed.
- **magnetic levitation:** The suspension of an object above a second object by means of magnetic repulsion.

References

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CONCEPT 3 Current and Magnetism

Students will learn to analyze and solve problems involving current carrying wires in magnetic fields.

Students will learn to analyze and solve problems involving current carrying wires in magnetic fields.

Key Equations

$F_{\text{wire}} = LIB\sin(\theta)$ Force on a Current Carrying Wire

In this equation, L refers to the length of the wire, I to the electric current, B the magnitude of the magnetic field and θ is the angle between the direction of the current and the direction of the magnetic field.

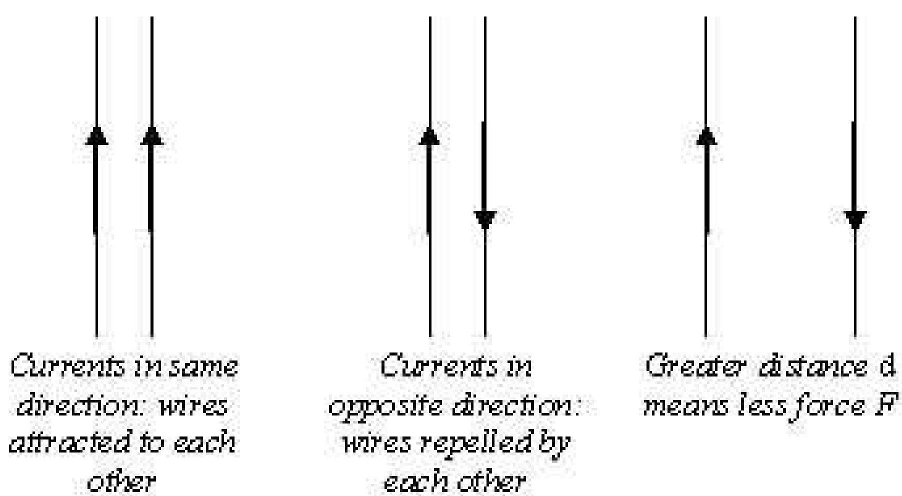
$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$ Magnetic field at a distance r from a current-carrying wire

Where $\mu_0 = 4\pi \times 10^{-7}$ Tm/A Permeability of Vacuum (approximately same for air also)

Force on a Wire

Since a wire is nothing but a collection of moving charges, the force it will experience in a magnetic field will simply be the vector sum of the forces on the individual charges. If the wire is straight — that is, all the charges are moving in the same direction — these forces will all point in the same direction, and so will their sum. Then, the direction of the force can be found using the second right hand rule, while its magnitude will depend on the length of the wire (denoted L), the strength of the current, the strength of the field, and the angle between their directions:

Two current-carrying wires next to each other each generate magnetic fields and therefore exert forces on each other:



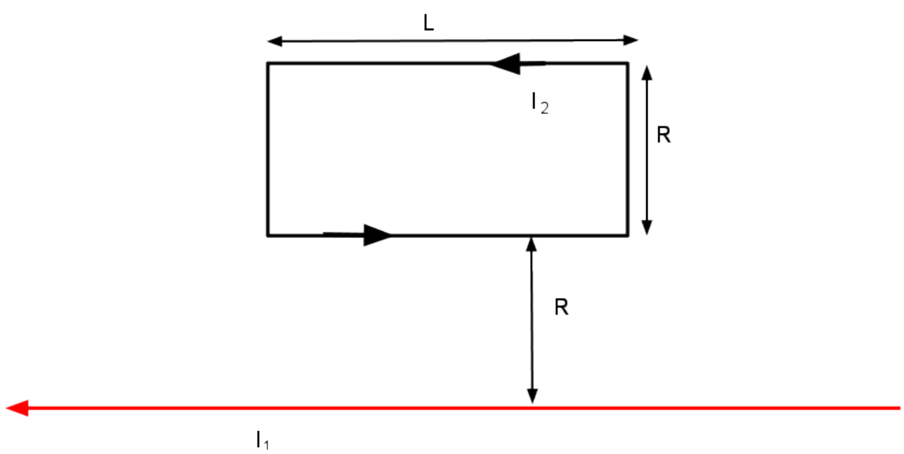
Example 1

MEDIA

Click image to the left for more content.

Example 2

A wire loop and an infinitely long current carrying cable are placed a distance r apart. The infinitely long wire is carrying a current I_1 to the left and the loop is carrying a current I_2 CCW. The dimensions of the wire loop are shown in the diagram illustrating the situation below. What is the magnitude and direction of the net force on the loop (the mass of the wires are negligible)?



Solution

In this problem, it is best to start by determining the direction of the force on each segment of the loop. Based on the first right hand rule, the magnetic field from the infinite cable points into the page where the loop is. This means that the force on the top segment of the loop will be down toward the bottom of the page, the force on the left segment will be to right, the force on the bottom segment will be toward the top of the page, and the force on the right segment will be to the left. The forces on the left and right segments will balance out because both segments are the same distance from the cable. The forces from the top and bottom section will not balance out because the wires are different distances from the cable. The force on the bottom segment will be stronger than the one on the top segment because the magnetic field is stronger closer to the cable, so the net force on the loop will be up, toward the top of the page.

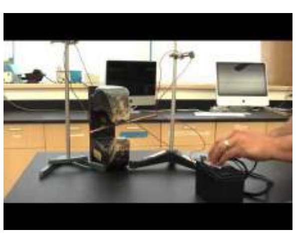
Now we will begin to calculate the force's magnitude by first determining the strength of the magnetic field at the bottom and top segments. All we really have to do is plug in the distances to each segment into the equation we already know for the magnetic field due to a current carrying wire.

$$B = \frac{\mu_o I}{2\pi r}$$
$$B_{bottom} = \frac{\mu_o I_1}{2\pi R}$$
$$B_{top} = \frac{\mu_o I_1}{2\pi 2R}$$

Now we will calculate the net force on the loop using the equation given above. We'll consider up the positive direction.

$\Sigma F = F_{bottom} - F_{top}$	start by summing the forces on the loop
$\Sigma F = I_2 L B_{bottom} - I_2 L B_{top}$	substitute in the values for each of the force terms
$\Sigma F = I_2 L (B_{bottom} - B_{top})$	factor the equation
$\Sigma F = I_2 L (\frac{\mu_o I_1}{2\pi R} - \frac{\mu_o I_1}{2\pi 2R})$	substitute in the values for the magnetic field
$\Sigma F = \frac{\mu_o I_1 I_2 L}{2\pi R}(1 - \frac{1}{2})$	factor the equation again
$\Sigma F = \frac{\mu_o I_1 I_2 L}{4\pi R}$	simplify to get the answer

Watch this Explanation

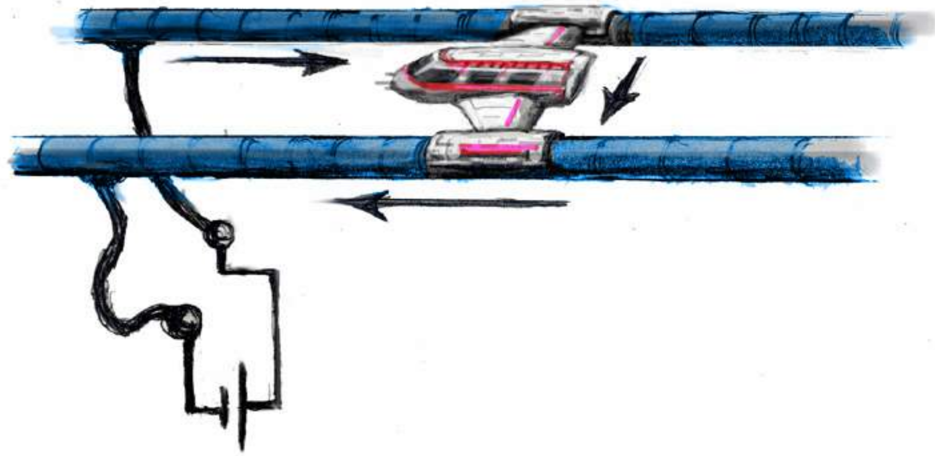


MEDIA

Click image to the left for more content.

Time for Practice

1. A vertical wire, with a current of 6.0 A going towards the ground, is immersed in a magnetic field of 5.0 T pointing to the right. What is the value and direction of the force on the wire? The length of the wire is 2.0 m .



2.

3. A futuristic magneto-car uses the interaction between current flowing across the magneto car and magnetic fields to propel itself forward. The device consists of two fixed metal tracks and a freely moving metal car (see illustration above). A magnetic field is pointing downward with respect to the car, and has the strength of 5.00 T . The car is 4.70 m wide and has 800 A of current flowing through it. The arrows indicate the direction of the current flow.
   a. Find the direction and magnitude of the force on the car.
   b. If the car has a mass of 2050 kg , what is its velocity after 10 s , assuming it starts at rest?
   c. If you want double the force for the same magnetic field, how should the current change?
4. A horizontal wire carries a current of 48 A towards the east. A second wire with mass 0.05 kg runs parallel to the first, but lies 15 cm below it. This second wire is held in suspension by the magnetic field of the first wire above it. If each wire has a length of half a meter, what is the magnitude and direction of the current in the lower wire?
5. Show that the formula for the force between two current carrying wires is $F = \frac{\mu_o L i_1 i_2}{2\pi d}$, where d is the distance between the two wires, i_1 is the current of first wire and L is the segment of length of the second wire carrying a current i_2 . (Hint: find magnetic field emanating from first wire and then use the formula for a wire immersed in that magnetic field in order to find the force on the second wire.)
6. Two long thin wires are on the same plane but perpendicular to each other. The wire on the $y-$ axis carries a current of 6.0 A in the $-y$ direction. The wire on the $x-$ axis carries a current of 2.0 A in the $+x$ direction. Point, P has the co-ordinates of $(2.0, 2, 0)$ in meters. A charged particle moves in a direction of 45^o away from the origin at point, P , with a velocity of 1.0×10^7 m/s.
   a. Find the magnitude and direction of the magnetic field at point, P .
   b. If there is a magnetic force of 1.0×10^{-6} N on the particle determine its charge.
   c. Determine the magnitude of an electric field that will cancel the magnetic force on the particle.
7. A long straight wire is on the $x-$ axis and has a current of 12 A in the $-x$ direction. A point P , is located 2.0 m above the wire on the $y-$ axis.
   a. What is the magnitude and direction of the magnetic field at P .
   b. If an electron moves through P in the $-x$ direction at a speed of 8.0×10^7 m/s what is the magnitude and direction of the force on the electron?

Answers to Selected Problems

1. Down the page; 60 N
2. a. To the right, 1.88×10^4 N b. 91.7 m/s c. It should be doubled
3. East 1.5×10^4 A
4. .
5. a. 8×10^{-7} T b. 1.3×10^{-6} C

6. a. 1.2×10^{-6} T, $+z$ b. 1.5×10^{-17} N, $-y$

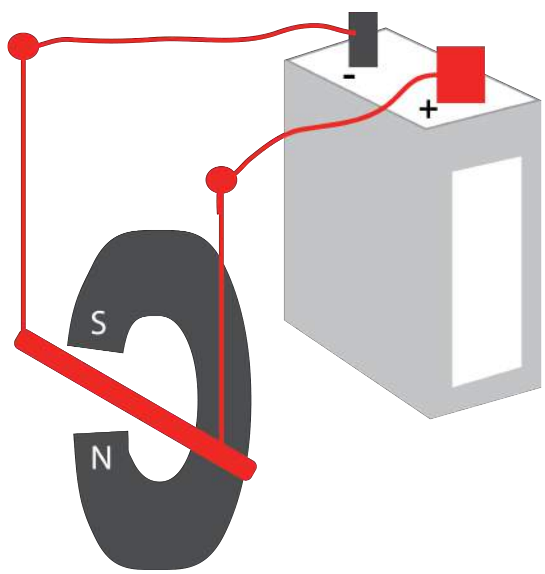
CONCEPT 4 Electric Motors

- Explain the design and operation of an electric motor.

As gas prices continue to rise, electric cars and hybrids are becoming increasingly popular. These cars are certainly a part of our future. On the left in the image above is an all-electric vehicle, and on the right is a hybrid vehicle that uses gas part time and electricity part time.

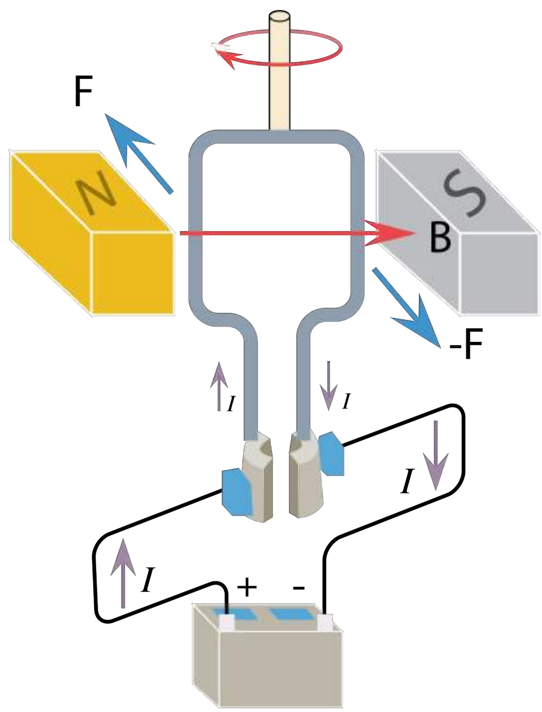
Electric Motors

In an earlier concept, we described and calculated the force that a magnetic field exerts on a current carrying wire. Since you are familiar with Newton's third law of motion, you know that if the magnetic field exerts a force on the current carrying wire, then the current carrying wire also exerts a force of equal magnitude and opposite direction on the magnetic field.



In the sketch above, a circuit is connected to a battery, with one part of the circuit placed inside a magnetic field. When current runs through the circuit, a force will be exerted on the wire by the magnetic field, causing the wire to move. If we choose to consider electron current in this case, the electrons flow from the back of the sketch to the front while the magnetic field is directed upward. Using the left hand rule for this, we find that the force on the wire is to the right of the page. Had we chosen to consider the current to be conventional current, then the current would be flowing from the front of the sketch to the back and we would use the right hand rule. The force on the wire would, once again, be toward the right. This movement is harnessed in **electrical motors** .

Electrical motors change electrical energy into mechanical energy. The motor consists of an electrical circuit with part of the wires inside a magnetic field. This can be seen below. Positive charges move through the circuit in the direction of the light purple arrows. When the charges move up through the part of the coil that is right next to the north pole, the right hand rule tells us that the wire suffers the force, F , pushing the wire in the direction of the blue arrow, toward the back of the sketch. On the other side of the coil, where the charges are moving down through the field, the right hand rule shows the force would push this side of the coil toward the front. These two forces are working together, rotating the coil in the direction of the circular red arrow.



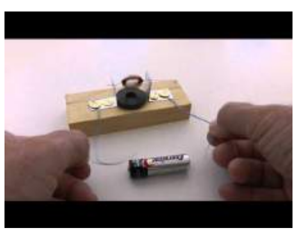
Where the rotating coil (in grey) meets the wires attached to the power source (black), we find a split ring **commutator.** The coil turns, but the commutator and power source do not. As the coil turns, it moves off of the blue box connector and as it continues to turn, it connects to the other blue box connector. As the coil turns, it reverses its connections to the external circuit. Therefore, inside the coil, the current is always flowing in the same direction because the left side of the coil is always attached to the left side of the external circuit. This allows the coil, or **armature,** to continue to spin the same direction all the time.

In electrical motors, these coils often consist of not just one, but many wires, as can be seen here:

Summary

- When current runs through the circuit, a force will be exerted on the wire.
- An electrical motor changes electrical energy into mechanical energy.
- A split ring commutator keeps the current in the coil flowing in the same direction even though the coil changes sides every half turn.

Practice



MEDIA
Click image to the left for more content.

http://www.youtube.com/watch?v=elFUJNodXps

In the video, a simple electrical motor is constructed.

Follow up questions.

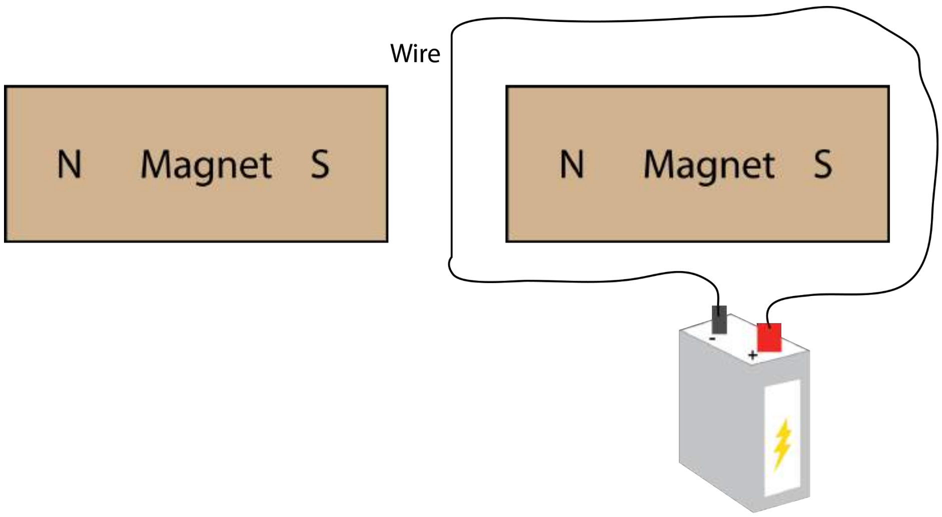
1. Who first built an electric motor?
2. What size battery was used in the video motor?
3. The rover Curiosity is on the surface of what body?

Review

1. Which way will the wire be pushed when current passes through the wire?

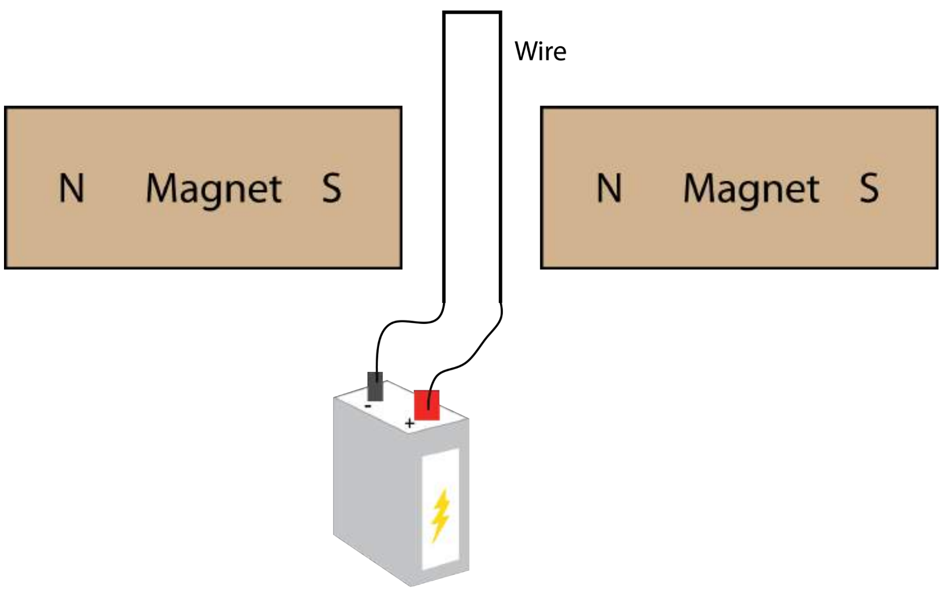
a. up
b. down

c. left
d. right
e. None of these.



2. Which way will the coil spin when current passes through the wire?

a. clockwise
b. counterclockwise



- **electric motor:** An electricmotor converts electricity into mechanical motion.
- **commutator:** A split - ring commutator (sometimes just called a commutator) is a simple and clever device for reversing the current direction through an armature every half turn.
- **armature:** A revolving structure in an electric motor or generator, wound with the coils that carry the current.

References

1. Myrtle Beach TheDigitel and User:Mariordo/Wikimedia Commons. http://commons.wikimedia.org/wiki/File:Nissan_Leaf_%26_Chevy_Volt_charging_trimmed.jpg . CC-BY 2.0

2. . . CC BY-NC-SA
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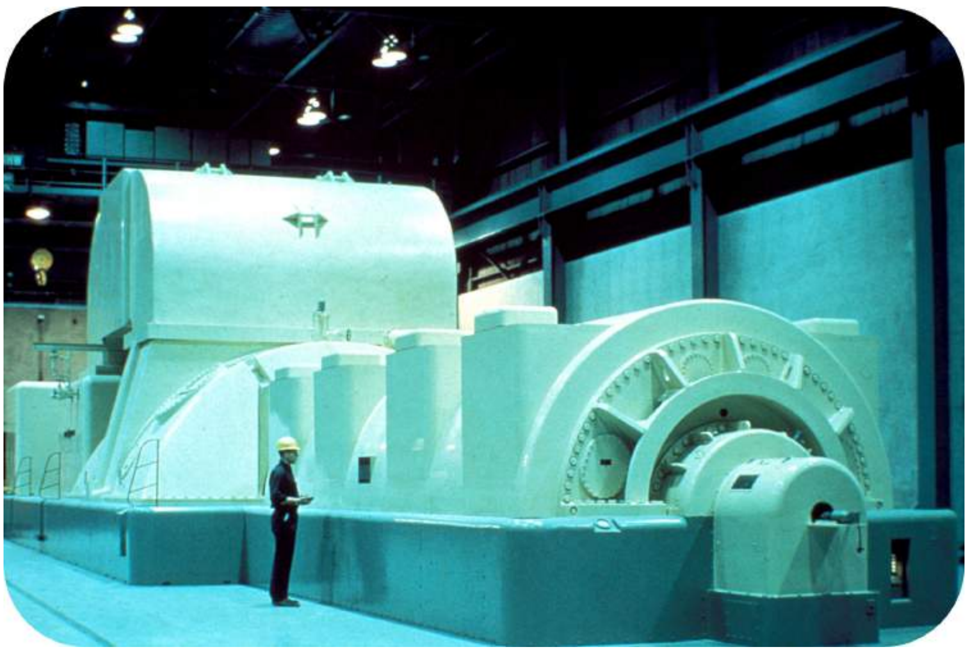
CONCEPT 5 Electromotive Force

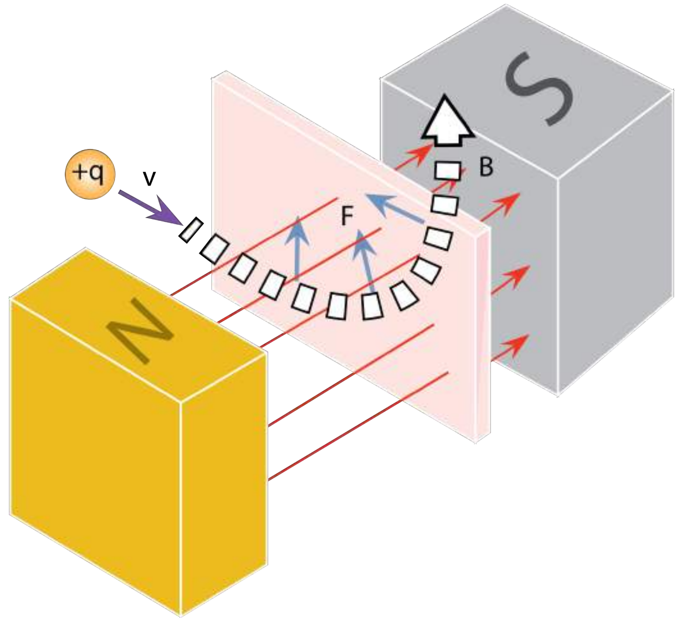
- Define EMF.
- Calculate EMF for a wire moving in a magnetic field.



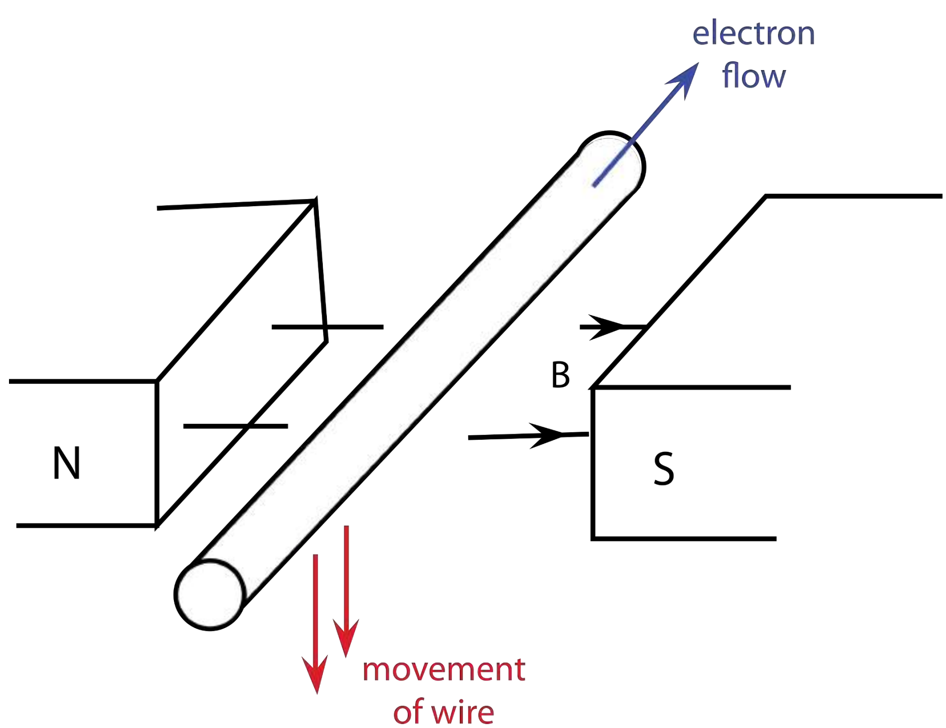
Electrical generators convert mechanical energy into electrical energy. Every electrical generator needs some method for spinning the coil inside the magnetic field. Hydroelectric generators use water pressure to spin the coil while windmills, of course, use the wind to spin the coil. The image here is a combination of steam turbine and generator. The steam can be produced by burning coal or diesel fuel or by a nuclear reaction and the steam then turns the coil and generates electricity.

Electromotive Force

When an individual charge flies through a magnetic field, a force is exerted on the charge and the path of the charge bends. In the case shown in the sketch below, the charge is positive and the right hand rule shows us the force will be upward, perpendicular to both the field and the path of the charge.



If a wire that is part of a complete circuit is moved through a magnetic field, the force on the individual electrons in the wire occurs in exactly the same manner. Since the electrons in the wire are negatively charged, the force would be in the opposite direction but otherwise the situation is the same. When the wire is pulled downward through the magnetic field, the force on the electrons cause them to move within the wire. Since the charges are negative, the left hand rule shows that the electrons would move as diagrammed in the sketch. (Point fingers in the direction of the magnetic field, point thumb in the direction of wire movement, and palm shows direction of electron flow.) No current will flow, of course, unless the section of wire is part of a complete circuit.



This process allows us to convert mechanical energy (the motion of the wire) into electrical energy (the current). This is the opposite of what happens in an electric motor where electrical energy is converted into mechanical energy.

In order to maintain a constant current flow, it is necessary to have a potential difference or voltage in the circuit. The voltage or potential difference is also frequently referred to **electromotive force** . The term electromotive force, like many historical terms, is a misnomer. Electromotive force is NOT a force, it is a potential difference or potential energy per unit charge and is measured in volts. The potential difference in the case of moving a wire through a magnetic field is produced by the work done on the charges by whatever is pushing the wire through the field.

The EMF (or voltage) depends on the magnetic field strength, B , the length of the wire in the magnetic field, l , and the velocity of the wire in the field.

$$EMF = Blv$$

This calculation is based on the wire moving perpendicularly through the field. If the wire moves an angle to the field, then only the component of the wire perpendicular to the field will generate EMF .

Example Problem: A 0.20 m piece of wire that is part of a complete circuit moves perpendicularly through a magnetic field whose magnetic induction is 0.0800 T. If the speed of the wire is 7.0 m/s, what EMF is **induced** in the wire?

Solution:
$EMF = Blv = (0.0800\ N/A \cdot m)(0.20\ m)(7.0\ m/s) = 0.11\ N \cdot m/C = 0.11\ J/C = 0.11\ V$

Summary

- If a wire that is part of a complete circuit is moved through a magnetic field, the magnetic field exerts a force on the individual electrons in the wire, which causes a current to flow.
- The potential difference in the case of moving a wire through a magnetic field is produced by the work done on the charges by whatever is pushing the wire through the field.
- The EMF (or voltage) depends on the magnetic field strength, B , the length of the wire in the magnetic field, l , and the velocity of the wire in the field, $EMF = Blv$.

Practice

MEDIA
Click image to the left for more content.

http://www.youtube.com/watch?v=0OHmMVBLXTI

Follow up questions.

1. We have been discussing the process of generating electricity by moving a wire through a magnetic field. What happens if the wire is held steady and the magnetic field moves instead?
2. When a loop of wire is turned circularly in a magnetic field, what type of current is produced?

Review

1. Which of the following units are equivalent to those of EMF produced in a generator?
   (a) $T \cdot m/s$
   (b) $V \cdot m^2/s$

   (c) J/s
   (d) $A \cdot \Omega$
   (e) $T \cdot m$

2. A straight wire 0.500 m long is moved straight up through a 0.400 T magnetic field pointed in the horizontal direction. The speed of the wire is 20.0 m/s.
   (a) What EMF is induced in the wire?
   (b) If the wire is part of a circuit with a total resistance of 6.00 Ω , what is the current in the circuit?
3. A straight wire, 25.0 m long, is mounted on an airplane flying at 125 m/s. The wire moves perpendicularly through earth's magnetic field $(B = 5.00 \times 10^{-5}\ T)$. What is the EMF induced in the wire?
4. A straight wire, 30.0 m long, moves at 2.00 m/s perpendicularly through a 1.00 T magnetic field.
   (a) What is the induced EMF ?
   (b) If the total resistance of the circuit is 15.0 Ω , what is the current in the circuit?

- **electromotive force:** The potential energy per unit charge that produces a flow of electricity in a circuit, expressed in volts.
- **induced current:** The electric current generated in a loop of conducting material by movement of the loop across a magnetic field.

References

1. Courtesy of the NRC. http://commons.wikimedia.org/wiki/File:Modern_Steam_Turbine_Generator.jpg . Public Domain
2. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0
3. CK-12 Foundation - Samantha Bacic. . CC-BY-NC-SA 3.0

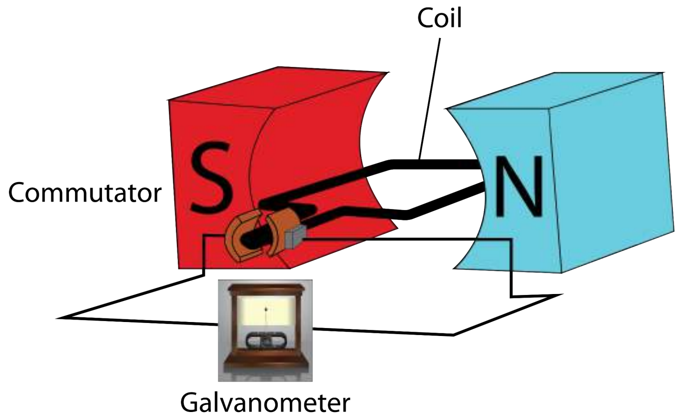
CONCEPT 6 Electric Generators

- Explain how an electric generator works.
- Explain the difference between an electric generator and an electric motor.
- Explain the difference between peak and effective voltage and current from a generator.

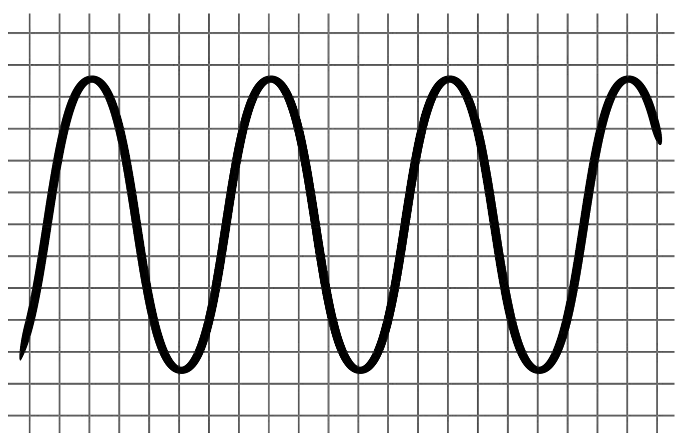
These large machines are electric generators. This particular row of generators is installed in a hydroelectric power station. The insides of these generators are coils of wire spinning in a magnetic field. The relative motion between the wire and the magnetic field is what generates electric current. In all generators, some mechanical energy is used to spin the coil of wire in the generator. In the case of hydroelectric power, the coil of wire is spun by water falling from higher PE to lower PE . Windmills and steam turbines are used in other types of power generators to spin the coil.

Electric Generators

Electric generators convert mechanical energy to electric energy. The generator consists of some number of wire loops wrapped around an iron core and placed in a strong magnetic field. The loops of wire and the iron core are called the **armature** . The armature is mounted so that it can rotate freely inside the magnetic field. Mechanical energy is used to spin the armature in the field so that the wire loops cut across the field and produce electric current. The EMF of this current is calculated by $EMF = Blv$.



Consider the coil and magnetic field sketched above. When the right hand side of the coil moves up through the field, the left hand rule indicates that the electron flow will be from the front to the back in that side of the coil. The current generated will have the greatest EMF as the wire is cutting perpendicularly across the field. When the wire reaches the top of its arc, it is moving parallel to the field and therefore, not cutting across the field at all. The EMF at this point will be zero. As that same wire then cuts down through the field as it continues to spin, the left hand rule indicates that the electron flow will be from the back to the front in that side of the coil. In this second half of the arc, the direction of the electron flow has reversed. The magnitude of the EMF will reach maximum again as the wire cuts perpendicularly down through the field and the EMF will become zero again as the wire passes through the bottom of the arc. The current produced as the armature goes around will resemble a sine wave where the EMF reaches a maximum in one direction, then goes to zero, then goes to a maximum in the other direction. This type of current is called **alternating current** . By having more and more loops of wire on the armature, the crests and troughs overlap and fill in until a constant current is produced.



A **direct current** is one that always flows in the same direction rather than alternating back and forth. Batteries produce direct currents. A generator can also produce direct current by using a split ring commutator that changes external connections every half turn of the armature so that even though the current in the coil changes direction, every time the current in the coil changes direction, the external connection switches so that the external current always goes in the same direction.

Generators and motors are almost identical in construction but convert energy in opposite directions. Generators convert mechanical energy to electrical energy and motors convert electrical energy to mechanical.

Because of the alternating direction in alternating current, the average value is less than the power supplied by a direct current. In fact, the average power of an AC current is one-half its maximum power and one-half the power of an equivalent DC current. The effective current of an AC generator is 0.707 times its maximum current. The same is true for the effective voltage of an AC generator.

$I_{\text{eff}} = 0.707\ I_{\text{max}}$

$V_{\text{eff}} = 0.707\ V_{\text{max}}$

Example Problem: An AC generator develops a maximum voltage of 34.0 V and delivers a maximum current of 0.170 A.

a. What is the effective voltage of the generator?
b. What is the effective current delivered by the generator?
c. What is the resistance in the circuit?

Solution:

a. $V_{\text{eff}} = 0.707\ \ V_{\text{max}} = (0.707)(34.0\ V) = 24.0\ V$
b. $I_{\text{eff}} = 0.707\ \ I_{\text{max}} = (0.707)(0.17\ A) = 0.120\ A$
c. $R = \frac{V}{I} = \frac{24.0\ V}{0.120\ A} = 200.\ \Omega$

Summary

- Electric generators convert mechanical energy to electric energy.
- The generator consists of some number of wire loops wrapped around an iron core and placed in a strong magnetic field.
- The loops of wire and the iron core are called the armature.
- The armature is mounted so that it can rotate freely inside the magnetic field.
- Mechanical energy is used to spin the armature in the field so that the wire loops cut across the field and produce electric current.
- The current produced as the armature goes around will resemble a sine wave where the EMF reaches a maximum in one direction, then goes to zero, then goes to a maximum in the other direction. This type of current is called alternating current.
- A generator can also produce direct current by using a split ring commutator that changes external connections every half turn of the armature so that even though the current in the coil changes direction, every time the current in the coil changes direction, the external connection switches so that the external current always goes in the same direction.
- The effective current of an AC generator is 0.707 times its maximum current.
- The effective voltage of an AC generator is 0.707 times its maximum voltage.

Practice

MEDIA
Click image to the left for more content.

http://www.youtube.com/watch?v=RFOMpOM1WHQ

Follow up questions.

1. Which of the two generators in the video (an AC generator and a DC generator) involves a magnetic field?
2. Which of the two generators in the video involves a wire-wrapped armature?
3. What is the difference between the DC generator and the AC generator?

Review

1. What three things are necessary to produce EMF mechanically?
   (a) magnet, force lines, and magnetic field
   (b) EMF , conductor, and magnetic field
   (c) conducting wire, magnetic field, and relative motion
   (d) conducting wire, electrical field, and relative motion
   (e) none of these will produce EMF mechanically
2. Increasing which of the following will increase the output of a generator?
   (a) EMF
   (b) strength of the magnetic field
   (c) resistance of the conductor
   (d) load on the meter
   (e) none of these
3. The current in the rotating coil of all generators is
   (a) AC
   (b) DC
   (c) pulsating AC
   (d) pulsating DC
4. A generator in a power plant develops a maximum voltage of 170. V.
   (a) What is the effective voltage?
   (b) A 60.0 W light bulb is placed across the generator. A maximum current of 0.70 A flows through the bulb. What effective current flows through the bulb?
   (c) What is the resistance of the light bulb when it is working?
5. The effective voltage of a particular AC household outlet is 117 V.
   (a) What is the maximum voltage across a lamp connected to the outlet?
   (b) The effective current through the lamp is 5.50 A. What is the maximum current in the lamp?

- **direct current:** An electric current flowing in one direction only.
- **alternating current:** An electric current that reverses direction in a circuit at regular intervals.
- **electric generator:** An electric generator is a device that converts mechanical energy to electrical energy.
- **armature:** The rotating part of a generator, consisting essentially of copper wire wound around an iron core.

References

1. Image copyright James L. Davidson, 2013. http://www.shutterstock.com . Public Domain
2. Galvanometer: Image copyright scropy, 2013; composite created by CK-12 Foundation. http://www.shutterstock.com . Used under license from Shutterstock.com
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Concept 7 Transformers

- Describe the function of a transformer.
- Explain the relationship between turns and voltage ratio.
- Solve mathematical problems involving transformers.

Power loss in long transmission lines is related to the magnitude of the current. Specifically, the power loss can be decreased by decreasing the magnitude of the current.

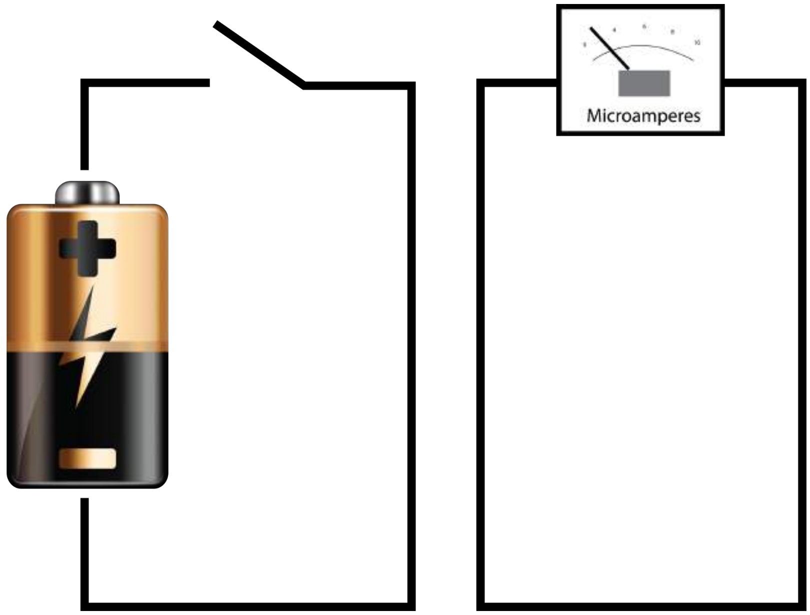
The amount of power passed through transmission lines can be calculated by multiplying voltage by current. The same power can be transmitted using a very high voltage and a very low current as with a low voltage and high current.

Since power companies do not wish to waste power as it is transmitted to homes and businesses, they deliberately 'step up' the voltage and reduce the current before transmitting the power over extended distances. That type of power transmits well without great loss of energy but it cannot be used in household appliances. It becomes necessary to convert it back ('step down') to low voltage and high current for household use. That is the job of electrical transformers – those big gray barrels you see on power poles.

Transformers

When we move a wire through a magnetic field, a force is exerted on the charges in the wire and a current is induced. Essentially the same thing happens if we hold the wire steady and move the magnetic field by moving the magnet. Yet a third way of causing relative motion between the charges in a wire and a magnetic field is to expand or contract the field through the wire.

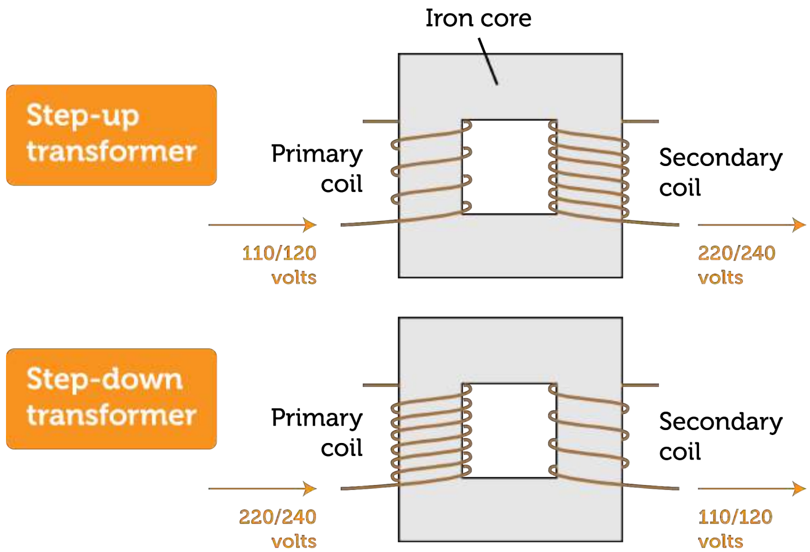
When a current begins to flow in a wire, a circular magnetic field forms around the wire. Within the first fractional second when the current begins to flow, the magnetic field expands outward from the wire. If a second wire is placed nearby, the expanding field will pass through the second wire and induce a brief current in the wire.



Consider the sketch above. When the knife switch is closed, current begins to flow in the first circuit and therefore, a magnetic field expands outward around the wire. When the magnetic field expands outward from the wire on the right side, it will pass through the wire in the second circuit. This relative motion between wire and field induces a current in the second circuit. The magnetic field expands outward for only a very short period of time and therefore, only a short jolt of current is induced in the second circuit. You can leave the knife switch closed and the current will continue to flow in the first circuit but no current is induced in the second circuit because the field is constant and therefore there is no relative motion between the field and the wire in the second circuit. When the knife switch is opened, the current in the first circuit ceases to flow and the magnetic field collapses back through the wire to zero. As the magnetic field collapses, it passes through the wire and once again we have relative motion between the wire in the second circuit and the magnetic field. Therefore, we once again have a short jolt of current induced in the second circuit. This second jolt of induced current will be flowing in the opposite direction of the first induced current. We can produce an alternating current in the second circuit simply by closing and opening the knife switch continuously in the first circuit.

Obviously, a transformer would have little use in the case of DC current because current is only induced in the second circuit when the first circuit is started or stopped. With AC current, however, since the current changes direction 60 times per second, the magnetic field would constantly be expanding and contracting through the second wire.

A **transformer** is a device used to increase or decrease alternating current voltages. They do this with essentially no loss of energy. A transformer has two coils, electrically insulated from each other as shown in the sketch. One coil is called the **primary coil** and the other is called the **secondary coil** . When the primary coil is connected to a source of AC voltage, the changing current creates a varying magnetic field. The varying magnetic field induces a varying EMF in the secondary coil. The EMF induced in the secondary coil is called the secondary voltage and is proportional to the primary voltage. The secondary voltage also depends on the ratio of turns on the secondary coil to turns on the primary coil.



$$\frac{\text{secondary voltage}}{\text{primary voltage}} = \frac{\text{number of turns on secondary}}{\text{number of turns on primary}}$$
$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

If the secondary voltage is larger than the primary voltage, the transformer is called a **step-up transformer** . If the voltage out of the transformer is smaller than the voltage in, then the transformer is called a **step-down transformer** .

In an ideal transformer, the electric power put into the primary equals the electric power delivered by the secondary.

$$V_P I_P = V_S I_S$$

The current that flows in the primary depends on how much current is required by the secondary circuit.

$$\frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S}$$

Example Problem: A particular step-up transformer has 200 turns on the primary coil and 3000 turns on the secondary coil.

a. If the voltage on the primary coil is 90.0 V, what is the voltage on the secondary coil?
b. If the current in the secondary circuit is 2.00 A, what is the current in the primary coil?
c. What is the power in the primary circuit?
d. What is the power in the secondary circuit?

Solution:

a. $\frac{V_P}{V_S} = \frac{N_P}{N_S}$ $\quad \frac{90.0\ V}{V_S} = \frac{200}{3000}$ $\quad V_S = \frac{(90.0\ V)(3000)}{(200)} = 1350\ V$
b. $\frac{I_S}{I_P} = \frac{N_P}{N_S}$ $\quad \frac{2.00\ A}{I_P} = \frac{200}{3000}$ $\quad I_P = 30.0\ A$
c. $P_P = V_P I_P = (90.0\ V)(30.0\ A) = 2700\ W$
d. $P_S = V_S I_S = (1350\ V)(2.00\ A) = 2700\ W$

Summary

- When a current begins to flow in a wire, a circular magnetic field forms around the wire.
- Within the first fractional second when the current begins to flow, the magnetic field expands outward from the wire.
- If a second wire is placed nearby, the expanding field will pass through the second wire and induce a brief current in the wire.
- A transformer is a device used to increase or decrease alternating current voltages.
- A transformer has two coils, electrically insulated from each other. One coil is called the primary coil and the other is called the secondary coil.
- The varying magnetic field induces a varying EMF in the secondary coil.
- The EMF induced in the secondary coil is called the secondary voltage and is proportional to the primary voltage. The secondary voltage also depends on the ratio of turns on the secondary coil to turns on the primary coil.

$$\frac{\text{secondary voltage}}{\text{primary voltage}} = \frac{\text{number of turns on secondary}}{\text{number of turns on primary}}$$

Practice

MEDIA

Click image to the left for more content.

http://www.youtube.com/watch?v=-v8MYAFl7Mw

Follow up questions.

1. What type of transformer is used at the power station where the electric power is generated?
2. What type of transformer is used at power sub-stations?
3. What type of transformer is used inside cell phone chargers?

Review

1. A step-down transformer has 7500 turns on its primary and 125 turns on its secondary. The voltage across the primary is 7200 V.
   (a) What is the voltage across the secondary?
   (b) The current in the secondary is 36 A. What current flows in the primary?
2. The secondary of a step-down transformer has 500 turns. The primary has 15,000 turns.
   (a) The EMF of the primary is 3600 V. What is the EMF of the secondary?
   (b) The current in the primary is 3.0 A. What is the current in the secondary?
3. An ideal step-up transformer's primary coil has 500 turns and its secondary coil has 15,000 turns. The primary EMF is 120 V.
   (a) Calculate the EMF of the secondary.
   (b) If the secondary current is 3.0 A, what is the primary current?
   (c) What power is drawn by the primary?

- **transformer:** An electric device consisting essentially of two or more windings wound on the same core, which by electromagnetic induction transforms electric energy from one circuit to another circuit such that the frequency of the energy remains unchanged while the voltage and current usually change.
- **primary coil:** A coil to which the input voltage is applied in a transformer.
- **secondary coil:** The coil in a transformer where the current is induced.
- **step-up transformer:** A step-up transformer is one that increases voltage.
- **step-down transformer:** A step-down transformer is one that decreases voltage.

References

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